

# FALTINGS' THEOREM

College Seminar  
Summer 2015  
Wednesdays 13.15-15.00 in 1.023

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The main goal of the semester is to understand some aspects of Faltings' proofs of some far-reaching finiteness theorems about abelian varieties over number fields, the highlight being the Tate conjecture, the Shafarevich conjecture, and the Mordell conjecture. There are a variety of references, including:

- (A) G. Faltings. *Endlichkeitssätze für abelsche Varietäten über Zahlkörpern*. We won't actually be using this.
- (B) L. Szpiro (editor). *Seminaire sur les pinceaux arithmétiques: la conjecture de Mordell*. Treats everything in detail.
- (C) G. Faltings and G. Wüstholz (editors). *Rational points*. Lots of detail on the representation theory side of things.

Deligne and Szpiro also have Bourbaki exposés overviewing the proof. For general background on abelian varieties, see

- (D) D. Mumford. *Abelian varieties*.
- (E) J. S. Milne. *Abelian varieties*, available at [www.jmilne.org/math/](http://www.jmilne.org/math/).

Milne also treats the finiteness theorems. For the most part we'll follow C, though we'll invert the order slightly.

## OUTLINE

- 15.4.2015 **Overview.**
- 22.4.2015 **Abelian varieties over arbitrary fields.** Isogenies. Tate module. (D).
- 29.4.2015 **Tate module.** More on the Tate module. Weil pairing. Zarhin's trick. (D, E).
- 6.5.2015 **Tate and Shafarevich conjectures from finiteness.** (C §IV.2 or E §IV.2,3).
- 13.5.2015 **Group schemes.** Néron models. Semistable reduction. (C §IV.3 also E §I.17).
- 20.5.2015 **Heights.** Isogeny formula. (C §III,IV.3).
- 27.5.2015 **Ramification of  $p$ -divisible groups.** (C §III).
- 3.6.2015 **Proof of the Tate conjecture.** (C § IV.3).
- 10.6.2015 **Ramification of finite group schemes.** Raynaud's theorem. (C § III.4).
- 17.6.2015 **Finiteness of isogeny classes.** (C §V).
- 24.6.2015 **Siegel moduli space: overview.** (C §I or any other reference).
- 1.7.2015 **Faltings height.** Hermitian line bundles and heights. (C §II or E § IV.6).
- 8.7.2015 **Height comparison.** (C §II).
- 15.7.2015 **Mordell conjecture.** Parshin's argument. Utah? (C §V.4 or E §IV.5).