

ABELIAN VARIETIES AND FOURIER–MUKAI TRANSFORMS

College Seminar
Winter 2014
Wednesdays 13.15-15.00 in 1.023

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During the semester we'll develop the theories of abelian varieties and derived categories as a geometric invariant in parallel; about halfway through, the two theories will converge, and we'll see that many of the classical results on abelian varieties are more naturally expressed and proved using the derived category. This will culminate in a classification due to Orlov and Polishchuk of those abelian varieties that have isomorphic derived categories, as well as a complete understanding of the group of derived autoequivalences.

The main references we'll be using are:

- A. Polishchuk. *Abelian varieties, Theta functions, and the Fourier–Mukai transform.*
- D. Mumford. *Abelian varieties.*
- D. Huybrechts. *Fourier–Mukai transforms in algebraic geometry.*

The following may also prove of use:

- J. S. Milne. *Abelian varieties*, available at www.jmilne.org/math/.
- S. Gelfand and Y. Manin. *Methods of homological algebra.*

There is a seminar at Bonn this semester on the derived category (though not focusing on abelian varieties) that will be covering much of what we're doing, and might also prove a useful reference:

- www.math.uni-bonn.de/people/lombardi/fourier-mukai.html.

OUTLINE

- 15.10.2014 **Overview.**
- 22.10.2014 **Triangulated categories.** Generalities. Abelian categories. Homotopy category of complexes. (Huybrechts Chapters 1,2; Gelfand–Manin Chapter III,IV).
- 29.10.2014 **Complex tori.** Cohomology. Riemann forms. (Mumford Chapter 1; Milne §2).
- 5.11.2014 **Derived categories.** Derived categories of abelian categories. (Huybrechts Chapter 2; Gelfand–Manin Chapter III).
- 12.11.2014 **Abelian varieties.** Definition. Rigidity. Theorem of the cube. (Polishchuk Chapter 8; Mumford Chapter II,III; Milne §1,4).
- 19.11.2014 **Derived categories of varieties.** Serre functors. Reconstruction theorem. (Huybrechts Chapter 3,4).
- 26.11.2014 **Line bundles on abelian varieties.** Projectivity. Line bundles I. Isogenies. (Polishchuk Chapter 8; Mumford Chapter II; Milne §5,6)
- 3.12.2014 **Exact functors.** Derived functors. Fourier–Mukai transforms I. (Huybrechts Chapter 4).
- 10.12.2014 **The dual abelian variety.** Construction of the dual. (Polishchuk Chapter 9; Milne §7).
- 17.12.2014 **Mukai's theorem.** The Poincaré bundle. Line bundles II. Homogeneous bundles. (Polishchuk Chapter 11).
- 7.1.2015 **Orlov's criterion.** Fourier–Mukai transforms II. Proof of Mukai's theorem. (Huybrechts Chapter 5).

- 14.1.2015 **The derived category of an elliptic curve.** Stable sheaves. SL_2 action.
(Polishchuk Chapter 14).
- 21.1.2015 **Derived equivalences of abelian varieties I.** (Huybrechts Chapter 9).
- 28.1.2015 **Derived equivalences of abelian varieties II.** (Huybrechts Chapter 9).
- 4.2.2015 **Autoequivalences of abelian varieties.** (Huybrechts Chapter 9).
- 11.2.2015 **Further topics.**