Mean-Risk Optimization of Electricity Portfolios Using Multiperiod Polyhedral Risk Measures

Andreas Eichhorn, Werner Römisch, and Isabel Wegner
Humboldt-University Berlin
Department of Mathematics
10099 Berlin, Germany

Abstract— **We present an applied mathematical model with stochastic input data for mean-risk optimization of electricity portfolios containing electricity futures as well as several components to satisfy a stochastic electricity demand: electricity spot market, two different types of supply contracts offered by a large power producer, and a combined heat and power production facility with limited capacity. Stochasticity enters the model via uncertain electricity demand, heat demand, spot prices, and future prices. The model is set up as a decision support system for a municipal power utility (price taker) and considers a medium term optimization horizon of one year in hourly discretization. The objective is to maximize the expected overall revenue and, simultaneously, to minimize risk in terms of multiperiod risk measures. Such risk measures take into account intermediate cash values in order to avoid uncertainty and liquidity problems at any time. We compare the effect of different multiperiod risk measures taken from the class of polyhedral risk measures which was suggested in our earlier work.**

I. Introduction

The deregulation of energy markets has lead to an increased awareness of the need for profit maximization with simultaneous consideration of financial risk, adapted to individual risk aversion policies of market participants. Mathematical modeling of such optimization problems with uncertain input data results in mixed-integer large-scale stochastic programming models with a risk measure in the objective. For the case that a medium-term planning horizon is considered, one is faced with consecutive decisions based on consecutive observations, thus, the stochastic programs have to be of the multi-stage type, cf. [1], [2]. We refer to a wide range of literature dealing with simultaneous optimization of power production and electricity trading, e.g. [3], [4], [5], [6], [7].

The multi-stage stochastic optimization model presented in this paper is tailored to the requirements of a typical Germnan municipal power utility, which has to serve an electricity demand and a heat demand of customers in a city and its vicinity. The power utility owns a *combined heat and power* (CHP) facility that can serve the heat demand completely and the electricity demand partly. Further electricity can be obtained by purchasing volumes for each hour at the (day-ahead) spot market of the European Energy Exchange (EEX), or by signing a supply contract for a medium term horizon with a larger power producer. The latter possibility is suspected to be expensive, but relying on the spot market only is known to be extremely risky. Spot price risk, however, may be reduced by obtaining electricity futures at EEX.

Futures at EEX are purely financial contracts relating to a specified delivery period in the future. Obtaining a future at a certain market value results, at the end of the corresponding delivery period, in a compensation of the difference between this market value and the average spot price in the delivery period. Hence, the question arises, whether the utility should sign a delivery contract or rely on spot and future market. That decision will be an output of the optimization which aims to maximize the mean overall revenue and, simultaneously, to minimize a risk measure.

To put this in concrete terms, we take a hourly discretization and an optimization horizon of one year as a basis. We suppose that two types of contracts are available: a fix contract (fix delivery schedule, fix price), and a flexible contract. The latter is based on the same delivery schedule, but, at the end of each month, it is allowed to alter these pre-arranged volumes for each hour of the following month by a certain percentage and, in addition, to realter these new volumes in a day-ahead manner by another percentage. The price of this contract may depend on the overall volume and on the maximum power (demand rate). Since electricity production together with contract volumes might exceed the demand, we also allow for selling at EEX spot and future market.

Due to the medium term horizon, we slightly simplified the technical restrictions of the CHP facility in the model such that no integer variables appear, we only impose that the heat and the electricity production are within certain interdependent bounds and that the electricity production of two consecutive time-steps must not differ more than a given delta (cf. section IV). Furthermore, we assume linear production cost. For the spot market, we restrict ourselves to price-independent bids. This guarantees full volume safety. We fully incorporate the trading rules of EEX including transaction costs, day-ahead offering, and initial and variation margins[1] for futures. We consider monthly base and peak futures for each month within the optimization horizon, i.e., we neglect futures for quarters and their cascading. We allow for rebalancing the future stock on every trading day at 12 am.

Electricity demand and heat demand as well as spot and future prices are not known in advance, but statistical information is available due to historical observations. We set up a

[1] When a future is obtained from EEX, a deposit, the initial margin, has to be payed rather than the market value. As long as the future is held, changes of the market value have to be compensated immediately (variation margin).

time series model for these processes and we derive a *scenario tree* from it by means of techniques according to [8], [9]. A scenario tree is a specific form of a discrete approximation of the time series model. The tree structure reflects the fact that information is revealed over time. Future prices are assumed to be so-called *fair prices* with respect to the spot prices of the corresponding delivery period.

Risk aversion is achieved by including a multiperiod risk measure in the objective. Such risk measures do not focus on the terminal wealth only, but also take into account the wealth at intermediate points in time in order to avoid liquidity problems at all time [10]. Here, we apply the risk measures to the cash values at the end of each month within the optimization horizon. We use risk measure taken from the class of *polyhedral risk measures*, that have been shown to be particularly suitable for being optimized in a stochastic program [11], [12]. They are, basically, multiperiod extentions of the Conditional-Value-at-Risk ($CVaR$), which is, in turn, an improvement of the well-known Value-at-Risk (VaR), that is known to have certain drawbacks [13]. The key-idea of polyhedral risk measures is that they can be written as a specific stochastic minimization problem with an expectation as objective, hence, minimizing such a risk measure is in many respects equivalent to minimizing an expectation.

The remaining paper is organized as follows: First, we describe the statistical models and the procedure of generating a scenario tree therewith. Then, in section IV, we formalize the above optimization model. Together with the scenario tree, the model can be seen as a linear program. This so-called *deterministic equivalent* is solved with a commercial LP solver and simulation results are presented in section V.

II. Modeling stochastic input data

First, we develop a time series model, i.e., a statistical model, for the random input data of the optimization model consisting of electricity demand, heat demand, and spot prices. (The future prices will be derived directly from the spot prices, cf. section III-B.) For each of these processes we have access to a historical time series in hourly discretization to which our time series model will be adapted.

The core of this model is a trivariate ARMA model for the daily mean values. A general ARMA(p,q) model for a stationary multivariate process $\mathbf{X}$ is based on finite order linear difference equations with constant coefficient matrices, i.e.,

$$\begin{aligned}&\mathbf{X}_k - \phi_1 \mathbf{X}_{k-1} - \ldots - \phi_p \mathbf{X}_{k-p}\\ &= \mathbf{Z}_k + \theta_1 \mathbf{Z}_{k-1} + \ldots + \theta_q \mathbf{Z}_{k-q}, \quad \mathbf{Z}_k \sim \mathrm{WN}(0, \Sigma)\end{aligned} \tag{II.1}$$

with a multivariate white noise process $\mathbf{Z}$ and suitable matrices ϕ_j, θ_j, Σ. Such a multivariate model is capable to describe the correlation between the components of $\mathbf{X}_k$ appropriately [14].

However, since ARMA models can only be applied to *stationary* time series, several preprocessing methods have to be applied in order to transform the originally non-stationary series into stationary ones. In addition, for the diurnal profiles, cluster analyses are performed for all of the processes, respectively.

A. Preprocessing electricity demand and heat demand

The historical data for the electricity demand D^e_t (electrical load) and heat demand D^h_t (thermal load), which have been provided by a German municipal utility, represents an one-year period of hourly loads, hence, $t = 1, ..., 365 \cdot 24$. Due to climatic influence, such demands are characterized by typical yearly cycles with high demand during winter time and low demand during summer time. Further, the electricity demand shows weekly cycles based on varying consumption behavior of private and industrial customers on working days and weekends. Moreover, the diurnal demand profiles reflect a characteristic consumption behavior of the customers with seasonal differences. Outliers can be observed on public holidays, on days between holidays, and on days with extreme climatic conditions.

Following our earlier studies [15], we suggest a decomposition approach for the electricity demand data and a separate handling of daily mean load data $\{\bar{D}^e_k, k = 1, \ldots, 365\}$ and adjusted diurnal demand profiles

$$\{\boldsymbol{Y}^e_k = (Y^e_{24(k-1)+1}, \ldots, Y^e_{24k}), \quad k = 1, \ldots, 365\}$$

such that $D^e_t = \bar{D}^e_{\lceil t/24 \rceil} \cdot Y^e_t$. For the heat demand we proceed analogously with $D^h_t = \bar{D}^h_{\lceil t/24 \rceil} \cdot Y^h_t$.

The diurnal demand profiles are modeled by standard clustering algorithms. We applied two specific types of cluster analysis methods successively, joining (tree clustering) and k-means clustering. The tree clustering method links together objects of increasing dissimilarity or distance successively. This suggests a suitable number k of clusters. The k-means algorithm aims to find the optimum partition for dividing a number of objects into k clusters. This procedure will move objects around from cluster to cluster aiming to minimize the within cluster variance and to maximize the between cluster variance.

Applying these methods to the electricity demand data, one can identify 9 clusters, namely for normal working days, Saturdays and Sundays - finer classified depending on the season. Outliers are assigned to the Sunday cluster of the same season. Thus, we are able to assign a cluster to every day of the year. For the heat demand, a number of 7 clusters turns out to be appropriate. In this case, the clusters reflect the dependence on the season and - in winter - on the day type.

For the daily means of the electricity demand data, we start with the construction of a robust (deterministic) trend function reflecting the varying consumption behavior of customers during the year and the typical weekly cycles. An initial trend estimation is calculated from the empirical means m^e_{ij} of the $\bar{D}^e_k$ values at week-day i in month j ($i = 1, \ldots, 7$, $j = 1, \ldots, 12$). For each day $k = 1, ..., 365$ of the year let $i(k)$ and $j(k)$ denote the week-day and the month, respectively. Then, a primary trend is given by $m^e_k = m^e_{i(k),j(k)}$ with $k = 1, \ldots, 365$.

Next, we improve this estimation by the extraction of large outliers. Note that, for these demand series, outliers are values lying fairly *below* the usual magnitude. We consider the relative differences between $\bar{D}_k^e$ and m_k^e ($k = 1, ..., 365$). For each k we replace $\bar{D}_k^e$ by $\bar{D}_k^{e\prime} := m_k^e$ if this respective difference is considered as large. For the case that the difference is small, we set $\bar{D}_k^{e\prime} := \bar{D}_k^e$. From this modified series $\bar{D}_k^{e\prime}$ ($k = 1, ..., 365$), we calculate the empirical means $m_{ij}^{e\prime}$ of week-day i and month j in the same manner as above. Then the second version of the trend estimation is given by $m_k^{e\prime} := m_{i(k),j(k)}^{e}{}'$, $k = 1, ..., 365$. Analogously to the empirical means $m_{ij}^{e\prime}$, we calculate the empirical standard deviations $v_{ij}^{e\prime}$ an set $v_k^{e\prime} := v_{i(k),j(k)}^{e}{}'$.

For the final trend function $m_k^{e\prime\prime}$ we consider the truncated series $\bar{D}_k^{e\prime\prime}$ defined by $\bar{D}_k^{e\prime\prime} := \bar{D}_k^e$ if $\bar{D}_k^e \geq m_k^{e\prime} - 3 \cdot v_k^{e\prime}$ and $\bar{D}_k^{e\prime\prime} := m_k^{e\prime} - 3 \cdot v_k^{e\prime}$ else. The values $m_k^{e\prime\prime}$ are again calculated as the empirical means of the the series $\bar{D}_k^{e\prime\prime}$ depending on week-day and month.

Now, the time series $\{X_k^e := \bar{D}_k^{e\prime\prime} - m_k^{e\prime\prime}, \quad k = 1, \ldots, 365\}$ turns out to be stationary, hence, it is suitable for being described by an ARMA model. Processing the heat demand series $\bar{D}_k^h$ in the same manner yields an analogous stationary time series $\{X_k^h, \quad k = 1, \ldots, 365\}$.

B. Preprocessing Spot prices

The spot market differs from other commodity markets fundamentally. Electricity is a hardly storable product, hence, there is the necessity of a balanced production and consumption. Clearly, spot prices are affected by local characteristics of different markets (climatic conditions, economic activities, characteristics of local power producers, behavior of customers, political events), hence, an all-embracing modeling is hardly possible. The problem of spot price modeling is studied in a wide range of literature, see, e.g., [16], [17], [18], [19].

Like the electricity demand, spot prices are characterized by typical yearly cycles with high prices during winter time and lower prices during summer time. Further, the spot prices show weekly and daily cycles. The midday load profiles reflect the characteristic consumption behavior of the customers with seasonal differences (evening price peak during winter time and a midday peak during summer time). Further, one can observe outliers on public holidays, on days with extreme climatic conditions, and as a result of transmission shortages or power plant outages.

In [16] a simultaneous modeling of the typical features of the price process by combining an AR-GARCH process with a jump-diffusion model is proposed. The model is calibrated to historical data by using the maximum likelihood method. But, facing the number of parameters that have to be estimated, this seems to be an ambitious task in practice. Furthermore, for the EEX spot price data, it can be observed that large price spikes are followed by a fast reversion to the long term mean. But, due to the high degree of autocorrelation in the price series, the integrated model leads, in contrast to the former observation, to a slow downward movement to the normal price level.

Hence, we suggest an easily adjustable discrete model for daily mean spot prices $\bar{C}_k^s$ inspired by the above model but with fast reversion behavior. It consists of a deterministic component $m_k^{s\prime\prime}$ capturing trend and seasonality, the stationary random component X_k^s from the multivariate ARMA model (II.1) reflecting autocorrelation and mean reversion, and a simplified jump diffusion model $O_k \cdot Z_k$ representing occasional price spikes. We allow for nonconstant jump rates and frequencies by a parameterization depending on season:

$$\bar{C}_k^s = m_k^{s\prime\prime} + X_k^s + O_k Z_k \qquad \text{(II.2)}$$

$$\begin{aligned} O_k &= \begin{cases} 0 & \text{with probability} \quad 1 - \lambda_k \\ 1 & \text{with probability} \quad \lambda_k \end{cases} \\ Z_k &\sim N(\mu_k, \sigma_k) \\ \lambda_k &\in (0, 1) \\ (\mu_k, \sigma_k, \lambda_k) &= (\mu^w, \sigma^w, \lambda^w), \text{ if } k \in \text{winter} \\ (\mu_k, \sigma_k, \lambda_k) &= (\mu^s, \sigma^s, \lambda^s), \text{ if } k \in \text{summer} \end{aligned}$$

The distribution of jumps is modeled with a Bernoulli distribution with jump rate λ_k and the random jump magnitude is normally distributed.

To estimate the deterministic component, we proceed analogously as for the demand series. First, we calculate yearly trend estimations m_k^s and $m_k^{s\prime}$ and standard deviations $v_k^{s\prime}$ depending on week-day and month in the same manner as in section II-A. Then, extreme price outliers have to be identified. Note that, here, outliers are values lying fairly *above* the usual magnitude. Therefore, we define the outlier series by $O_k = \max\{0, O_k{}'\}$ with $O_k{}' := \bar{C}_k^s - (m_k^{s\prime} + 2 \cdot v_k^{s\prime})$. The final trend estimation $m_k^{s\prime\prime}$ is calculated from the adjusted series $\bar{C}_k^s - O_k$ as the empirical means of the respective week-day $i(k)$ in the respective month $j(k)$.

The parameters of the outlier process $(\mu^w, \sigma^w, \lambda^w)$ and $(\mu^s, \sigma^s, \lambda^s)$ are estimated by the empirical means, the empirical standard deviations, and the relative outlier frequencies in summer and winter time, respectively.

As for the demand profiles, we carried out a cluster analysis with adjusted diurnal price profiles $\mathbf{Y}_k^s = (Y_{24(k-1)+1}^s, \ldots, Y_{24k}^s)$ to model the intra daily price behavior. In comparison to the electricity demand, there is a higher variability among the profiles. Thus, the required number of cluster is greater in this case. One can identify 10 clusters that distinguish between working days, Saturdays, and Sundays on the one hand, and between different seasons on the other hand. Further, we operate with 3 outlier clusters for working days with middle, high and extreme daily maximum for summer and winter time, respectively. In addition, there is an outlier cluster for weekends during winter time.

In the overall simulation procedure, the mapping between days and profile clusters is a canonical one as long as $O_k = 0$, i.e., as long as the simulation doesn't yield an outlier for day k, cf. (II.2). If, however, $O_k = 1$ for some day k, then we have to assign an outlier cluster type. The choice among the available outlier cluster in this case is made according to the simulated outlier intensity Z_k. For illustration, consider winter time and let c_1, c_2, c_3 denote the number of elements in the clusters for moderate, middle, and large outliers, respectively. With the model parameters $(\mu^w, \sigma^w, \lambda^w)$ from (II.2) we calculate limits b_1 and b_2 such that the relative numbers of outlier cluster elements is approximately maintained. Since Z_k is normally distributed, this is achieved, if we chose b_1 and b_2 such that

$$F(b_1,\mu^w,\sigma^w) - F(\mu^w,\mu^w,\sigma^w) = \frac{1}{2}\cdot\frac{c_1}{c_1+c_2+c_3}$$
$$F(b_2,\mu^w,\sigma^w) - F(b_1,\mu^w,\sigma^w) = \frac{1}{2}\cdot\frac{c_2}{c_1+c_2+c_3}$$

with $F(y,\mu^w,\sigma^w)$ denoting the normal distribution function with mean μ^w and variance σ^w evaluated at y. Hence, if $O_k = 1$, we assign for day k the cluster of moderate outliers if $0 < Z_k \leq b_1$, the middle outlier cluster if $b_1 < Z_k \leq b_2$, and the cluster of large outliers if $Z_k > b_2$, respectively.

C. Calibrating the ARMA model

The calibration of the multivariate ARMA(p,q)-model requires the determination of orders p and q as well as the estimation of the model coefficients and the variance of the noise process. All parameters can be estimated by the Hannan-Rissanen-Algorithm simultaneously. The method generates different ARMA models for specified model order upper bounds, which are tested on stationarity and invertibility.

Then we have to decide among the remaining models for the optimal one in the sense of a statistical information criterion (Akaike, Bayes). After that, the efficiency of coefficient and variance estimators may be improved by the conditional maximum likelihood method. The analysis is completed by some tests of goodness of fit of the residuals (Portmanteau, Turning-Point, Difference-Sign), which shall show the behavior of a white noise process with zero mean and constant variance.

This calibration procedure applied to the preprocessed series

$$\{\mathbf{X}_k = (X_k^e, X_k^h, X_k^s) \quad k = 1, ..., 365\},$$

which is free of trend, seasonal patterns and extreme outliers, yields a stationary invertible ARMA model, which passes all the residual tests.

From this ARMA model we derive sample paths by simulating the white noise process $\mathbf{Z}_k$ in (II.1) repeatedly. After composing these sample paths with the respective trend functions and the jump process, diurnal profiles are added using a bootstrap simulation procedure. Finally, we delete all nonpositive scenarios.

III. Scenario trees

The formulation of the optimization model is based on the input scenario tree $\mathcal{T}$, which consists of the tree structure (nodes $n \in \mathcal{N}$ and predecessor mapping), node probabilities π_n, and the random data (D_n^e, D_n^h, C_n^s) for $n \in \mathcal{N}$. The scenario tree is constructed from a large number of sample paths from section II via specialized scenario reduction algorithms, that aim to minimize the approximation error with respect to the optimal value and the solution set of the stochastic optimization problem [9], [8].

The nodes of $\mathcal{T}$ are numbered successively beginning with the root node 1. Every node $n \in \mathcal{N} \setminus \{1\}$ has a unique predecessor denoted by $n-$ and a unique corresponding time-step $t(n) \in \{1, ..., 365 \cdot 24\}$. Furthermore, we set $\text{path}(n) =$

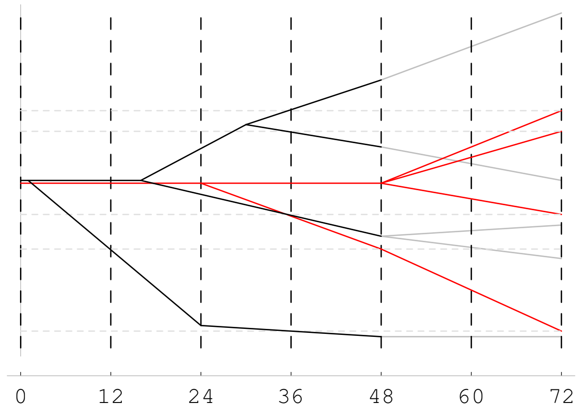

Fig. 1. The (beginning of the) original scenario tree $\mathcal{T}$ (black) and the trading tree $\mathcal{T}^{\text{trade}}$ (red/gray) which is derived by delaying branching in $\mathcal{T}$ until the beginning of the next trading day ($t = 24$, $t = 48$).

$\{n, n-, ..., 1\}$ the set of all nodes between n and the root node. The node probabilities π_n are understood unconditional, i.e., for each time-step t it holds that $\sum_{\{n\in\mathcal{N}:t(n)=t\}} \pi_n = 1$.

Beside the random input data, also the decision variables are defined on the scenario tree. This implies the *non-anticipativity* of the decisions, i.e., the requirement that decisions at time t must be based on the observations until time t only.

A. Derived trees

To formulate the optimization model, it is useful to introduce further (smaller) trees derived from $\mathcal{T}$ by delaying branching points or by eliminating time-steps. These trees reflect further non-anticipativity restrictions, e.g. day-ahead requirements for spot market decisions. All decision variables are defined on the nodes of the trees. The nodes of the trees are numbered in the same way as for the original scenario tree:

- Future tree $\mathcal{T}^{\text{fut}}$: based on the original scenario tree, the number of time-steps and, hence, the number of nodes is reduced such that there is one time-step at each trading day at 12 am. In addition, there are time-steps (and nodes) for the final billing of the futures at the end of each month (11 pm). Every node $d \in \mathcal{N}^{\text{fut}}$ has a unique corresponding node $n(d) \in \mathcal{N}$ in $\mathcal{T}$.
- Trading day tree $\mathcal{T}^{\text{trade}}$: based on the original scenario tree. For every day and every scenario, branching between 12 am, previous day, and 12 am, current day, is delayed in time until the beginning of the next trading day (mon-fri and not a holiday), cf. Fig. 1. Each node $n \in \mathcal{N}$ of the original scenario tree has a unique corresponding node $j(n) \in \mathcal{N}^{\text{trade}}$ such that for the time-steps of the nodes it holds that $t(n) = t(j(n))$.
- Contract tree $\mathcal{T}^{\text{contr}}$: based on $\mathcal{T}^{\text{trade}}$, branching is (further) delayed to the 1st day of the following month. For each node j of $\mathcal{T}^{\text{trade}}$ there is a unique corresponding node $l(j) \in \mathcal{N}^{\text{contr}}$.

Note that the decision about the contract alternatives (fix, flexible, on none) has to be made already at the beginning, i.e., the respective decision variable would be defined on the root node 1 rather than on one of the above trees.

B. Prices for electricity futures

A future for a month m expires at the end of this month. Then, the final future price is fixed to the average electricity spot price in this month m. Note that, for peak futures, only the hours between 8 am and 8 pm on trading days contribute to the respective average, whereas, for base futures, every hour of month m is taken into account. Hence, for the price of a future for month m before the end of this month, it is natural to assume so-called *fair prices*, i.e., the market value of the future at some point in time $t < \mathrm{end}(m)$ is given by the conditional expectation of the (temporal) average of the (stochastic) spot prices with respect to the information that is available at this time t. This approach guarantees the future prices to be arbitrage-free.

IV. Optimization model

A. Parameters

The scenario tree data can be understood as parameters indexed by node numbers. All the other parameters are indexed by time-step or they are not indexed at all:

D_n^e, D_n^h: Demand of electricity, heat at node $n \in \mathcal{N}$ in MW
C_n^s: Spot price cots for electricity in Euro/MWh ($n \in \mathcal{N}$)
$C_n^{fb,m}$, $C_n^{fp,m}$: Prices for base, peak futures in Euro/MWh
$C^{s,\mathrm{trans}} = 0.04\,\mathrm{Euro/MWh}$: Spot market transaction cost
$C^{f,\mathrm{trans}} = 0.02\,\mathrm{Euro/MWh}$: Future market transaction cost
$C^{f,\mathrm{imar}} = 2.0\,\mathrm{Euro/MWh}$: Initial margin for futures
C^{pe}: Cost factor for electricity production in Euro/MWh
C^{ph}: Cost factor for production of heat in Euro/MWh
δ^{pe}: Maximum gradient for electricity production in MW
P^e: Selling price for electricity in Euro/MWh
P^h: Selling price for heat in Euro/MWh
V_t^c: Pre-arranged contract volumes ($t = 1, ..., 365 \cdot 24$)
$C^{c,\mathrm{fix}}$: Energy rate for fix contract in Euro/MWh
$C^{c,\mathrm{flex},p}$: Peak energy rate, flexible contr. in Euro/MWh
$C^{c,\mathrm{flex},o}$: Off-peak energy rate in Euro/MWh
$C^{c,\mathrm{flex},d}$: Maximum demand rate in Euro/MW

B. Decision variables

Decision variables will be denoted by the letter x. All of them are defined on one of the trees described in the previous section and, hence, are indexed by the respective node number:

Future stock for month m (base): $x_d^{fb,m} \in \mathbb{R}$, $d \in \mathcal{N}^{\mathrm{fut}}$
Future stock for month m (peak): $x_d^{fp,m} \in \mathbb{R}$, $d \in \mathcal{N}^{\mathrm{fut}}$
Spot market volumes: $x_j^s \in \mathbb{R}$, $j \in \mathcal{N}^{\mathrm{trade}}$
Power production, electricity: $x_n^{pe} \in \mathbb{R}_+$, $n \in \mathcal{N}$
Power production, heat (thermal): $x_n^{ph} \in \mathbb{R}_+$, $n \in \mathcal{N}$
Power production: $x_n^p = (x_n^{pe}, x_n^{ph}) \in \mathbb{R}^2$, $n \in \mathcal{N}$
Monthly declared contr. volumes: $x_l^{c,\mathrm{flex,decl}} \in \mathbb{R}_+$, $l \in \mathcal{N}^{\mathrm{contr}}$
Contract volumes: $x_j^c \in \mathbb{R}_+$, $j \in \mathcal{N}^{\mathrm{trade}}$

C. Restrictions

For the future trading variables, we impose that the initial future stock is empty and that, after future for month m has expired, the respective amount of futures is zero:

$x_1^{fb,m} = x_1^{fp,m} = 0$ for $m = 1, ..., 12$,
$x_d^{fb,m} = x_d^{fp,m} = 0$ if $t(d) \geq \mathrm{end}(m)$ for $m = 1, ..., 12$.

For the CHP facility we impose a gradient restriction for the production of electricity, the heat demand satisfaction restriction, and that, for all time-steps, the two-dimensional vector x_n^p lies within some given bounded polyhedron in $\mathbb{R}^2$ that is given through a matrix A^p and a vector b^p:

$|x_n^{pe} - x_{n-}^{pe}| \leq \delta^{pe}$ for $n \in \mathcal{N} \setminus \{1\}$,
$x_n^{ph} \geq D_n^h$ for $n \in \mathcal{N}$,
$A^p \cdot x_n^p \leq b^p$ for $n \in \mathcal{N}$.

For the contract volumes we have that $x_j^c = 0$ if no contract is purchased and, if the fix contract is included, $x_j^c = V_{t(j)}^c$ for $j \in \mathcal{N}^{\mathrm{trade}}$. For the case that the flexible contract is chosen, the monthly declared volumes and the effective volumes, respectively, have to satisfy:

$$\begin{array}{lcl} x_l^{c,\mathrm{flex,decl}} & \in & [(1-\alpha) \cdot V_{t(l)}^c, (1+\alpha) \cdot V_{t(l)}^c] \\ x_j^c & \in & [(1-\beta) \cdot x_{l(j)}^{c,\mathrm{flex,decl}}, (1+\beta) \cdot x_{l(j)}^{c,\mathrm{flex,decl}}] \end{array}$$

for $l \in \mathcal{N}^{\mathrm{contr}}$, $j \in \mathcal{N}^{\mathrm{trade}}$ with some given percentages α, β.

For the spot market, no further restrictions are imposed. It remains to require the satisfaction of the electricity demand:

$$x_{j(n)}^s + x_n^{pe} + x_{j(n)}^c \geq D_n^e, \; n \in \mathcal{N} \qquad \text{(IV.3)}$$

D. Cash values

For formulating the objective, we introduce auxiliary variables z_n ($n \in \mathcal{N}$) that represent the wealth at time $t(n)$ in the respective scenario, i.e., the accumulated revenues. These *cash values* are composed of the revenues from satisfying the demands, the cost of power production and contracts, and the cash flows caused by spot market activity and future trading:

$$\begin{array}{rcl} z_n & = & z_{n-} + P^e \cdot D_n^e + P^h \cdot D_n^h \\ & + & z_n^p + z_n^c + z_n^s \\ & + & \sum_{m=1}^{12} z_n^{fb,m} + \sum_{m=1}^{12} z_n^{fp,m} \end{array} \qquad \text{(IV.4)}$$

Note that the z variables depend on the decisions. The cash flows for power production and spot market are given by

$$\begin{array}{rcl} z_n^p & = & -C^{pe} \cdot x_n^{pe} - C^{ph} \cdot x_n^{ph} \\ z_n^s & = & -x_{j(n)}^s \cdot C_n^s - |x_{j(n)}^s| \cdot C^{s,\mathrm{trans}}, \end{array}$$

respectively. Because we allow for future trading only on trading days at noon, $z_n^{fb,m} = z_n^{fp,m} = 0$ if $t(n)$ does not correspond to such point in time. If $t(n)$ does correspond to 12 am on a trading day, i,e., if there is a corresponding node $d(n) \in \mathcal{N}^{\mathrm{fut}}$, then

$$\begin{array}{rcl} z_n^{fb,m} & = & x_{d(n)-}^{fb,m} \cdot \left(C_{d(n)}^{fb,m} - C_{d(n)-}^{fb,m} \right) \\ & - & \left(|x_{d(n)}^{fb,m}| - |x_{d(n)-}^{fb,m}| \right) \cdot C^{f,\mathrm{imar}} \\ & - & \left| x_{d(n)}^{fb,m} - x_{d(n)-}^{fb,m} \right| \cdot C^{f,\mathrm{trans}} \cdot \mathbf{1}_{\{t(n) \neq \mathrm{end}(m)\}} \end{array}$$

for base futures of month $m = 1, ..., 12$. The first and the second summand in the above equation represent the variation

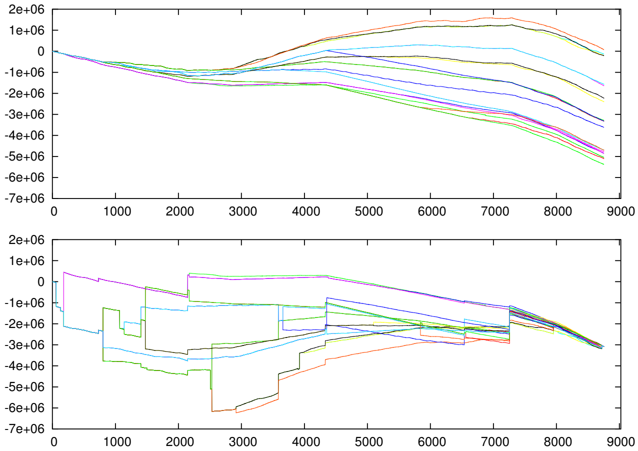

Fig. 2. Resulting optimal cash values over time for the case that no contract is included and without incorporating multiperiod risk measures. Top: Only $\mathbb{E}[z_T]$ is optimized. Bottom: $0.9 \cdot CVaR(z_T) - 0.1 \cdot \mathbb{E}[z_T]$ is minimized. There is considerably high spreading and many scenarios reach fairly low cash values at the end or in the meantime, respectively.

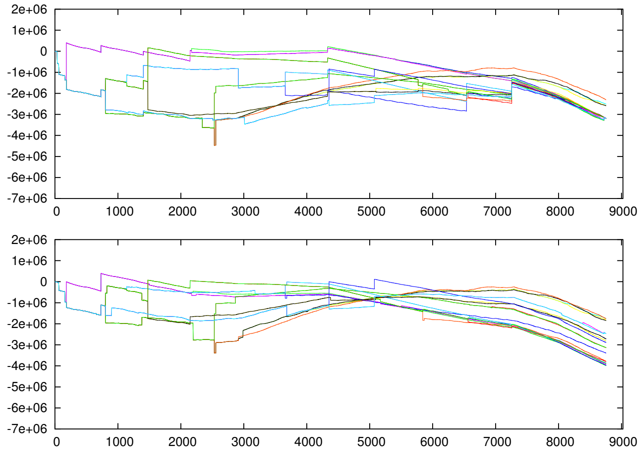

Fig. 3. Resulting optimal cash values over time for the case that no contract is included. Two different multiperiod polyhedral risk measures are optimized, ρ_2 (top) and ρ_4 (bottom), cf. [11], [12]. Obviously, multiperiod risk measures aim to reduce spreading at all time. Roughly speaking, ρ_4 tries to reach equal spread at all time, whereas ρ_2 tries to find a maximal level that is not underrun.

margin and the initial margin, respectively. The indicator function in the third summand reflects the fact, that transaction cost don't need to be payed when the future contract expires. For peak futures, the cost functions $z_n^{fp,m}$ are analogous.

For the contracts cash flow z_n^c, we have to distinguish between the fix and the flexible contract. For both of them, there is a volume dependent price to be payed, but for the latter, there is, in addition, an extrapolated demand rate $z_n^{c,\text{flex},d}$ depending on the maximum demand within the elapsed time, which is to be payed at the end of each month.

$$z_n^c = \begin{cases} -x_{j(n)}^c \cdot C^{c,\text{fix}} & \text{for the fix contract} \\ -x_{j(n)}^c \cdot C_{t(n)}^{c,\text{flex}} - z_n^{c,\text{flex},d} & \text{for the flexible contract} \end{cases}$$

The monthly demand rate is adapted such that, at the end of the term, the overall payment is proportional to the overall maximum power, hence,

$$\sum\nolimits_{\{\tilde{n}\in\text{path}(n)\}} z_{\tilde{n}}^{c,\text{flex},d} = C^{c,\text{flex},d} \cdot \max\nolimits_{\{j\in\text{path}(j(n))\}} x_j^{c,\text{flex}}$$

for all leaves n, i.e., for $n \in \mathcal{N}$ such that $t(n) = T$. Note that $z_{\tilde{n}}^{c,\text{flex},d} = 0$ if $t(\tilde{n})$ is not the end of a month.

E. Objective

The above cash values z_n can be understood, together with the node probabilities π_n, as discrete random variables z_t, $t = 1, ..., T$, with $T = 365 \cdot 24$ and $z_t = (z_n)_{\{n\in\mathcal{N}:t(n)=t\}}$. Thus, the overall expected revenue is given by $\mathbb{E}[z_T]$ and the multiperiod risk measure ρ applied to the time-steps $t_1, ..., t_{T'}$ reads $\rho(z_{t_1}, ..., z_{t_{T'}})$. Hence, the objective can be written as

$$\min\ \gamma \cdot \rho(z_{t_1}, ..., z_{t_{T'}}) - (1-\gamma) \cdot \mathbb{E}[z_T] \qquad \text{(IV.5)}$$

with some weighting parameter $\gamma \in [0, 1]$. The minimization is over all the x variables from section IV-B with respect to the constraints from section IV-C. For the simulations, we used $\gamma = 0.9$ and for $t_1, ..., t_{T'}$ we took the end of each month within the optimization horizon.

V. Simulation results

The model is implemented and solved with ILOG CPLEX 8.1, the ILOG Concert Technology 13 library, and GNU C++ on a 2 GHz Linux PC with 1 GB memory. We used a scenario tree with 21 scenarios, $365 \cdot 24 = 8760$ time-steps, and 98016 nodes.

We ran the simulation successively for the case that the fix contract, the flexible one, or no contract at all is included. We separated this decision from the rest of the optimization model, because all the remaining decision variables are continuous, hence, the three remaining (sub-) problems are purely linear programs. Time for solution is in either case around two hours. We optimized with $CVaR$, with 2 multiperiod risk measures, and without risk measure and obtained:

	no contract	fix contr.	flexible contr.
$CVaR(z_T)$	$3,088,140$	$3,135,020$	$3,354,510$
$\mathbb{E}[z_T]$	$-3,088,140$	$-3,135,020$	$-3,354,510$
opt. value	$3,088,140$	$3,135,020$	$3,354,510$
$\rho_2(z)$	$3,192,280$	$3,670,050$	$4,129,490$
$\mathbb{E}[z_T]$	$-3,086,200$	$-3,128,670$	$-3,347,280$
opt. value	$3,181,670$	$3,615,910$	$4,051,270$
$\rho_4(z)$	$1,264,820$	$1,382,560$	$1,562,660$
$\mathbb{E}[z_T]$	$-3,085,480$	$-3,133,950$	$-3,365,940$
opt. value	$1,446,890$	$1,557,700$	$1,742,990$
$\mathbb{E}[z_T]$	$-3,072,310$	$-3,120,760$	$-3,339,230$

These values suggest, that going without any contract is the best alternative in terms of expected revenue and, surprisingly, in terms of risk, too. Note that the absolute values of the risk measures may not have a significant meaning, but can be compared for the three contract alternatives.

Beside the (optimal) magnitude of the risk measure and the expected terminal wealth, the shape of the cash values over all time-steps seems to be the most relevant output information. For the case that no contract is considered, the effect of different risk measures can be observed very well, cf. Fig. 2

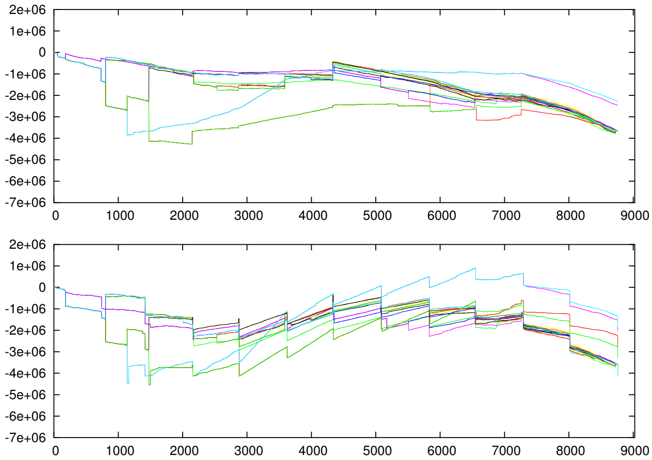

Fig. 4. Resulting cash values over time using the multiperiod risk measure ρ_2 for the case that the fix contract (top) or the flexible contract (bottom) is included. In the latter case, there are jumps due to the monthly demand rate.

and Fig. 3. The different shapes of the curves are achieved by different policies of future trading. Future trading is revealed through the jumps in the curves. If no risk is considered (Fig. 2 top), then there is no future trading at all. For the case that a delivery contract is considered, future trading activity is reduced, cf. Fig. 4.

VI. Conclusions and Outlook

Regarding the optimal values, relying on spot and future market appears to be the better choice than purchasing one of the available delivery contracts. However, the situation may be different if the conditions, i.e., the parameters, are changed, or if we no longer assume fair prices for the futures. Due to this fair prices assumptions, futures are almost too perfectly capable of reducing spot price risk.

The model could be adapted and improved in numerous directions, e.g. by allowing for price-dependent spot market bids or by introducing integer variables into the CHP production facility model. Moreover, another goal is to enlarge the number of scenarios in order to approximate the uncertainty more accurately. Therefore, one would need more efficient solution methods. Currently, we are working on a decomposition approach based on Lagrangian relaxation of the coupling constraint (IV.3) and the coupling induced by the non-linearity of the polyhedral risk measures.

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Biographies

Andreas Eichhorn is a research assistant at the Department of Mathematics, Humboldt-University Berlin, Germany. He graduated in technical mathematics in 1999 at the University of Karlsruhe, Germany.
E-Mail: `eichhorn@math.hu-berlin.de`
Web: `http://www.math.hu-berlin.de/˜eichhorn/`

Werner Römisch is a Full Professor at the Department of Mathematics, Humboldt-University Berlin, Germany. His current research interests are in theory and solution methods for mixed-integer stochastic programming problems. He is actively working on several applications, especially in the electric power industry. He is co-editor of the *Stochastic Programming E-Print Series* (`http://www.speps.info`).
E-Mail: `romisch@math.hu-berlin.de`
Web: `http://www.math.hu-berlin.de/˜romisch/`

Isabel Wegner is also a research assistant at the Department of Mathematics, Humboldt-University Berlin, Germany, where she graduated in 1999.
E-Mail: `isabel@math.hu-berlin.de`
Web: `http://www.math.hu-berlin.de/˜isabel/`