Recent Progress in Stochastic Programming and Applications in Energy

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Introduction

Practical optimization models often contain uncertain parameters or stochastic processes. In many cases it is not appropriate to replace the uncertain parameters by their mean values or some other statistical estimate. Alternatives are robust/worst case optimization models or, if statistical data is available, modeling the relevant stochastic process by a finite number of scenarios with given probabilities and incorporating them into the optimization model. This leads to stochastic optimization models having the advantages:

- Solutions are robust with respect to uncertain changes of the data.
- The risk of decisions can be measured and managed.
- Simulation studies show that "stochastic solutions" may be advantageous compared to deterministic ones.

The presentation will focus on

- Modeling stochastic programs (two- or multi-stage, or probabilistic (chance) constraints ?
- Chance constraints: State-of-the-art
- Two-stage stochastic programs: Theory, approximations and algorithms are (almost) complete.
- Mixed-integer two-stage stochastic programs: State-of-the-art
- Approximations and scenario trees for multi-stage stochastic programs.
- Decomposition methods for (multi-stage) stochastic programs.
- Stochastic optimization models for electricity portfolio management and their solution by Lagrangian relaxation.
- Measuring and managing risk, in particular, in electricity portfolio management models.

Modeling

Assumptions: Information on the underlying probability distribution is available (e.g., statistical data) and the distribution does not depend on decisions.

Modeling questions: Are recourse actions available if uncertainty influences decisions ? Is the decision process based on recursive observations of the uncertainty ?

- No recourse actions available: **Chance constraints**.
- Recourse actions available, but no recursive observations: **Two-stage stochastic programs** (possibly multi-period).
- Recursive observation and decision process: **Multi-stage stochastic programs**.

Integer variables should be incorporated if they are model-important.

Chance constraints

Let us consider the (linear) chance constrained model

$$\min\{\langle c,x\rangle : x \in X, P(\{\xi \in \Xi : T(\xi)x \geq h(\xi)\}) \geq p\},$$

where $c \in \mathbb{R}^m$, X and Ξ are polyhedra in $\mathbb{R}^m$ and $\mathbb{R}^s$, respectively, $p \in (0,1)$, P is a probability measure on Ξ, i.e., $P \in \mathcal{P}(\Xi)$, and the right-hand side $h(\xi) \in \mathbb{R}^d$ and the (d,m)-matrix $T(\xi)$ are affine functions of ξ.

Challenges:
Although the sets $H(x) = \{\xi \in \Xi : T(\xi)x \geq h(\xi)\}$ are (convex) polyhedral subsets of Ξ, the function

$$x \to P(H(x))$$

is, in general, non-concave and non-differentiable on $\mathbb{R}^m$, hence, the optimization model is nonconvex.
Approximations by discrete probability measures lead to mixed-integer linear programs.

Theory and Algorithms:

Convexity results for probability distributions satisfying certain concavity properties (e.g., normal distributions), bounds for chance constraints, Monte-Carlo type methods inside nonlinear programming algorithms (Prekopa 95), well-developed stability analysis (Römisch 03, Henrion-Römisch 04).

More recently: Convex approximations (Nemirovski-Shapiro 06), extension of convexity results (Henrion-Strugarek 06).

Recent motivation: Optimization of Value-at-Risk objectives, where

$$VaR_\alpha(z) := \inf\{x \in \mathbb{R} : \mathbb{P}(z \leq x) \geq \alpha\}.$$

Challenge: Dimension of ξ !

Two-stage stochastic programs

$$\min \left\{ \langle c, x \rangle + \int_{\Xi} \Phi(\xi, x) P(d\xi) : x \in X \right\},$$

where

$$\Phi(\xi, x) := \inf\{\langle q(\xi), y \rangle : y \in Y, W(\xi)y = h(\xi) - T(\xi)x\}$$

$P := \mathbb{P}\xi^{-1} \in \mathcal{P}_2(\Xi)$ is the probability distribution of the random vector ξ, $c \in \mathbb{R}^m$, $X \subseteq \mathbb{R}^m$ is a bounded polyhedron, $q(\xi) \in \mathbb{R}^{\overline{m}}$, $Y \in \mathbb{R}^{\overline{m}}$ is a polyhedral cone, $W(\xi)$ a $r \times \overline{m}$-matrix, $h(\xi) \in \mathbb{R}^r$ and $T(\xi)$ a $r \times m$-matrix. We assume that $q(\xi)$, $h(\xi)$, $W(\xi)$ and $T(\xi)$ are affine functions of ξ.

Theory and Algorithms: The function $\Phi : \Xi \times X \to \overline{\mathbb{R}}$ is well understood for fixed recourse (i.e., $W(\xi) \equiv W$) (Walkup-Wets 69). Convexity, optimality and duality results, decomposition methods, Monte-Carlo type methods (Wets 74, Kall 76, Ruszczyński-Shapiro 03), scenario reduction (Heitsch-Römisch 07) and stability analysis (Rachev-Römisch 02, Römisch-Wets 07) are well developed.

Mixed-integer two-stage stochastic programs

$$\min\left\{\langle c,x\rangle + \int_{\Xi}\Phi(q(\xi),h(\xi)-T(\xi)x)P(d\xi) : x\in X\right\},$$

where Φ is given by

$$\Phi(u,t) := \inf\left\{\langle u_1,y\rangle + \langle u_2,\bar{y}\rangle : Wy+\bar{W}\bar{y}\le t, y\in\mathbb{Z}^{\hat{m}}, \bar{y}\in\mathbb{R}^{\bar{m}}\right\}$$

for all pairs $(u,t)\in\mathbb{R}^{\hat{m}+\bar{m}}\times\mathbb{R}^r$, and $c\in\mathbb{R}^m$, X is a closed subset of $\mathbb{R}^m$, Ξ a polyhedron in $\mathbb{R}^s$, W and $\bar{W}$ are $(r,\hat{m})$- and $(r,\bar{m})$-matrices, respectively, $q(\xi)\in\mathbb{R}^{\hat{m}+\bar{m}}$, $h(\xi)\in\mathbb{R}^r$, and the (r,m)-matrix $T(\xi)$ are affine functions of ξ, and $P\in\mathcal{P}_2(\Xi)$.

Theory and Algorithms: The function Φ is well understood (Blair-Jeroslow 77, Bank-Mandel 88), nonconvex optimization models, structural analysis (Schultz 95, van der Vlerk 95), scenario decomposition (Carøe-Schultz 99), decomposition methods (surveys: Schultz 03, Sen 05), sampling methods (Shapiro 03, Eichhorn-Römisch 07), stability analysis (Schultz 95, 96, Römisch-Vigerske 07), scenario reduction (Henrion-Küchler-Römisch 07).

Multistage stochastic programs

Let $\{\xi_t\}_{t=1}^T$ be a discrete-time stochastic data process defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and with ξ_1 deterministic. The stochastic decision x_t at period t is assumed to be measurable with respect to $\mathcal{F}_t(\xi) := \sigma(\xi_1, \ldots, \xi_t)$ (nonanticipativity).

Multistage stochastic optimization model:

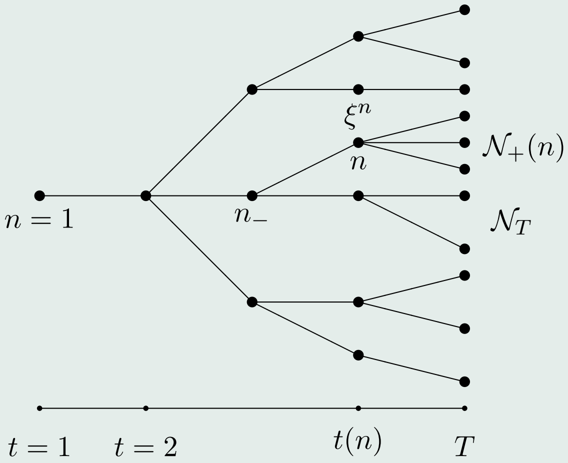
$$\min\left\{\mathbb{E}\left[\sum_{t=1}^{T}\langle b_t(\xi_t), x_t\rangle\right] \middle| \begin{array}{l} x_t \in X_t, x_t \text{ is } \mathcal{F}_t(\xi)\text{-measurable}, t = 1, .., T \\ A_{t,0}x_t + A_{t,1}(\xi_t)x_{t-1} = h_t(\xi_t), t = 2, .., T \end{array}\right\}$$

where X_t, $t = 1, \ldots, T$, are polyhedral, the vectors $b_t(\cdot)$, $h_t(\cdot)$ and $A_{t,1}(\cdot)$ are affine functions of ξ_t, where ξ varies in a polyhedral set Ξ.

If the process $\{\xi_t\}_{t=1}^T$ has a finite number of scenarios, they exhibit a scenario tree structure.

Data process approximation by scenario trees

The process $\{\xi_t\}_{t=1}^T$ is approximated by a process forming a scenario tree being based on a finite set $\mathcal{N} \subset \mathbb{N}$ of nodes.



Scenario tree with $T = 5$, $N = 22$ and 11 leaves

$n = 1$ root node, n_- unique predecessor of node n, $\text{path}(n) = \{1, \ldots, n_-, n\}$, $t(n) := |\text{path}(n)|$, $\mathcal{N}_+(n)$ set of successors to n, $\mathcal{N}_T := \{n \in \mathcal{N} : \mathcal{N}_+(n) = \emptyset\}$ set of leaves, $\text{path}(n)$, $n \in \mathcal{N}_T$, scenario with (given) probability π^n, $\pi^n := \sum_{\nu \in \mathcal{N}_+(n)} \pi^\nu$ probability of node n, ξ^n realization of $\xi_{t(n)}$.

Tree representation of the optimization model

$$\min\left\{\sum_{n\in\mathcal{N}}\pi^n\langle b_{t(n)}(\xi^n),x^n\rangle\,\middle|\,\begin{array}{l}x^n\in X_{t(n)},n\in\mathcal{N}\\ A_{t(n),0}x^n+A_{t(n),1}x^{n-}=h_{t(n)}(\xi^n),n\in\mathcal{N}\end{array}\right\}$$

How to solve the optimization model ?

- Standard software (e.g., X-PRESS, CPLEX)

- Decomposition methods for (very) large scale models (Ruszczyński 03)

Open questions:

- Which decomposition scheme should be used ?

- How to generate scenario trees for multi-stage models ?

- How to model and incorporate risk ?

Decomposition of (convex) stochastic programs

Direct or primal decomposition approaches:

- starting point: Benders decomposition based on both *feasibility* and *objective* cuts;
- variants: regularization to avoid an explosion of the number of cuts; nesting when applied to solve the dynamic programming equations on subtrees recursively; stochastic cuts.

Dual decomposition approaches:

(i) Scenario decomposition by Lagrangian relaxation of nonanticipativity constraints (solving the dual by bundle subgradient methods, augmented Lagrangian decomposition, splitting methods);
(ii) nodal decomposition by Lagrangian relaxation of dynamic constraints (same variants as in (i));
(iii) geographical decomposition by Lagrangian relaxation of coupling constraints (same variants as in (i)).

Mostly used for convex models: nested Benders decomposition, stochastic dual dynamic programming, stochastic decomposition and scenario decomposition.

Geographical decomposition

In electricity optimization the tree representation of the multistage stochastic program often has block separable structure

$$\min\left\{\sum_{n\in\mathcal{N}}\pi^n\sum_{i=1}^{k}\langle b^i_{t(n)}(\xi^n),x^n_i\rangle\;\middle|\;\begin{array}{l}x^n_i\in X^i_{t(n)}\\ \sum_{i=1}^{k}B^i_{t(n)}(\xi^n)x^n_i\geq g_{t(n)}(\xi^n)\\ A^i_{t(n),0}x^n_i+A^i_{t(n),1}x^{n-}_i=h^i_{t(n)}(\xi^n)\\ i=1,\dots,k,n\in\mathcal{N}\end{array}\right\}$$

Lagrange relaxation of coupling constraints: $L(x,\lambda)=$

$$\sum_{n\in\mathcal{N}}\pi^n\Big(\sum_{i=1}^{k}\langle b^i_{t(n)}(\xi^n),x^n_i\rangle+\langle\lambda^n,(g_{t(n)}(\xi^n)-\sum_{i=1}^{k}B^i_{t(n)}(\xi^n)x^n_i)\rangle\Big)$$

The dual problem

$$\max_{\lambda\geq 0}\;\inf_{x}L(x,\lambda)$$

decomposes into k geograhical subproblems and is solved by bundle subgradient methods. For nonconvex models the duality gap is typically small allowing for Lagrangian heuristics.

Stability and approximations

To have the model well defined, we assume $x \in L_{r'}(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^m)$ and $\xi \in L_r(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^s)$, where $r \geq 1$ and

$$r' := \begin{cases} \frac{r}{r-1} & , \text{ if only costs are random} \\ r & , \text{ if only right-hand sides are random} \\ 2 & , \text{ if costs and right-hand sides are random} \\ \infty & , \text{ if all technology matrices are random and } r = T. \end{cases}$$

Then nonanticipativity may be expressed as

$$x \in \mathcal{N}_{r'}(\xi)$$

$$\mathcal{N}_{r'}(\xi) = \left\{ x \in \times_{t=1}^{T} L_{r'}(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^{m_t}) : x_t = \mathbb{E}[x_t | \mathcal{F}_t(\xi)] \,, \, \forall t \right\},$$

i.e., as a subspace constraint, by using the conditional expectation $\mathbb{E}[\,\cdot\,|\mathcal{F}_t(\xi)]$ with respect to the σ-algebra $\mathcal{F}_t(\xi)$.

For $T = 2$ we have $\mathcal{N}_{r'}(\xi) = \mathbb{R}^{m_1} \times L_{r'}(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^{m_2})$.

$\longrightarrow$ infinite-dimensional optimization problem

Let F denote the objective function defined on $L_r(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^s) \times L_{r'}(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m) \to \mathbb{R}$ by $F(\xi, x) := \mathbb{E}[\sum_{t=1}^{T} \langle b_t(\xi_t), x_t \rangle]$, let

$$\mathcal{X}_t(x_{t-1}; \xi_t) := \{x_t \in X_t : A_{t,0}x_t + A_{t,1}(\xi_t)x_{t-1} = h_t(\xi_t)\}$$

denote the t-th feasibility set for every $t = 2, \ldots, T$ and

$$\mathcal{X}(\xi) := \{x \in L_{r'}(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^m) : x_1 \in X_1, x_t \in \mathcal{X}_t(x_{t-1}; \xi_t)\}$$

the set of feasible elements with input ξ.
Then the multi-stage stochastic program may be rewritten as

$$\min\{F(\xi, x) \,:\, x \in \mathcal{X}(\xi) \cap \mathcal{N}_{r'}(\xi)\}.$$

Let $v(\xi)$ denote its optimal value and, for any $\alpha \geq 0$,

$$\begin{aligned} S_\alpha(\xi) &:= \{x \in \mathcal{X}(\xi) \cap \mathcal{N}_{r'}(\xi) : F(\xi, x) \leq v(\xi) + \alpha\} \\ S(\xi) &:= S_0(\xi) \end{aligned}$$

denote the α-approximate solution set and the solution set of the stochastic program with input ξ.

Assumptions:

(A1) $\mathbb{E}[|\xi|^r] < \infty$,

(A2) The optimization model has relatively complete recourse,

(A3) The objective function is level-bounded locally uniformly at ξ.

Theorem: (Heitsch-Römisch-Strugarek 06)

Let (A1) – (A3) be satisfied and X_1 be bounded.

Then there exist positive constants L and δ such that

$$|v(\xi) - v(\tilde{\xi})| \leq L(\|\xi - \tilde{\xi}\|_r + d_{\mathrm{f},T-1}(\xi, \tilde{\xi}))$$

holds for all $\tilde{\xi} \in L_r(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^s)$ with $\|\tilde{\xi} - \xi\|_r \leq \delta$.

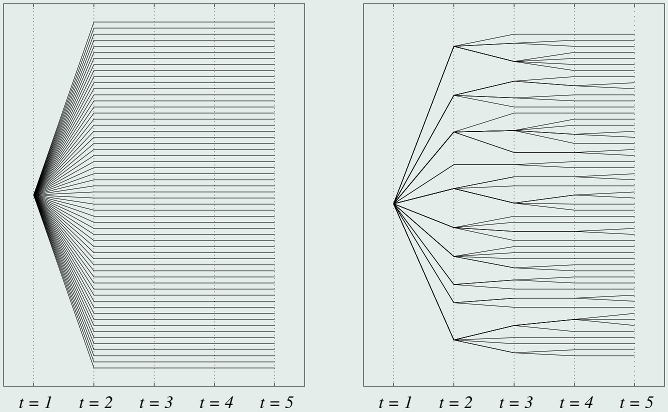
If $1 < r' < \infty$ and $(\xi^{(n)})$ converges to ξ in L_r and with respect to $d_{\mathrm{f},T}$, then any sequence $x_n \in S(\xi^{(n)})$, $n \in \mathbb{N}$, contains a subsequence converging weakly in $L_{r'}$ to some element of $S(\xi)$.

Here, $d_{\mathrm{f},\tau}(\xi, \tilde{\xi})$ denotes the filtration distance of ξ and $\tilde{\xi}$ defined by

$$d_{\mathrm{f},\tau}(\xi, \tilde{\xi}) := \sup_{\|x\|_{r'} \leq 1} \sum_{t=2}^{\tau} \|\mathbb{E}[x_t|\mathcal{F}_t(\xi)] - \mathbb{E}[x_t|\mathcal{F}_t(\tilde{\xi})]\|_{r'}.$$

Consequences for designing scenario trees

- If ξ_{tr} is a scenario tree process approximating ξ, one has to take care that $\|\xi - \xi_{\mathrm{tr}}\|_r$ and $d_{\mathrm{f},T}(\xi, \xi_{\mathrm{tr}})$ are small. This is achieved for the generation of scenario trees by recursive scenario reduction (Heitsch-Römisch 05).



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- Specific approximations $\tilde{\xi}$ of ξ are characterized such that an estimate of the form $|v(\xi) - v(\tilde{\xi})| \leq L \|\xi - \tilde{\xi}\|_r$ is valid (Küchler 07). Approximation schemes developed by Kuhn 05, Pennanen 05, Hochreiter-Pflug 07, Mirkov-Pflug 07 are based on approximating conditional distributions and also avoid filtration distances.

Risk functionals

Let $\mathcal{Z}$ denote a linear space of real random variables on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, e.g., $\mathcal{Z} = L_r(\Omega, \mathcal{F}, \mathbb{P})$, $1 \le r \le +\infty$. A functional $\mathcal{A} : \mathcal{Z} \to \overline{\mathbb{R}}$ is called a acceptability functional if it satisfies the following conditions for all z, $\tilde{z} \in \mathcal{Z}$:

(i) Monotonicity: $\mathcal{A}(z) \le \mathcal{A}(\tilde{z})$ if $z \le \tilde{z}$ $\mathbb{P}$-a.s.

(ii) Equivariance: $\mathcal{A}(z + r) = \mathcal{A}(z) + r$ for every $r \in \mathbb{R}$.

(iii) Concavity of $\mathcal{A}$ on $\mathcal{Z}$.

An acceptability functional is called coherent if it is positively homogeneous, i.e., $\rho(\lambda z) = \lambda \rho(z)$ for all $\lambda \ge 0$ and $z \in \mathcal{Z}$. Functionals $\rho := -\mathcal{A}$ and $\mathcal{D} = \mathbb{E} - \mathcal{A}$ are called capital and deviation risk functionals, if $\mathcal{A}$ is an acceptability functional.

Example: Average Value-at-Risk Rockafellar-Uryasev 02

$$AVaR_\alpha(z) = \frac{1}{\alpha} \int_0^\alpha VaR_x(z) dx = \max \left\{ x - \frac{1}{\alpha} \mathbb{E}([z - x]^-) : x \in \mathbb{R} \right\}$$

(Artzner-Delbaen-Eber-Heath 99, Föllmer-Schied 02, Pflug-Römisch 07)

Multiperiod (polyhedral) risk functionals

When a stochastic process $z = \{z_t\}_{t=1}^{T}$ in $\mathcal{Z} = \times_{t=1}^{T} L_r(\Omega, \mathcal{F}_t, \mathbb{P})$, $1 \leq r \leq +\infty$, is considered that evolves over time and unveils the available information with the passing of time, it may become necessary to use multiperiod risk functionals. Then we need to consider the filtration of σ-fields adapted to z, i.e., $\mathcal{F}_t = \sigma\{z_1, \ldots, z_t\}$, $t = 1, \ldots, T$, where $\mathcal{F}_1 = \{\emptyset, \Omega\}$.

A functional $\mathcal{A} : \mathcal{Z} \to \overline{\mathbb{R}}$ is called multi-period acceptability functional if for all $z, \tilde{z} \in \mathcal{Z}$

(i) Monotonicity: $\mathcal{A}(z) \leq \mathcal{A}(\tilde{z})$ if $z \leq \tilde{z}$ $\mathbb{P}$-a.s.

(ii) Equivariance: $\mathcal{A}(z_1, \ldots, z_t + c_t, \ldots, z_T) = \mathcal{A}(z_1, \ldots, z_T) + \mathbb{E}(c_t)$ for every $\mathcal{F}_{t-1}$-measurable c_t, $t = 2, \ldots, T$.

(iii) Concavity of $\mathcal{A}$ on $\mathcal{Z}$.

Example: Multi-period Average Value-at-Risk

$$mAVaR_{\alpha,\gamma}(z) = \sum_{t=2}^{T} \gamma_t \mathbb{E}(AVaR_{\alpha_t}(z_t | \mathcal{F}_{t-1}))$$

Definition: A multi-period acceptability functional $\mathcal{A}$ on $\mathcal{Z}$ is called polyhedral if there are $k_t \in \mathbb{N}$, $c_t \in \mathbb{R}^{k_t}$, $w_{t\tau} \in \mathbb{R}^{k_{t-\tau}}$, $\tau = 0, ..., t-1$, and polyhedral cones $V_t \subset \mathbb{R}^{k_t}$, $t = 1, \ldots, T$, such that

$$\mathcal{A}(z) = \sup \left\{ \mathbb{E}\left[\sum_{t=1}^{T} \langle c_t, v_t \rangle\right] \middle| \begin{array}{c} v_t \in L_p(\Omega, \mathcal{F}_t, \mathbb{P}; \mathbb{R}^{k_t}),\ v_t \in V_t \\ \sum_{\tau=0}^{t-1} \langle w_{t,\tau}, v_{t-\tau} \rangle = z_t,\ \ t = 1, \ldots, T \end{array} \right\}.$$

Remark: A convex combination of expectation and a multi-period polyhedral acceptability functional is again a multi-period polyhedral risk functional.

Polyhedral acceptability functionals preserve linearity and decomposition structures of optimization models.

(Eichhorn-Römisch 05, Pflug-Römisch 07)

Example: (Multi-period acceptability functional)

The following functional is polyhedral, satisfies (i) and (iii), but a weaker equivariance property.

$$\mathcal{A}_2(z) = \sup_{x \in \mathbb{R}} \left\{ x - \sum_{t=2}^{T} \frac{1}{\alpha_t} \mathbb{E}[(z_t - x)^-] \right\}.$$

Electricity Portfolio Management

We consider the electricity portfolio management of an electric power company. Its portfolio consists of the following positions:

- power production (based on company-owned thermal units),
- bilateral contracts,
- (physical) (day-ahead) spot market trading (e.g., EEX) and
- (financial) trading of derivatives (here, futures).

The time horizon is discretized into hourly intervals. The underlying stochasticity consists in a multivariate stochastic load and price process that is approximately represented by a finite number of scenarios. The objective is to maximize the total expected revenue. The portfolio management model is a large scale mixed-integer multistage stochastic program.

Objective: Maximizing the expected revenue and/or the acceptability of its production and trading decisions.

Electricity portfolio management

Stochastic process: $\{\xi_t = (d_t, \gamma_t, \alpha_t, \beta_t, \zeta_t)\}_{t=1}^T$
(electrical load, inflows, (fuel or electricity) prices) given as a (multivariate) scenario tree.
Mixed-integer programming problem:

$$\min \sum_{n\in\mathcal{N}} \pi^n \sum_{i=1}^{I} [C_i^n(p_i^n, u_i^n) + S_i^n(u_i)] \quad \text{s.t.}$$

$$p_{it(n)}^{\min} u_i^n \le p_i^n \le p_{it(n)}^{\max} u_i^n, \quad u_i^n \in \{0,1\}, \quad n \in \mathcal{N},\ i = 1,\dots,I,$$

$$u_i^{n-\tau} - u_i^{n-(\tau+1)} \le u_i^n, \quad \tau = 1,\dots,\bar{\tau}_i - 1,\ n \in \mathcal{N},\ i = 1,\dots,I,$$

$$u_i^{n-(\tau+1)} - u_i^{n-\tau} \le 1 - u_i^n, \quad \tau = 1,\dots,\underline{\tau}_i - 1,\ n \in \mathcal{N},\ i = 1,\dots,I,$$

$$0 \le v_j^n \le v_{jt(n)}^{\max},\ 0 \le w_j^n \le w_{jt(n)}^{\max},\ 0 \le l_j^n \le l_{jt(n)}^{\max},\ n \in \mathcal{N},\ j = 1,\dots,J,$$

$$l_j^n = l_j^{n-} - v_j^n + \eta_j w_j^n + \gamma_j^n, \quad n \in \mathcal{N},\ j = 1,\dots,J,$$

$$l_j^0 = l_j^{\text{in}}, \quad l_j^n = l_j^{\text{end}}, \quad n \in \mathcal{N}_T,\ j = 1,\dots,J,$$

$$\sum_{i=1}^{I} p_i^n + \sum_{j=1}^{J} (v_j^n - w_j^n) \ge d^n, \quad n \in \mathcal{N},$$

$$\sum_{i=1}^{I} (u_i^n p_{it(n)}^{\max} - p_i^n) \ge r^n, \quad n \in \mathcal{N}.$$

Here C_i^n are fuel or trading costs and S_i^n start-up costs of unit i at node $n \in \mathcal{N}$:

$$C_i^n(p_i^n, u_i^n) := \max_{l=1,\dots,\bar{l}} \{\, \alpha_{il}^n p_i^n + \beta_{il}^n u_i^n \,\} \qquad S_i^n(u_i) := \max_{\tau=0,\dots,\tau_i^c} \zeta_{i\tau}^n (u_i^n - \sum_{\kappa=1}^{\tau} u_i^{n-\kappa})$$

Electricity portfolio management: statistical models and scenario trees (Eichhorn-Römisch-Wegner 05)

For the stochastic input data of the optimization model here (yearly electricity and heat demand, and electricity spot prices), a statistical model is employed. It is adapted to historical data as follows:

- cluster classification for the intra-day (demand and price) profiles

- 3-dimensional time series model for the daily average values (deterministic trend functions, a trivariate ARMA model for the (stationary) residual time series)

- simulation of an arbitrary number of three dimensional sample paths (scenarios) by sampling the white noise processes for the ARMA model and by adding on the trend functions and matched intra-day profiles from the clusters afterwards.

- generation of scenario trees as in Heitsch-Römisch 05.

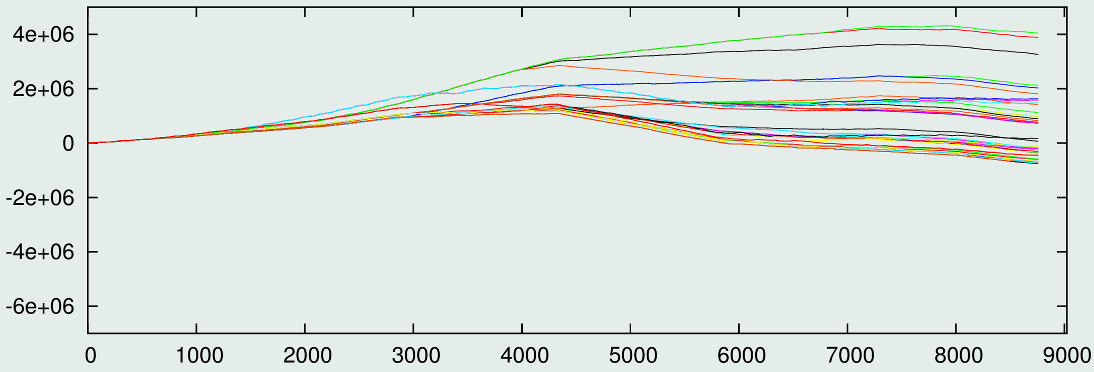
Electricity portfolio management: Results

Test runs were performed on real-life data of the utility DREWAG Stadtwerke Dresden GmbH leading to a linear program containing $T = 365 \cdot 24 = 8760$ time steps, a scenario tree with 40 demand-price scenarios and about $N = 150.000$ nodes. The objective function is of the form

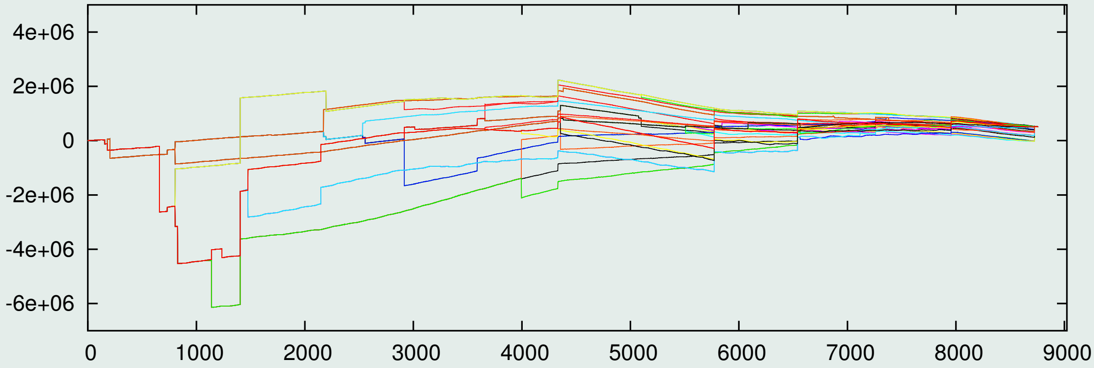
$$\text{Maximize} \quad \gamma \mathcal{A}(z) + (1-\gamma)\mathbb{E}(z_T)$$

with a (multiperiod) acceptability functional $\mathcal{A}$ and coefficient $\gamma \in [0,1]$ ($\gamma = 0$ corresponds to no risk). $\mathbb{E}(z_T)$ denotes the overall expected revenue.

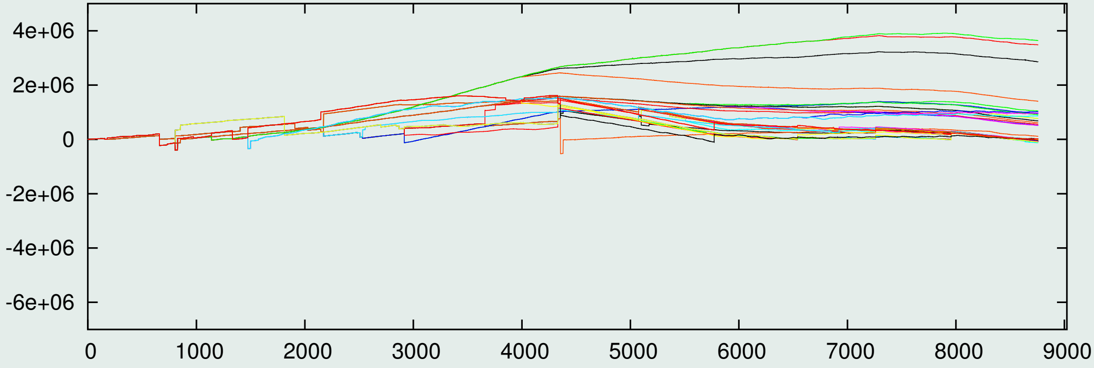
The model is implemented and solved with ILOG CPLEX 9.1 on a 2 GHz Linux PC with 1 GB memory.



Total revenue and $\gamma = 0$



Total revenue with $AVaR_{0.05}$ and $\gamma = 0.9$



Total revenue with $\mathcal{A}_2$ and $\gamma = 0.9$

Some further developments and challenges

- Decomposition of multistage stochastic programs with recombining scenario trees (within a non-Markovian framework) (Küchler-Vigerske 07).
- Stochastic dominance constraints as alternatives of risk functionals in stochastic programs (Dentcheva-Ruszczyński 03, Gollmer-Neise-Schultz 07).
- Structural properties, stability and scenario trees for mixed-integer multi-stage stochastic programs.

References:

Artzner, P., Delbaen, F., Eber, J.-M., Heath, D.: Coherent measures of risk, *Mathematical Finance* 9 (1999), 203–228.

Bank, B., Mandel, R.: *Parametric Integer Optimization*, Akademie Verlag, Berlin 1988.

Blair, C. E., Jeroslow, R. G.: The value function of a mixed integer program I, *Discrete Mathematics* 19 (1977), 121–138.

Carøe, C., Schultz, R.: Dual decomposition in stochastic integer programming, *Operations Research Letters* 24 (1999), 37–45.

Dentcheva, D., Ruszczyński, A.: Stochastic optimization with dominance constraints, *SIAM Journal on Optimization* 14 (2003), 548–566.

Dupačová, J., Consigli, G., Wallace, S. W.: Scenarios for multistage stochastic programs. *Annals Operations Research* 100 (2000), 25–53.

Eichhorn, A., Römisch, W.: Polyhedral risk measures in stochastic programming, *SIAM Journal on Optimization* 16 (2005), 69–95.

Eichhorn, A., Römisch, W.: Stochastic integer programming: Limit theorems and confidence intervals, *Mathematics of Operations Research* 32 (2007), 118–135.

Eichhorn, A., Römisch, W., Wegner, I.: Mean-risk optimization of electricity portfolios using multiperiod polyhedral risk measures, *IEEE St. Petersburg Power Tech* 2005.

Föllmer, H., Schied, A.: Stochastic Finance: An Introduction in Discrete Time, de Gruyter, Berlin 2004 (Second Edition 2004).

Gollmer, R., Neise, F., Schultz, R.: Stochastic Programs with first-order dominance constraints induced by mixed-integer linear recourse, *Stochastic Programming E-Print Series* 4 (2007) `www.speps.org`.

Heitsch, H., Römisch, W., Strugarek, C.: Stability of multistage stochastic programs, *SIAM Journal Optimization* 17 (2006), 511–525.

Heitsch, H., Römisch, W.: Scenario tree modeling for multistage stochastic programs, Preprint 296, DFG Research Center MATHEON, Berlin 2005, and submitted to *Mathematical Programming*.

Heitsch, H., Römisch, W.: Stability and scenario trees for multistage stochastic programs, Preprint 324, DFG Research Center MATHEON, Berlin 2006, and submitted to Stochastic Programming - The State of the Art (G. Dantzig, G. Infanger eds.).

Heitsch, H., Römisch, W.: A note on scenario reduction for two-stage stochastic programs, Operations Research Letters (2007) (to appear).

Henrion, R., Küchler, C., Römisch, W.: Scenario reduction in stochastic programming with respect to discrepancy distances, Stochastic Programming E-Print Series 13 (2006) and *Computational Optimization and Applications* (to appear).

Henrion, R., Römisch, W.: Hölder and Lipschitz stability of solution sets in programs with probabilistic constraints, *Mathematical Programming* 100 (2004), 589–611.

Henrion, R., Strugarek, C.: Convexity of chance constraints with independent random variables, Stochastic Programming E-Print Series 9 (2006) and *Computational Optimization and Applications* (to appear).

Hochreiter, R., Pflug, G. Ch.: Financial scenario generation for stochastic multi-stage decision processes as facility location problem, *Annals of Operations Research* 152 (2007), 257–272.

Kall. P.: *Stochastic Linear Programming*, Springer, Berlin, 1976.

Küchler, C.: On stability of multistage stochastic programs, Preprint, Humboldt-University Berlin, 2007.

Küchler, C., Vigerske, S.: Decomposition of multistage stochastic programs with recombining scenario trees, Preprint, Humboldt-University Berlin, 2007.

Kuhn, D.: *Generalized Bounds for Convex Multistage Stochastic Programs*, Lecture Notes in Economics and Mathematical Systems, Vol. 548, Springer, 2005.

Mirkov, R., Pflug, G. Ch.: Tree approximations for dynamic stochastic programs, Preprint, University of Vienna, 2007.

Nemirovski, A., Shapiro, A.: Convex approximations of chance constrained programs, *SIAM J. Optim.* 17 (2006), 969–996.

Pennanen, T.: Epi-convergent discretizations of multistage stochastic programs, *Mathematics of Operations Research* 30 (2005), 245–256.

Pflug, G. Ch., Römisch, W.: Modeling, Measuring and Managing Risk, World Scientific, Singapore, 2007 (to appear).

Prekopa, A.: *Stochastic Programming*, Kluwer, Dordrecht, 1995.

Rachev, S. T., Römisch, W.: Quantitative stability in stochastic programming: The method of probability metrics, *Mathematics of Operations Research* 27 (2002), 792–818.

Rockafellar, R. T., Uryasev, S.: Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance* 26 (2002), 1443–1471.

Römisch, W.: Stability of stochastic programming problems, in *Stochastic Programming* (Ruszczyński, A., Shapiro, A., eds.), Elsevier, Amsterdam, 2003, 483–554.

Römisch, W., Wets, R. J-B: Stability of ε-approximate solutions to convex stochastic programs, *SIAM Journal on Optimization* (to appear).

Römisch, W., Vigerske, S.: Quantitative stability of fully random mixed-integer two-stage stochastic programs, Preprint 367, DFG Research Center MATHEON, Berlin 2007.

Ruszczyński, A.: Decomposition methods, in *Stochastic Programming* (Ruszczyński, A., Shapiro, A., eds.), Elsevier, Amsterdam, 2003, 141–211.

Ruszczyński, A., Shapiro, A. (eds.): *Stochastic Programming*, Handbook of Operations Research and Management Science Vol. 10, Elsevier, Amsterdam, 2003.

Schultz, R.: On structure and stability in stochastic programs with random technology matrix and complete integer recourse, *Mathematical Programming* 70 (1995), 73–89.

Schultz, R.: Rates of convergence in stochastic programs with complete integer recourse, *SIAM Journal on Optimization* 6 (1996), 1138–1152.

Schultz, R.: Stochastic programming with integer variables, Mathematical Programming 97 (2003), 285–309.

Sen, S.: Algorithms for stochastic mixed-integer programming models, in *Discrete Optimization* (K. Aardal, G. L. Nemhauser, R. Weissmantel eds.), Handbooks in Operations Research and Management Science Vol. 12, Elsevier, Amsterdam 2005, 515–558.

Shapiro, A.: Monte Carlo sampling methods, in *Stochastic Programming* (Ruszczyński, A., Shapiro, A., eds.), Elsevier, Amsterdam, 2003, 353–425.

van der Vlerk, M. H.: Stochastic programming with integer recourse, Ph.D. Thesis, University of Groningen, 1995.

Walkup, D., Wets, R. J-B: Lifting projections of convex polyhedra, *Pacific Journal of Mathematics* 28 (1969), 465–475.

Wets, R. J-B: Stochastic programs with fixed recourse: The equivalent deterministic program, *SIAM Review* 16 (1974), 309–339.