Uncertainty in Electricity Markets: Capacity decisions in electricity production under risk aversion & risk trading

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Review of perfectly competitive capacity equilibria
Risk aversion and risk trading
Example

Outline

ICCP 2014, Berlin | Uncertainty in Electricity Markets

Motivation from electricity capacity equilibria under uncertainty & perfect competition

Electricity capacity expansion is kind of stochastic equilibrium

Invest Today: In stage 1, generator (genco) makes investments in different technologies (power plants)

- Later consider 3 technologies: Coal Steam Turbine (CST), Combined Cycle Gas Turbine (CCGT), Gas Turbine (GT)
- (Can deal with any no. of gencos, consumers, technologies)

Operate in Uncertain Tomorrow: In stage 2, operating cost of portfolio of plants is stochastic, depends on scenario ω

- Fuel & C prices, weather (demand) depend on ω — stochastic data
- Perfect competition sets price P_ω that clears energy market
  - Endogenous to equilibrium
  - Agents do not see their affect on price: perfect competition

Start with review of two stage capacity equilibrium

This tutorial is confined to perfectly competitive (PC) markets.

1. PC capacity equilibrium — deterministic case
   - Economic interpretation of **two stage optimization** as **capacity equilibrium**
   - Basis for MARKAL — long term capacity planning
2. PC risk neutral (RN) capacity equilibrium
   - To assess uncertain outcomes, take an **average**
   - Economic interpretation of **two stage stochastic optimization** as **stochastic capacity equilibrium**
   - Basis for **stochastic** MARKAL

What this tutorial doesn't cover

This tutorial could be extended, in various ways, to discuss

- electricity over a congested transmission network
- markets in commodities other than electricity
- multi stage capacity or other staged stochastic equilibrium problems
- Cournot rather than perfectly competitive markets

The analysis presented here won't extend naturally to any kind of nonconvexity

- nonconvex production cost
- strategic capacity decisions, eg, multi leader multi follower games or EPECs

Outline

Genco's two stage capacity problem — deterministic case

Genco minimises **Investment** + **Operating** costs:

$$\min_x \sum_j \mathbf{I_j}(\mathbf{x_j}) + \mathbf{Q_g}(\mathbf{x}, \mathbf{p}) \quad \text{s.t.} \quad x \in \mathcal{X}$$

Stage 1, investment

- There are $j = 1, \ldots, J$ energy technologies (plant types)
- $I_j(x_j) :=$ convex investment cost of plant j, capacity x_j
- $\mathcal{X} :=$ closed convex set of feasible technology designs in $\mathbb{R}^J$, any $x = (x_j)_j \in \mathcal{X}$ specifies portfolio of plants

Stage 2, cost of production

- $p =$ future price
- $Q_g(x, p) :=$ generator's operating costs net of revenue, or negative profit

Stage 2: Spot market — deterministic case

Fix plant capacities $x = (x_j)_j \geq 0$.

Genco optimises production $y = (y_j)_j$ given capacity x, price p

$$\mathbf{Q_g}(\mathbf{x}, \mathbf{p}) := \min_y \sum_j c_j(y_j) - p \sum_j y_j \quad \text{s.t.} \quad y_j \quad 0 \leq y_j \leq x_j, \forall j$$

where $c_j(y_j) :=$ convex production cost of technology j

Consumer optimises unserved (or lost) load u given price p

$$\min_u (\hat{p} - p)u \quad \text{s.t.} \quad u \geq 0$$

where $\hat{p} :=$ positive price cap or Value of Lost Load (exog. data)
$d :=$ inelastic demand (exog. data)

Price of electricity p clears the market given y, u

$$0 \leq \sum_j y_j + u - d \quad \perp \quad p \geq 0$$

Stage 2: Explore meaning of Perfect Competition (PC) for producer

Exercise 1. Write down stationary conditions of genco's problem

$$\min_{y} \sum_j c_j(y_j) - p \sum_j y_j \quad \text{s.t.} \quad 0 \le y \le x$$

Hint: Introduce a KKT multiplier $\mu = (\mu_j)_j$ for $y \le x$.
But you don't need a multiplier for $y \ge 0$, just complementarity conditions

Stage 2: Notes on Perfect Competition

Suppose capacity $x = (x_j) > 0$ and market price $p > 0$ are known.
PC means:

- Each agent only responds to price
  - Doesn't respond to actions of others
  - Doesn't try to affect price
- Genco wants to produce as much as possible up to the point where the last unit of production costs as much as p
  - Marginal cost pricing
  - Relies on marginal production cost $c_j'(y_j)$ being non-decreasing in $y_j \ge 0$ ($\Leftrightarrow$ convexity of c_j).
  - May give different production level for each technology j

Stage 2: A system view of production & consumption

A system or social planning view:
Minimize cost of meeting demand (allowing for lost load),

$$\begin{array}{ll} \min\limits_{y,u} & \sum_j c_j(y_j) + \hat{p}u \\ \text{s.t.} & 0 \le y \le x, \;\; 0 \le u \\ & 0 \;\le\; \sum_j y_j \,+\, u \,-\, d \end{array}$$

Exercise 2. Write down stationary conditions for system problem.
Compare this KKT/complementarity conditions for spot market ...

Stage 2: Deterministic spot market $\Leftrightarrow$ System optimisation

Assume c_j is convex; $\hat{p} > 0$; $d > 0$

Economics 101: Perfectly competitive market
Standard welfare economics [Samuelson-47, Tirole-98] says

Theorem (Spot equilibrium $\Longleftrightarrow$ Spot cost minimization)

For fixed capacities $x \ge 0$:

y, u, p is spot equilibrium in PC market $\Longleftrightarrow$ y, u solve

$$\begin{array}{ll} \min\limits_{y,u} & \sum_j c_j(y_j) + \hat{p}u \\ \text{s.t.} & 0 \le y \le x, \;\; 0 \le u \\ & 0 \;\le\; \sum_j y_j \,+\, u \,-\, d \end{array}$$

and p is KKT multiplier for demand constraint.

Notes. 1. Existence of a solution follows ...
2. Thm. extends to any no. of commodities with convex costs

Perfectly competitive capacity equilibrium

Assume $\mathcal{X} \subset \mathbb{R}^J_+$ is nonempty, closed & convex
Assume c_j is convex; $\hat{p} > 0$; $d > 0$

Economics 102: PC two stage capacity equilibrium

Genco sets investment $x = (x_j)$ & production $y = (y_j)$ given p:

$$\min_{x,y} \sum_j I_j(x_j) + \sum_j c_j(y_j) - p \sum_j y_j \quad \text{s.t.} \quad x \in \mathcal{X},\ 0 \le y \le x$$

Two stage stochastic program with recourse

Consumer sets unserved load u given p:

$$\min_u (\hat{p} - p)u \quad \text{s.t.} \quad u \ge 0$$

where $\hat{p}$:= price cap or Value of Lost Load (exog. data)
d := inelastic demand (exog. data)

Price of electricity p clears the market given y, u

$$0 \le \sum_j y_j + u - d \quad \perp \quad p \ge 0$$

PC capacity equilibrium ⇔ PC capacity optimization

Assume $\mathcal{X} \subset \mathbb{R}^J_+$ is nonempty, closed & convex
Assume c_j is convex; $\hat{p} > 0$; $d > 0$

Economics 102: PC two stage capacity equilibrium

Genco sets investment x & production y given p

Consumer sets unserved load u given p

Spot price p clears market given y, u

Standard welfare economics says

Theorem (Capacity equilibrium ⇔ Capacity optimization)

x is a capacity equilibrium (for some y, u, p) $\Longleftrightarrow$ x solves

$$\begin{aligned} \min_{x,y,u} \quad & \textstyle\sum_j I_j(x_j) + \sum_j c_j(y_j) + \hat{p}u \\ \text{s.t.} \quad & 0 \le y \le x,\ \ 0 \le u \\ & 0 \ \le\ \textstyle\sum_j y_j + u - d \quad \text{s.t.} \quad x \in \mathcal{X} \end{aligned}$$

and p is KKT multiplier for demand constraint.

Proof of theorem

Exercise 3.
Write down the KKT conditions for the system capacity optimization problem. Observe that these comprise
(i) KKT conditions for the genco,
(ii) KKT conditions for the consumer, and
(iii) market clearing (pricing).

QED

Outline

Genco's two stage RN capacity problem

Genco minimises **Investment** + **Average Operating** costs:

$$\min_x \sum_j \mathbf{I_j}(\mathbf{x_j}) + \mathbb{E}_{\mathbf{\Pi_0}}\left[\mathcal{Q}_\mathbf{g}(\mathbf{x},\mathbf{P})\right] \quad \text{s.t.} \quad x \in \mathcal{X}$$

Stage 1, investment

- There are $j = 1, \ldots, J$ energy technologies (plant types)
- $I_j(x_j) :=$ convex investment cost of plant j, capacity x_j
- $\mathcal{X}$:= closed convex set of feasible technology designs, any $x = (x_j) \in \mathcal{X}$ specifies portfolio of plants

Stage 2, uncertain cost of production

- $\Pi_0 = (\Pi_{0\omega})_\omega$ = probability density (PD) over scenarios $\omega = 1, .., K$
- $P = (P_\omega)_\omega$ = prices in all future scenarios
- $\mathcal{Q}_g(x,P) := (\mathcal{Q}_{g\omega}(x,P_\omega))_\omega$ has expectation $\mathbb{E}_{\Pi_0}[\mathcal{Q}_g(x,P)]$
  $\mathcal{Q}_{g\omega}(x,P_\omega) :=$ generator's operating costs net of revenue, or negative profit, in scenario ω

Stage 2 in scenario ω: Deterministic spot market

Fix plant capacities $x \geq 0$ and spot market scenario ω.

Genco optimises production $Y_\omega = (Y_{j\omega})_j$ given cap. x, price P_ω

$$\mathcal{Q}_{\mathbf{g}\omega}(\mathbf{x},\mathbf{P}_\omega) := \min_{Y_\omega} \sum_j C_{j\omega}(Y_{j\omega}) - P_\omega \sum_j Y_{j\omega} \quad \text{s.t.} \quad 0 \leq Y_{j\omega} \leq x_j, \forall j$$

where $C_{j\omega}(y_j) :=$ convex production cost of technology j

Consumer optimises unserved (lost) load U_ω given price P_ω

$$\mathcal{Q}_{\mathbf{c}\omega}(\mathbf{P}_\omega) := \min_{U_\omega} (\hat{p} - P_\omega)U_\omega \quad \text{s.t.} \quad U_\omega \geq 0$$

where $\hat{p}$:= positive price cap or Value of Lost Load (exog. data)
D_ω := inelastic demand (exog. data)

Price of electricity P_ω clears the market given Y_ω, U_ω

$$0 \leq \sum_j Y_{j\omega} + U_\omega - D_\omega \quad \perp \quad P_\omega \geq 0$$

Stage 2 in scenario ω: Spot market ⇔ System optimisation

Assume C_{jw} as convex; $\hat{p} > 0$; $D_\omega > 0$

Economics 101: Perfectly competitive market
This is the *deterministic case*, as above, with subscript ω:

Theorem (Spot equilibrium $\iff$ Spot cost minimization)

For fixed plant capacities $x \geq 0$ and spot market scenario ω: $Y_\omega = (Y_{j\omega})_j$, U_ω, P_ω is spot equilibrium $\iff$ Y_ω, U_ω solve

$$\begin{aligned} \mathcal{Q}_{\mathbf{s}\omega}(\mathbf{x}) \; := \; \min_{Y_\omega,U_\omega} \quad & \textstyle\sum_j C_{j\omega}(Y_{j\omega}) + \hat{p}U_\omega \\ \text{s.t.} \quad & 0 \leq Y_\omega \leq x, \; 0 \leq U_\omega \\ & 0 \leq \textstyle\sum_j Y_{j\omega} + U_\omega - D_\omega \end{aligned}$$

and P_ω is KKT multiplier for demand constraint.

RN generator's capacity problem

Assume $\mathcal{X} \subset \mathbb{R}^J_+$ is nonempty, closed & convex
Assume $C_{j\omega}$ is convex; $\hat{p} > 0$; $D_\omega > 0$ ($\forall\, j, \omega$)

Genco chooses levels of investment $x = (x_j)_j$ and production $Y_\omega = (Y_{j\omega})_j$ for all j & ω, given all future prices $P = (P_\omega)$:

$$\begin{aligned} \min_{x,Y=(Y_\omega)} \quad & \sum_j I_j(x_j) + \sum_\omega \Pi_{0\omega}\left(\sum_j C_{j\omega}(Y_{j\omega}) - P_\omega \sum_j Y_{j\omega}\right) \\ \text{s.t.} \quad & x \in \mathcal{X} \\ & 0 \leq Y_\omega \leq x \end{aligned}$$

Or, more compactly:

$$\min_x \sum_j I_j(x_j) + \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_g(x,P)\right] \quad \text{s.t.} \quad x \in \mathcal{X}$$

where $\mathcal{Q}_g(x,P) = (\mathcal{Q}_{g\omega}(x,P_\omega))_\omega$

RN capacity equilibrium ⇔ RN capacity optimization

Assume $\mathcal{X} \subset \mathbb{R}^J_+$ is nonempty, closed & convex
Assume $C_{j\omega}$ is convex; $\hat{p} > 0$; $D_\omega > 0$ $(\forall\ j, \omega)$

Economics 102: PC RN two stage capacity equilibrium

Genco sets investment x & production Y_ω $\forall\omega$ given $P = (P_\omega)$:

Consumer sets unserved load U_ω for all ω given P

Spot price P_ω clears market for all ω given all Y_ω, U_ω

Theorem (RN capacity equilibrium ⇔ RN capacity optimization)

Then x is a RN capacity equilibrium (for some (Y_ω), (U_ω), (P_ω)) $\iff$ x solves

$$\min_x \textstyle\sum_j I_j(x_j) + \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_s(x)\right] \quad \text{s.t.} \quad x \in \mathcal{X}$$

where $\mathcal{Q}_s(x) \ = \ \left(\mathcal{Q}_{s\omega}(x)\right)_\omega$

This is explored in detail in [Gurkan-etal-13]

Proof of theorem — try on your own

Exercise 4.

1. Write RN capacity optimization as 2 stage stochastic program
   - Use formulation of $\mathcal{Q}_{s\omega}(x)$ above
   - Variables of 2 stage problem are x, Y_ω and U_ω $\forall\omega$
2. Write down the KKT conditions for two stage optimization problem.
3. Observe that these comprise (i) KKT conditions for the Generator, (ii) KKT conditions for the Consumer in each scenario ω, and (iii) market clearing (spot pricing) in each scenario ω.

QED

Why is optimization important?

Optimization is important because — in the convex case (here) — it leads to **tractable** problems
Economic consistency of social planning, or system optimization, with agents' investment decisions makes it **credible**

MARKAL, MARket ALlocation, [Fishbone-Abilock-81] is prototype software package implementing the theorem above

- Long term planning model under perfect competition
- Deterministic stagewise linear program (optimization) when functions are piecewise linear
- googlescholar: 4.3k publications mention MARKAL
- Stochastic MARKAL [Kanudia-Loulou-98]
- General MARKAL review [Seebregts-etal-01]

The tutorial changes gears

So far, we have looked at models and mechanics of PC markets

- Review of RN capacity equilibria
  - Theme: equilibrium ⇔ system optimization
  - Including deterministic spot market

From here on we present various equilibria models under risk aversion and sketch results

- Risk aversion via coherent risk measures (CRMs)
  - Model of capacity equilibria under risk aversion
  - Model of risk markets
- Risky capacity equilibrium, a models that combines
  - CRMs
  - risk trading

Outline

Risk averse agents

In risk neutral capacity equilibrium, given x and $P = (P_\omega)_{\omega=1}^K$,

- cost to generator of 2nd stage $= \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_g(x, P)\right]$
- cost to consumer of 2nd stage $= \mathbb{E}_{\Pi_0}\left[\mathcal{Q}_c(P)\right]$

where Π_0, $\mathcal{Q}_g(x, P)$, $\mathcal{Q}_c(P)$ have dimension K

What if expectation is replaced by **coherent risk measure** (CRM), $r : \mathbb{R}^K \to \mathbb{R}$?

[Artzner-et-al-99] characterise r as a **worst case expectation**:

- $r(Z) = \max_{\Pi \in \mathcal{D}} \mathbb{E}_\Pi[Z]$ for any cost $Z \in \mathbb{R}^K$ . . . risk averse
- $\mathcal{D}$ is nonempty closed convex set of PDs; **risk set** of r
- CVaR/AVaR/E Tail Loss is famous CRM in optimization [Rock-Uryas-00]
- Good Deal is CRM adapted from [Cochrane-Saa-Requejo-00]

Example of risk neutral genco

Suppose

- Genco's plant portfolio (x) is fixed
- There are 4 equally likely market scenarios
- Genco's uncertain cost $Z = (-6, -2, -1, 1)$.
  $\Rightarrow$ mean cost $\mathbb{E}[Z] = -8/4 = -2$ ... or profit is $+2$.

Mean = value for **fully diversified** or **risk neutral** plant owner:

- Many investments hence if lose on one then win on another
- C.f., Central Limit Theorem: with many uncertain investments, actual outcome $\approx$ mean with high confidence

Example of risk averse genco: $\mathrm{cV@R}$

What about **risk**?
Eg, manager of genco is risk averse if bonused on annual results

- Genco's uncertain cost is $Z = (-6, -2, -1, 1)$ with equally likely outcomes
  - $\mathrm{mean}(Z) = -2$ . . . positive profit
- Suppose genco's **risked valuation** is mean of worst two outcomes. This is 50% **conditional value at risk**
- Risked cost is $\mathrm{cV@R}_{0.5}(Z) = (-1 + 1)/2 = 0$.
  Portfolio is **breakeven**

More generally, $\mathrm{cV@R}_\gamma$ takes conditional value over worst $\gamma\%$ of outcomes, given $\gamma \in (0, 1)$.

Exercises to try on your own

Assume uniform distribution over 4 outcomes $\left(\Pi_0 = \frac{1}{4}(1,\,1,\,1,\,1)\right)$.
Take any $Z \in \mathbb{R}^4$.

Exercise, c.f. [Rock-Uryas-00].
Show that linear program below has optimal value $= \mathrm{cV@R}_{0.5}(Z)$:

$$\begin{array}{cl} \min\limits_{t\in\mathbb{R},\,\eta\in\mathbb{R}^4} & t + \dfrac{1}{2}\displaystyle\sum_{\omega=1}^{4} \eta_\omega \\ \text{s.t.} & \eta_\omega \geq 0,\ \eta_w \geq Z_w - t,\ \forall\omega \end{array}$$

Exercise.
Show that $\mathrm{cV@R}_{0.5}$ $\left(\text{or } \mathrm{cV@R}_{\Pi_0,0.5}\right)$ has risk set

$$\mathcal{D} \;=\; \mathrm{conv}\{\tfrac{1}{2}(1,1,0,0),\ \tfrac{1}{2}(1,0,1,0),\ldots,\ \tfrac{1}{2}(0,0,1,1)\}.$$

That is, $\mathrm{cV@R}_{0.5}(Z) \;=\; \max_{\Pi\in\mathcal{D}}(Z)$.

Suggestion.
Look up $\mathrm{cV@R}$ in [Rock-Uryas-00] and see how it is analysed in [Rusz-Shap-06]. Caution, these papers use very different notation

Review of perfectly competitive capacity equilibria
Risk aversion and risk trading
Example
Coherent risk measures
Risk trading and risk markets
Risky capacity equilibria in a complete risk market

Risk averse capacity equilibrium

Risk averse agents

- Genco's CRM is $r_g(.) = \max_{\Pi\in\mathcal{D}_g} \mathbb{E}_\Pi[\cdot]$
- Consumer's CRM is $r_c(.) = \max_{\Pi\in\mathcal{D}_c} \mathbb{E}_\Pi[\cdot]$

Risk averse capacity equilibrium:

Genco sets investment x & production $Y_\omega\ \forall\omega$ given $P = (P_\omega)$:

$$\min_x \textstyle\sum_j I_j(x_j) + \mathbf{r_g}\Big(\mathcal{Q}_g(x,P)\Big) \quad \text{s.t.} \quad x \in \mathcal{X}$$

Consumer sets unserved load U_ω for all ω given P

- $\mathbf{r_c}\big(\mathcal{Q}_c(P)\big)$ is risked cost of consumption over all ω

Spot price P_ω clears market for all ω given all Y_ω, U_ω

This is **inescapably equilibrium not convex optimization**

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There exists a solution of a risk averse capacity equilibrium

Theorem (Ehrenmann-Smeers-11a)

Under the same conditions given for RN case:
There exists an investment equilibrium x, along with spot market equilibria (Y_ω), (U_ω) (P_ω), of the risk averse capacity equilibrium.

Proof is via Kakutani's fixed point theorem.

But **equilibrium solutions are tricky**

- How does a solution relate to risk neutral (optimization) case?
- How to find/interpret multiple equilibria?
- Computationally can be hard to find any solution
  - Use `PATH`: Write equilibrium as large complementarity problem (use KKT conditions for genco & consumer)
  - Diagonalisation/Round Robin/Jacobi iteration: solve each equilibrium condition in turn and update [recent Ferris-Wathen]

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Outline

Introducing trading of financial products

Suppose capacity $x = (x_j) > 0$ and market prices $P = (P_\omega)$ are known.
Genco has risky cost $Z_g = \mathcal{Q}_g(x, P)$. **How to manage risk?**

- Genco is risk averse: $r_g(Z_g) = \max_{\Pi \in \mathcal{D}_g} \mathbb{E}_\Pi[Z_g]$.
- What if Genco could buy contracts or securities $W_g \in \mathbb{R}^K$ to change $r_g(Z_g)$ to $r_g(Z_g - W_g)$
  - Eg, natural gas futures to hedge cost of CCGT or GT
  - May buy a bundle of hedges: W_g is a vector

Eg, there are $K = 4$ equally likely scenarios, and $Z_g = (-6, -2, -1, 1)$
- Taking $W_g = (0, 0, 0, 2)$ gives $Z_g - W_g = (-6, -2, -1, \mathbf{-1})$
  Ie, your worst outcome is not so bad
- Value gained is $r_g((-6, -2, -1, 1)) - r_g((-6, -2, -1, \mathbf{-1}))$
- **Q: What is this worth?**

A market for risk

Consumer has similar question:
- $Z_c = \mathcal{Q}_c(P)$, quantified as $r_c(Z_c)$
- What to pay for W_c to change $r_c(Z_c)$ to $r_c(Z_c - W_c)$?

Economics answer:
A market gives **price of risk** $P^r \in \mathbb{R}^K$,
- Genco pays $P^r[W_g] := \sum_\omega P^r_\omega W_{g\omega}$ & consumer pays $P^r[W_c]$
- Trades balance: $W_g + W_c = 0$

A risk market puts a price on risk (securities)

Where does price of risk P^r come from? Risk market:
- Genco finds W_g: $\min_{W_g} P^r[W_g] + r_g(Z_g - W_g)$
- Consumer finds W_c: $\min_{W_c} P^r[W_c] + r_c(Z_c - W_c)$
- Price of risk P^r clears market: $W_g + W_c = 0$

[Arrow-60] applied **Economics 101** to the risk market to show risk equilibrium ⇔ system risk minimization,

$$r_s(Z_g, Z_c) := \min_{W_g, W_c} r_g(Z_g - W_c) + r_c(Z_c - W_c) \quad \text{s.t.} \quad W_g + W_c = 0$$

where P^r is Lagrange multiplier of trade balance constraint

Note. A basic difference between this and previous theorem on spot market is that financial products can have negative quantities

Risk market under CRMs

Risk market:
- Genco finds W_g: $\min_{W_g} P^r[W_g] + r_g(Z_g - W_g)$
- Consumer finds W_c: $\min_{W_c} P^r[W_c] + r_c(Z_c - W_c)$
- Price of risk P^r clears market: $W_g + W_c = 0$

Finance lit. starting with [Heath-Ku-04], c.f. [R-Smeers-11a], gives

Theorem (Finance: Risk market under CRMs $\iff$ System CRM)

Let $r_g = \max_{\Pi \in D_g} \mathbb{E}_\Pi[\cdot]$ and $r_c = \max_{\Pi \in D_c} \mathbb{E}_\Pi[\cdot]$, ie, CRMs,
Finding risk equilibrium $\implies$ evaluating system risk using
system CRM $\quad r_s(Z_g + Z_c)$
where $\quad r_s(\cdot) := \max_{\Pi \in \mathcal{D}_s} \mathbb{E}_\Pi[\cdot]$
and **system risk set** $\quad \mathcal{D}_s := \mathcal{D}_g \cap \mathcal{D}_c$ *is nonempty.*
Converse holds under mild technical conditions,
eg, risk sets polyhedral or relative interiors intersect

Complete risk market

The last result assumes any uncertainty is priced in risk market

- Terminology: Risk market is **complete**
- Mathematical meaning: $W_g, W_c \in \mathbb{R}^K$
- Practical implication: all significant risks can be contracted or covered by mixing contracts

We'll return to this assumption later . . .

Outline

Risky capacity equilibrium

Introduce risk trading into capacity equilibrium

Risky capacity equilibrium:

Genco sets $W_g \in \mathbb{R}^K$ and $x, Y_\omega \in \mathbb{R}^J \ \forall\omega$ given $P = (P_\omega)$:

$$\min_{x,W_g} \sum_j I_j(x_j) + P^{\mathrm{r}}[W_g] + r_g\Big(\mathcal{Q}_g(x,P) - W_g\Big) \quad \text{s.t.} \quad x \in \mathcal{X}$$

Consumer set $W_c \in \mathbb{R}^K$ and $U_\omega \in \mathbb{R} \ \forall\omega$ given P:

$$\min_{W_c} P^{\mathrm{r}}[W_c] + r_c\big(\mathcal{Q}_c(P) - W_c\big)$$

Spot price P_ω clears spot market $\forall\omega$ given all Y_ω, U_ω

Price of risk P^{r} clears risk market: $W_g + W_c = 0$

Risky capacity equilibria $\Longleftrightarrow$ Risky capacity optimization

Some work that combines

- economic theory of RN capacity equilibria
- finance theory of complete risk markets with CRMs

Theorem (Ehren-Smeers-11b, R-Smeers-11b . . .)

Under assumptions for RN case + completeness of risk market:

x solves risky capacity equilibrium (for some (Y_ω), (U_ω), (P_ω), (W_g, W_c), P^{r}) $\Longrightarrow$ x solves

$$\min_x \sum_j I_j(x_j) + r_s\Big[Q_s(x)\Big] \quad \text{s.t.} \quad x \in \mathcal{X}$$

Converse holds under mild technical conditions.

This has exactly same form as Risk Neutral case:

- equilibrium $\Leftrightarrow$ convex optimization

Cf related two stage structure [Philpott-Ferris-Wets] in progress

Sketch proof of theorem when using cV@R

1. Write risky system capacity problem as a two stage stochastic LP using [Rock-Uryas-00]
   - Variables are x_g, W_g, W_c, t, η;and Y_ω, U_ω $\forall\omega$
2. Write down the KKT conditions for this two stage LP
   - Take P^r as Lagrange multiplier for $W_g + W_c = 0$
   - For each ω take P_ω as KKT multiplier for spot demand constraint $\sum_j Y_{j\omega} + U_\omega - D_\omega \geq 0$
3. Observe that KKT conditions comprise
   (i) KKT conditions for the genco's risky capacity problem,
   (ii) KKT conditions for the consumer's risk capacity problem,
   (iii) spot market clearing (spot pricing) in each scenario ω, and
   (iv) balancing risk trades (risk pricing)

QED

Completeness?

But **energy generation markets are far from complete**!

- Can contract fuel (coal, natural gas) prices out many months, even several years
- Can contract electricity prices somewhat into future
- Cannot contract price or penalty of C or other emissions

What if a major uncertainty is not priced in risk market?

- We'll look at range between "worst case" of no risk trading and "best case" of complete risk trading
- Range indicates uncertainty in long term capacity planning

Summary of capacity equilibria under uncertainty

3 different cases of capacity equilibria, from **worst to best**

1. **No risk trading**: Risk averse capacity equilibrium
2. **Complete risk trading**: Risky capacity equilibrium
3. **Risk neutral:** RN capacity equilibrium using PD Π_0

Corollary (Easy)

Provided RN probability density Π_0 lies in $\mathcal{D}_s$:

Social cost at equilibrium:

No Risk Trading $\geq$ Complete RT $\geq$ Risk Neutral

Or, welfare at equilibrium:

No Risk Trading $\leq$ Complete RT $\leq$ Risk Neutral

Outline

2 stage capacity equilibrium

Stage 1 Generator sets capacity x of future electricity plants

- There are $J = 3$ technologies: $j = 1$ Coal Steam Turbine, $j = 2$ Combined Cycle Gas Turbine, $j = 3$ Gas Turbine
- $x = (x_j)_j$ specifies plant capacities, so $x \in \mathcal{X} := \mathbb{R}^3_+$
- Cost of capacity is $I(x) := I_1(x_1) + I_2(x_2) + I_3(x_3)$
  $I_3(1) \ \leq \ I_2(1) \ \leq \ I_1(1)$ in ratio $1 \ : \ 1.5 \ : \ 3$

Stage 2, scenario ω

- There are $K = 15$ scenarios, $\omega \in \{1, .., 15\}$
  Fuel prices of Coal & Natural Gas are random (exog. data)
  Price of C emissions is also highly uncertain (exog. data)
  Demand split into 8 random load segments (exog. data)
- CST runs cheaper than CCGT except when high coal & high C prices
- GTs are "peakers", expensive to run

Good deal risk measure

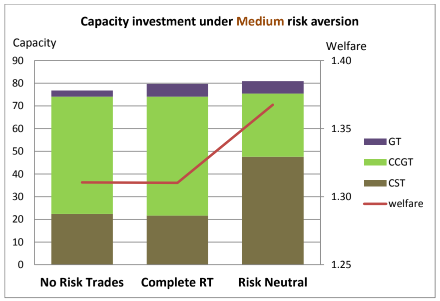
Adapt *Good Deal* risk measure from [Cochrane-Saa-Requejo-00]

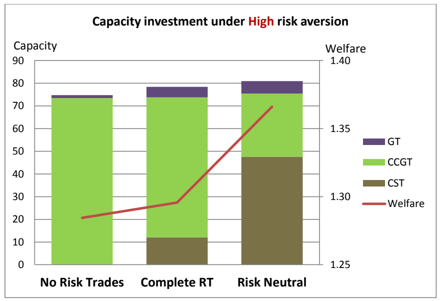
- Given "base" PD Π_0 and scalar $\nu > 0$, define risk set
  $$\mathcal{D}^{\mathrm{GD}}_{\nu} \ := \ \left\{ \zeta\Pi_0 \in \mathcal{P} \ : \ \mathbb{E}_{\Pi_0}\left[\zeta^2\right] \leq \nu^2 \right\}$$
  where $\zeta\Pi_0 = \left(\zeta_\omega \Pi_{0\omega}\right)$ and $\zeta^2 = \left(\zeta^2_\omega\right)$
- Taking $\nu = 1$ gives risk neutral case with respect to Π_0
- As ν increases above 1, risk aversion also increases

In results to follow,

- Both generator and consumer use same Good Deal risk set
  - Π_0 is uniform $(1/15, .., 1/15)$
  - ν is 1 (**Risk Neutral**) or 1.2 (**Medium** risk aversion) or 2 (**High**)
- There are approx 500 variables/constraints
- Use CONOPT & PATH: Tried EMP but need more smarts ...

2 stage capacity equilibrium results





- With respect to equilibrium Welfare (negative system cost),

No risk trading $\ \leq \ $ Complete risk trading $\ \leq \ $ Risk neutral

- Split between CST, CCGT and GT shows fear of high C price

CONCLUSION

1. **Managing risk** of **physical** assets with **financial** assets is exciting
   - Combines energy economics with financial markets
   - Risk neutral capacity equilibrium $\Leftrightarrow$ RN optimization
   - ... extends to risk averse case if all risks can be traded:
     Risky capacity equilibrium $\Leftrightarrow$ Risk averse optimization
2. Incomplete risk trading remains a challenge
3. Multi stage likewise challenging

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Example
Two stage capacity equilibrium

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