PREPRINT 94-1

44 Seiten

TED COX, ANDREAS GREVEN, TOKUZO SHIGA: Finite and Infinite Systems of Interacting Diffusions

Abstract: We study the problem of relating the long time behavior of finite and infinite systems of locally interacting components. The following class of models is considered in detail. Let $x(t) = \{x_i(t), i \in \mathbb{Z}^d\}$ be the solution of the system of stochastic differential equations

$$dx_i(t) = [\sum_{j \in \mathbb{Z}^d} a(i,j)x_j(t) - x_i(t)]dt + \sqrt{g(x_i(t))}dw_i(t), \quad i \in \mathbb{Z}^d.$$

We assume $a(i,j)$ is an irreducible random walk kernel on $\mathbb{Z}^d$, $g : \mathbb{R} \to \mathbb{R}^+$ satisfies certain regularity conditions, and $\{w_i(t), i \in \mathbb{Z}^d\}$ is a family of standard, independent Brownian motions on $\mathbb{R}$. $x(t)$ is an infinite system of interacting diffusions. The corresponding finite systems are $x^N(t) = \{x_i^N(t), i \in \Lambda_N\}$, which solve a similar system of equations, with $\Lambda_N = (-N, N]^d \cap \mathbb{Z}^d$, and $a(i,j)$ replaced by $a^N(i,j) = \sum_k a(i, j + 2Nk)$.
In the case $\hat{a}(i,j) = (a(i,j) + (a(j,i))/2$ is transient, the process $x(t)$ has a one-parameter family of extremal invariant measures $\{\nu_\theta, \theta \in \mathbb{R}\}$, with $E^{\nu_\theta} x_0 = \theta$. Let $\beta_N = (2N)^d$, let $g^*(\theta) = E^{\nu_\theta} g(x_0)$, and let $Z(t)$ be the diffusion $Z(t) = Z(0) + \int_0^t \sqrt{g^*(Z(s))}dw(s)$, where $w(t)$ is a Brownian motion. Let $Q_t(\rho, d\theta)$ be the probability transition function of $Z(t)$. A special case of our main result is the following. If $x_i^N(0) \equiv \rho$ for some constant ρ, then as $N \to \infty$,

$$|\Lambda_N|^{-1} \sum_{i \in \Lambda_N} x_i^N(s\beta_N) \Longrightarrow Z(s), \qquad Z(0) = \rho,$$

and

$$\mathcal{L}(x^N(s\beta_N)) \Longrightarrow \int_{\mathbb{R}} Q_s(\rho, d\theta)\nu_\theta.$$