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René Henrion: Topological Properties of the Approximate Subdifferential

Abstract: The approximate subdifferential introduced by Mordukhovich has attracted much attention in recent works on nonsmooth optimization. Potential advantages over other concepts of subdifferentiability might be related to its non-convexity. This motivates to study some topological properties more in detail. As the main result, it is shown that in a Hilbert space setting each weakly compact set may be obtained as the Kuratowski-Painlevé limit of the approximate subdifferentials of some family of Lipschitzian functions. As a consequence, apart from finiteness, there is no restriction on the number of connected components of the subdifferential. In the finite dimensional case, each topological type of a compact set may be realized by an approximate subdifferential of some Lipschitzian function. These are clear differences for instance to Clarke's subdifferential. The results stated above require the definition of Lipschitzian functions on a space which is enlarged by one extra dimension. Otherwise they would not hold true any longer since one can show, that for a real function the number of connected components of the approximate subdifferential is limited by two.