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DONALD A. DAWSON, ANDREAS GREVEN, JEAN VAILLANCOURT: Equilibria and quasiequilibria for infinite collections of interacting Fleming-Viot processes

Abstract: In this paper a class of infinite systems of interacting measure-valued diffusions each with state space $\mathcal{P}([0,1])$, the set of probability measures on $[0,1]$, is constructed and analysed (Fleming-Viot systems). These systems arise as diffusion limits of population genetics models with infinitely many possible types of individuals (labelled by $[0,1]$), spatially distributed over a countable collection of sites and evolving as follows. Individuals can migrate between sites and after an exponential waiting time a colony replaces its population by a new generation where the types are assigned by resampling from the empirical distribution of types at this site.
It is proved that depending on recurrence versus transience properties of the migration mechanism, the system either clusters as $t \to \infty$, that is, converges in distribution to a law concentrated on the states in which all components are equal to some δ_u, $u \in [0,1]$, or the system approaches a nontrivial equilibrium state. The properties of the equilibrium states, respectively the cluster formation, are studied by letting a parameter in the migration mechanism tend to infinity and explicitly identifying the limiting dynamics in a sequence of different space-time scales. These limiting dynamics have stationary states which are quasiequilibria of the original system, that is, change only in longer time scales. Properties of these quasiequilibria are derived and related to the global equilibrium process for large N. Finally we establish that the Fleming-Viot systems are the unique dynamics which remain invariant under the associated space-time renormalization procedure.