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MICHAEL CRANSTON, ANDREAS GREVEN: Coupling and harmonic functions in the case of continuous time Markov processes

Abstract: Consider two transient Markov processes $(X_t^\nu)_{t\in R^+}$, $(X_t^\mu)_{t\in R^+}$ with the same transition semigroup and initial distributions ν and μ. The probability spaces supporting the processes each are also assumed to support an exponentially distributed random variable independent of the process.
We show that there exist (randomized) stopping times S for (X_t^ν), T for (X_t^μ) with common final distribution, $\mathcal{L}(X_S^\nu \mid S<\infty) = \mathcal{L}(X_T^\mu \mid T<\infty)$, and the property that for $t<S$, resp. $t<T$, the processes move in disjoint portions of the state space. For such a coupling (S,T) it is shown

$$\text{Prob}\,(S=\infty) + \text{Prob}\,(T=\infty) = \max_{h\in\mathcal{H},|h|\le 1} \langle \nu-\mu, h\rangle\,,$$

where $\mathcal{H}$ denotes the bounded harmonic functions of the Markov transition semigroup. Extensions, consequences and applications of this result are discussed.