EXERCISE SHEET 2

GRAPH COMPLEXES, SUMMER 23, HU BERLIN

Please prepare to present your solutions in the exercise session on May 26th.

Exercise 1. Show that $d^2 = 0$ in GC_N, N even.

Exercise 2. Express the degree of a graph $|G|_N = e_G - Nh_G$ without using its loop number. (*Hint:* Ask Mr. Euler.) From this, try to come up with an alternative (combinatorial) notion of orientation for N odd.

Exercise 3.

(1) Draw all weighted stable graphs with 3 labeled legs and genus 1.

(2) Sketch a picture of $\Delta^0_{1,3}$ What space is it homeomorphic to? (If you can't "see" it, compute its cellular homology.)

Exercise 4.

(1) The moduli space of (normalized metric, unweighted) *holocolored* graphs with 3 labeled legs and 1 loop is defined as

$$MHG^0_{1,3} = \left\{ (G, \lambda, w, c) \;\middle|\; \begin{array}{l} \text{G stable with 3 legs, } h_G = 1,\ w = 0 \\ \text{vol}(\lambda) = 1,\ c : E_G \to \{1,2,3\} \text{ injective} \end{array} \right\} \Big/ {\sim} .$$

where $\sim$ is the equivalence relation introduced in the lecture. (in words, repeat the construction of $\Delta^0_{1,3}$ but now consider graphs with their edges distinctly colored by three colors $1, 2, 3$.). What space is it homeomorphic to?

(2) What happens if we drop the requirement of distinct coloring, i.e., consider

$$MCG^0_{1,3;2} = \left\{ (G, \lambda, w, c) \;\middle|\; \begin{array}{l} \text{G stable with 3 legs, } h_G = 1,\ w = 0 \\ \text{vol}(\lambda) = 1,\ c : E_G \to \{1,2\} \end{array} \right\} \Big/ {\sim} \quad ?$$

What is $H_2(MCG^0_{1,3;2}; \mathbb{Q})$?

(3) For the case of m-colored 1 loop graphs with 2 legs compute $H_k(MCG^0_{1,2;m}; \mathbb{Q})$.

Exercise 5. Another way to think about $\Delta^0_{1,n}$ is to view its points as specifying configurations of n marked points on a circle S^1. If we fix the first one, say at $1 \in \mathbb{C}$, then the other $n-1$ points are described by a vector of angles $(\theta_1, \dots, \theta_{n-1})$.

(1) Which familiar space X is this "set of angles" homeomorphic to?

(2) Which configurations describe isomorphic graphs? That is, find a group G such that $\Delta^0_{1,n} \cong X/G$.

(3) From this compute $H_k(\Delta^0_{1,n}; \mathbb{Q})$ for all $k, n \in \mathbb{N}$.