

## EXERCISE SHEET 3

GRAPH COMPLEXES, SUMMER 23, HU BERLIN

*Please prepare to present your solutions in the exercise session on June 16th.*

**Exercise 1.** Finish the proof of Theorem 4 (i).

**Exercise 2.** Show that the  $n$ -cube  $[0, 1]^n$  can be triangulated into  $n!$   $n$ -simplices. (If you need a hint, check page 112 in Hatcher's book 'Algebraic Topology'.)

**Exercise 3.** Prove the following lemma from the third lecture.

*Lemma.* Let  $S^n$  denote the  $n$ -sphere,  $D^n$  denote the  $n$ -disk. Let  $\Gamma$  be a finite linear group acting on  $S^n$  by permuting the coordinates of  $\mathbb{R}^{n+1}$ . Then

$$H_{\bullet}(S^n/\Gamma; \mathbb{Q}) = \begin{cases} H_{\bullet}(D^n; \mathbb{Q}) & \text{action induces orient.-reversing homeomorphisms,} \\ H_{\bullet}(S^n; \mathbb{Q}) & \text{else.} \end{cases}$$

**Exercise 4.** Find the dimensions of  $\Delta_{g,n}$ ,  $\Delta_{g,n}^{>0}$ ,  $\Delta_{g,n}^0$ , and  $S_{g,n}$ .