EXERCISE SHEET 4

GRAPH COMPLEXES, SUMMER 23, HU BERLIN

Please prepare to present your solutions in the exercise session on July 14th.

Exercise 1. Show property 4) of the invariant classes β_X^n,

$$\beta_{X^\top}^n = (-1)^{\frac{n(n-1)}{2}} \beta_X^n.$$

Exercise 2. Show that $\Lambda_G|_{x_e=0} = \Lambda_{G/e}$ whenever e is not a self-loop.

Exercise 3. Show the following factorization property of the graph polynomial Ψ_G:

If e is not a self-loop, then $\Psi_G = x_e \Psi_{G \setminus e} + \Psi_{G/e}$.

From this deduce that

$$V_G = \{\Psi_G = 0\} \cap C(G) = \bigcup_{\substack{\gamma \subset G \\ h_\gamma > 0}} F_\gamma$$

where $F_\gamma = \{x_e = 0 \mid e \in E_\gamma\} \cap C(G) \cong C(G/\gamma)$.

Exercise 4. Let G be the 'theta graph' ⬱. Analyse the behaviour of Ψ_G at the

three corners of the cell $C(G)$ by blowing up $\mathbb{P}^2$ in the three points $[1 : 0 : 0], [0 : 1 : 0], [0 : 0 : 1]$. Use this to verify assertion 2. in Thm. 1.