Exercises in Global Analysis II

University of Bonn, Winter Semester 2018-2019
Professor: Batu Güneysu
Assistant: Koen van den Dungen

UNIVERSITÄT BONN
Rheinische Friedrich-Wilhelms-Universität Bonn

Sheet 10: due on Friday 21 December at 12:00 in Room 1.032.

---

1 Properly supported pseudodifferential operators [7 points]

Let $k \in \mathbb{R}$. Show that every *properly supported* $P \in \Psi^k(\Omega; \mathbb{C}^l, \mathbb{C}^{l'})$ induces a continuous linear map

$$P : \mathscr{E}(\Omega, \mathbb{C}^l) \longrightarrow \mathscr{E}(\Omega, \mathbb{C}^{l'}).$$

2 The symbol of a differential operator [6 points]

Let $k \in \mathbb{N}$, and let $P = \sum_{|\alpha| \leq k} P_\alpha D^\alpha : \mathscr{D}(\Omega, \mathbb{C}^l) \longrightarrow \mathscr{E}(\Omega, \mathbb{C}^{l'})$ be a differential operator of order $\leq k$. Consider the full symbol $\sigma(P)$ and the principal symbol $\sigma_k(P)$, as defined in Definition 4.25 in the lecture notes. Prove that $\sigma(P)$ is represented by

$$\sigma(P)(x, \zeta) = \sum_{|\alpha| \leq k} P_\alpha(x) \zeta^\alpha,$$

and that $\sigma_k(P)$ is represented by

$$\sigma_k(P)(x, \zeta) = \sum_{|\alpha| = k} P_\alpha(x) \zeta^\alpha.$$

3 A pseudodifferential operator [7 points]

Let

$$P : \mathscr{D}(\Omega, \mathbb{C}^l) \longrightarrow \mathscr{E}(\Omega, \mathbb{C}^{l'}).$$

be a linear map which admits a $k \in \mathbb{R}$ such that for all $\phi_1, \phi_2 \in \mathscr{D}(\Omega)$ one has $\phi_1 P \phi_2 \in \Psi^k(\Omega; \mathbb{C}^l, \mathbb{C}^{l'})$. Show that then $P \in \Psi^k(\Omega; \mathbb{C}^l, \mathbb{C}^{l'})$.