

Exercises in Global Analysis II

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Sheet 13: due on Friday 25 January at 12:00 in Room 1.032.

1 Proof of Lemma 4.40 [10 points]

- (a) Assume $k \in \{-\infty\} \cup \mathbb{R}$ and $P \in \Psi^k(\Omega; \mathbb{C}^l, \mathbb{C}^{l'})$. Show that P is properly supported if and only if both P and P^\dagger are continuous maps

$$P: \mathcal{D}(\Omega, \mathbb{C}^l) \rightarrow \mathcal{D}(\Omega, \mathbb{C}^{l'}), \quad P^\dagger: \mathcal{D}(\Omega, \mathbb{C}^{l'}) \rightarrow \mathcal{D}(\Omega, \mathbb{C}^l).$$

Hint: First show that P is properly supported if and only if P^\dagger is properly supported.

- (b) Assume $k, r \in \{-\infty\} \cup \mathbb{R}$ and that $P \in \Psi^k(\Omega; \mathbb{C}^{l'}, \mathbb{C}^{l''})$ and $Q \in \Psi^r(\Omega; \mathbb{C}^l, \mathbb{C}^{l'})$ are properly supported. Show that the composition $PQ \in \Psi^{k+r}(\Omega; \mathbb{C}^l, \mathbb{C}^{l''})$ is also properly supported.

2 Sobolev mapping properties [10 points]

Assume $k, s \in \mathbb{R}$ and $P \in \Psi^k(\Omega; \mathbb{C}^l, \mathbb{C}^{l'})$. Show that P is a continuous map

$$P: H_c^s(\Omega, \mathbb{C}^l) \rightarrow H_{\text{loc}}^{s-k}(\Omega, \mathbb{C}^{l'}).$$