On the solution of affine generalized Nash games with shared constraints by Lemke's algorithm

Jong-Shi Pang

Department of Industrial and Systems Engineering

University of Southern California

presented at

6th International Conference on Complementarity Problems

Humboldt Universität zur Berlin, Germany

August 4–8, 2014

Contents of presentation

- The affine generalized Nash equilibrium problem
- The linear complementarity problem
- Variational equilibria
- Summary of results
- Details

D.A. Schiro, J.S. Pang, and U.V. Shanbhag. On the solution of affine generalized Nash equilibrium problems with shared constraints by Lemke's method. *Mathematical Programming, Series A* 146 (2013) 1–46.

The AGNEP

There are n players, each labeled by $\nu = 1, \cdots, n$.

Parameterized by rivals' strategies $x^{-\nu} \triangleq \left(x^{\nu'}\right)_{\nu \neq \nu'=1}^{n}$, player ν's optimization is a convex quadratic program:

$$\underset{x^\nu \in \Xi^\nu(x^{-\nu})}{\text{minimize}}\ \theta_\nu(x^\nu, x^{-\nu}) \triangleq \underbrace{\tfrac{1}{2}(x^\nu)^T H^{\nu\nu} x^\nu + (x^\nu)^T \left(h^\nu + \sum_{\nu \neq \nu'=1}^{n} H^{\nu\nu'} x^{\nu'} \right)}_{\text{convex quadratic in } x^\nu \text{ given } x^{-\nu}}, \quad \text{where}$$

$$\Xi^\nu(x^{-\nu}) \triangleq \left\{ x^\nu \in \mathbb{R}^{n_\nu}_+ \mid \underbrace{B^\nu x^\nu \geq f^\nu}_{\text{private constraint}}, \quad \underbrace{\sum_{\nu'=1}^{n} A^{\nu'} x^{\nu'} \geq b}_{\text{common, coupled constraint}} \right\}.$$

Remark: There is no player-dependent coupled constraint of the form:

$$\sum_{\nu'=1}^{n} A^{\nu\nu'} x^{\nu'} \geq b^{\nu}$$

LCP with no structural assumption on multipliers

Introducing multipliers $\lambda^{\nu,\mathrm{p}}$ (for private constraints) and $\lambda^{\nu,\mathrm{s}}$ (for shared constraints), we obtain a linear complementarity formulation of the game:

$$0 \leq z_{\mathrm{NE}} \perp w_{\mathrm{NE}} \triangleq q_{\mathrm{NE}} + M_{\mathrm{NE}} z_{\mathrm{NE}} \geq 0, \quad \text{with}$$

$$z_{\mathrm{NE}} \triangleq \begin{pmatrix} \begin{pmatrix} x^1 \\ \vdots \\ x^n \end{pmatrix} \\ --- \\ \begin{pmatrix} \lambda^{1,\mathrm{p}} \\ \vdots \\ \lambda^{n,\mathrm{p}} \end{pmatrix} \\ --- \\ \begin{pmatrix} \lambda^{1,\mathrm{s}} \\ \vdots \\ \lambda^{n,\mathrm{s}} \end{pmatrix} \end{pmatrix}, \quad w_{\mathrm{NE}} \triangleq \begin{pmatrix} \begin{pmatrix} y^1 \\ \vdots \\ y^n \end{pmatrix} \\ --- \\ \begin{pmatrix} s^{1,\mathrm{p}} \\ \vdots \\ s^{n,\mathrm{p}} \end{pmatrix} \\ --- \\ \begin{pmatrix} s^{1,\mathrm{s}} \\ \vdots \\ s^{n,\mathrm{s}} \end{pmatrix} \end{pmatrix}$$

$$
M_{\mathsf{NE}} \triangleq \left[\begin{array}{ccccccccccc}
H^{11} & \cdots & H^{1n} & | & -(B^1)^T & & & | & -(A^1)^T & & \\
\vdots & \ddots & \vdots & | & & \ddots & & | & & \ddots & \\
H^{n1} & \cdots & H^{nn} & | & & & -(B^n)^T & | & & & -(A^n)^T \\
-- & -- & -- & | & -- & -- & -- & | & -- & -- & -- \\
B^1 & & & | & & & & | & & & \\
& \ddots & & | & & & & | & & & \\
& & B^n & | & & & & | & & & \\
-- & -- & -- & | & -- & -- & -- & | & -- & -- & -- \\
A^1 & \cdots & A^n & | & & & & | & & & \\
\vdots & \cdots & \vdots & | & & & & | & & & \\
A^1 & \cdots & A^n & | & & & & | & & &
\end{array} \right]
$$

Note the structures in the last block in the first row (block diagonal) and the same block in last column (repeating rows).

In general, $H^{\nu\nu'} \neq H^{\nu',\nu}$ for $\nu \neq \nu'$; $H^{\nu\nu}$ is symmetric positive semidefinite.

A statement of Lemke's algorithm

The augmented LCP with covering vector $d > 0$:

$$0 \leq z \perp w \triangleq q + \tau d + Mz \geq 0.$$

3 possible outcomes of Lemke's algorithm: (a) cycling (degeneracy resolution), (b) ray termination, or (c) solution obtained.

(I) If the algorithm terminates at a secondary ray, then there exists a tuple $(w^*, \widetilde{w}, z_0^*, \widetilde{z}_0, z^*, \widetilde{z})$ with $z_0^* > 0$ and $\widetilde{z} \neq 0$ such that for all $\tau \geq 0$,

$$0 \leq z^* + \tau\widetilde{z} \perp w^* + \tau\widetilde{w} = q + d\,(z_0^* + \tau\widetilde{z}_0) + M(z^* + \tau\widetilde{z}) \geq 0. \qquad (1)$$

(II) If M is a semimonotone matrix, then the scalar $\widetilde{z}_0$ satisfying (1) must equal zero; hence, if for every scalar $z_0 > 0$, SOL$(q + dz_0, M)$ is bounded, then the LCP (q, M) has a solution that can be computed by Lemke's method with d as the covering vector (with a degeneracy resolution scheme). □

Specialization to the LCP $(q_{\text{NE}}, M_{\text{NE}})$

Let

$$J_{\text{NE}} \triangleq \left[\begin{array}{ccc|ccc} H^{11} & \cdots & H^{1n} & -(B^1)^T & & \\ \vdots & \ddots & \vdots & & \ddots & \\ H^{n1} & \cdots & H^{nn} & & & -(B^n)^T \\ -- & -- & -- & -- & -- & -- \\ B^1 & & & & & \\ & \ddots & & & & \\ & & B^n & & & \end{array} \right], \quad \mathbf{A} \triangleq \left[\begin{array}{ccc} A^1 & \cdots & A^n \end{array} \right].$$

- If J_{NE} is semimonotone, and if the nonzero entries in each row of the matrix $\mathbf{A}$ are of a single sign, then M_{NE} is semimonotone.

- Under further conditions:

$$\begin{aligned} \text{SOL}(0, J_{\text{NE}}) = \{\, 0 \,\} \quad &\Rightarrow \quad \text{SOL}(0, M_{\text{NE}}) = 0 \\ &\Rightarrow \quad \text{successful termination of Lemke} \\ & \quad \text{under degeneracy resolution} \end{aligned}$$

An illustrative example: degeneracy is real!

Consider a 2-person game with individual optimization problems as follows:

$$\begin{array}{ll} \underset{x_1 \geq 0}{\text{maximize}} & x_1(1 - 0.5x_1 - 0.5x_2) \\ \text{subject to} & 1 - x_1 - x_2 \overset{(\lambda_1)}{\geq} 0 \end{array}$$

$$\begin{array}{ll} \underset{x_2 \geq 0}{\text{maximize}} & x_2(2 - 0.5x_1 - 0.5x_2) \\ \text{subject to} & 1 - x_1 - x_2 \overset{(\lambda_2)}{\geq} 0 \end{array}$$

The following 2 tableaux detail the first pivot in Lemke's algorithm.

	q	z_0	x_1	x_2	λ_1	λ_2
w_1	-1	1	1	0.5	1	0
w_2	-2	**1**	0.5	1	0	1
s_1	1	1	-1	-1	0	0
s_2	1	1	-1	-1	0	0

	q	w_2	x_1	x_2	λ_1	λ_2
w_1	1	1	0.5	-0.5	1	-1
z_0	2	1	-0.5	-1	0	-1
s_1	3	1	-1.5	**-2**	0	-1
s_2	3	1	-1.5	**-2**	0	-1

- s_2 **made nonbasic:** pivots of $< s_2, x_2 >$; $< w_1, \lambda_2 >$; $< z_0, x_1 >$ terminate at the solution $(1, 0, 0, 1.5)$.
- s_1 **made nonbasic:** pivot of $< s_1, x_2 >$ leads to ray termination and the method fails.

2 remarks: (a) algorithm may fail, and (b) only one multiplier (λ_2) is positive.

LCP with common multipliers on shared constraints

Variational equilibria: $\lambda^{\nu,s} = \lambda^s$ for all ν; i.e., common multipliers of shared constraints.

$$z_{\text{VE}} \triangleq \begin{pmatrix} \begin{pmatrix} x^1 \\ \vdots \\ x^n \end{pmatrix} \\ - - - \\ \begin{pmatrix} \lambda^{1,\mathbf{p}} \\ \vdots \\ \lambda^{n,\mathbf{p}} \end{pmatrix} \\ - - - \\ \boxed{\begin{array}{c} \lambda^{\mathbf{s}} \\ \text{collapsed} \\ \text{into one} \end{array}} \end{pmatrix}$$

$$
M_{\mathsf{VE}} \triangleq \left[\begin{array}{ccc|ccc|c}
H^{11} & \cdots & H^{1n} & -(B^1)^T & & & -(A^1)^T \\
\vdots & \ddots & \vdots & & \ddots & & \vdots \\
H^{n1} & \cdots & H^{nn} & & & -(B^n)^T & -(A^n)^T \\
-- & -- & -- & -- & -- & -- & -- \\
B^1 & & & & & & \\
 & \ddots & & & & & \\
 & & B^n & & & & \\
-- & -- & -- & -- & -- & -- & -- \\
A^1 & \cdots & A^n & & & &
\end{array}\right]
$$

Excluding the upper left block, M_{VE} has a skew symmetry structure that is absent in the previous non-VE formulation. Let

$$
J_{\mathsf{VE}} \triangleq \begin{bmatrix} H^{11} & \cdots & H^{1n} \\ \vdots & \ddots & \vdots \\ H^{n1} & \cdots & H^{nn} \end{bmatrix}.
$$

- If J_{VE} is copositive, then so is M_{VE}.

Summary of results with Lemke's algorithm pertaining to multipliers of shared constraints

• Applied to the non-VE formulation, Lemke's algorithm, if successful, computes only one kind of NE, those for which at most one player has a non-zero multiplier associated with each shared constraint.

• Thus, many NE are elusive by this algorithm, motivating the need to modify it for (a) robustness, and (b) capability to compute NE of other kinds.

• Multipliers provide meaningful insights on the constraints; thus desirable to be able to compute solutions with different kinds of multipliers.

• Introduced the notion of a partial VE computable by a modified Lemke method; such a partial VE enforced multiplier consistency across all players for certain shared constraints.

• Rosen's VE can be approximated by specialized regularizations of the LCP formulation of the game.

• Introduced an equivalent reformulation of shared constraints and a parameterization idea, yielding computable NE with yet a different property of the multipliers of these constraints.

Structural property of multipliers in Lemke solutions

Proposition. If Lemke's method finds a solution of the LCP $(q_{\text{NE}}, M_{\text{NE}})$ of the non-VE formulation of the AGNEP, then for each shared constraint $\ell = 1, \cdots, m_s$, $\exists$ in that solution at most one $\nu \in \{1, \cdots, N\}$ such that $\lambda_\ell^{\nu,s} > 0$. $\square$

Thus, the only VE that can be computed by Lemke's algorithm when applied to the LCP $(q_{\text{NE}}, M_{\text{NE}})$ is the one with all multipliers equal to zero.

A modified Lemke algorithm for the AGNEP

Perform Lemke's method until the s-variables of shared constraint ℓ become blocking. Call this Tableau 1 and randomly choose $s_\ell^{\nu,\mathrm{s}}$ to pivot out of the basis so that the next entering variable is $\lambda_\ell^{\nu,\mathrm{s}}$. Define $I \triangleq \{\nu\}$.

• If ray termination occurs at any point after $s_\ell^{\nu,\mathrm{s}}$ is made nonbasic, choose $s_\ell^{\nu',\mathrm{s}}$ with $\nu' \notin I$ as the blocking variable in Tableau 1 so that $\lambda_\ell^{\nu',\mathrm{s}}$ is the next entering variable.

• Resume the usual operation of the algorithm using the new blocking variable in Tableau 1. Stop if a solution is found. Otherwise, either return to Step 1 with $I \triangleq I \cup \{\nu'\}$ or proceed to Step 2 following ray termination.

• If ray termination has occurred after all $s_\ell^{\bullet,\mathrm{s}}$ pivots, return to Tableau 1 and

- delete all but one row corresponding to the ℓth shared constraint and relabel the variable s_ℓ^{s};

- combine all $\lambda_\ell^{\bullet,\mathrm{s}}$ variables into a single multiplier labeled $\lambda_\ell^{\mathrm{s}}$;

- recalculate the $\lambda_\ell^{\mathrm{s}}$ column of the new tableau.

An illustration of the modified scheme

A river basin pollution game

(Haurie-Krawczyk 1997; Nabetani-Tseng-Fukushima 2011)

3 competitive players and 2 shared constraints:

$$\begin{array}{ll} \underset{x_\nu \geq 0}{\text{minimize}} & [\, \alpha_\nu x_\nu + 0.01\,(\, x_1 + x_2 + x_3\,) - \chi_\nu\,]\; x_\nu \\ \text{subject to} & -100 \leq -3.25 x_1 - 1.25 x_2 - 4.125 x_3 \\ \text{and} & -100 \leq -2.2915 x_1 - 1.5625 x_2 - 2.8125 x_3, \end{array}$$

with parameters $\alpha_1 = 0.01$, $\alpha_2 = 0.05$, $\alpha_3 = 0.01$, $\chi_1 = 2.9$, $\chi_2 = 2.88$, and $\chi_3 = 2.85$.

	q	z_0	x_1	x_2	x_3	$\lambda_1^{1,s}$	$\lambda_1^{2,s}$	$\lambda_1^{3,s}$	$\lambda_2^{1,s}$	$\lambda_2^{2,s}$	$\lambda_2^{3,s}$
w_1	-2.9	**1**	0.04	0.01	0.01	3.25	0	0	2.29	0	0
w_2	-2.88	1	0.01	0.12	0.01	0	1.25	0	0	1.56	0
w_3	-2.85	1	0.01	0.01	0.04	0	0	4.12	0	0	2.81
$s_1^{1,s}$	100	1	-3.25	-1.25	-4.12	0	0	0	0	0	0
$s_1^{2,s}$	100	1	-3.25	-1.25	-4.12	0	0	0	0	0	0
$s_1^{3,s}$	100	1	-3.25	-1.25	-4.12	0	0	0	0	0	0
$s_2^{1,s}$	100	1	-2.30	-1.51	-2.81	0	0	0	0	0	0
$s_2^{2,s}$	100	1	-2.30	-1.51	-2.81	0	0	0	0	0	0
$s_2^{3,s}$	100	1	-2.30	-1.51	-2.81	0	0	0	0	0	0

Table 0: The river basic game: Original formulation

	q	w_1	w_2	w_3	x_3	$\lambda_1^{1,s}$	$\lambda_{2,1}^{s}$	$\lambda_1^{3,s}$	$\lambda_2^{1,s}$	$\lambda_2^{2,s}$	$\lambda_2^{3,s}$
z_0	2.8	-0.3	-0.09	1.42	-0.05	1.08	0.1	-5.9	0.8	0.1	-4
x_1	1.7	33.3	0	-33	1	-108	0	138	-76.4	0	94
x_2	0.3	0	9.1	-9.1	0.3	0	-11.4	37.5	0	-14	26
$s_1^{1,s}$	97.1	-109	-11.5	121	**-7.8**	353	14	-500	249	18	-341
$s_1^{2,s}$	97.1	-109	-11.5	121	**-7.8**	353	14	-500	249	18	-341
$s_1^{3,s}$	97.1	-109	-11.5	121	**-7.8**	353	14	-500	249	18	-341
$s_2^{1,s}$	98.6	-76.7	-13.8	91.6	-5.6	249.3	17.3	-377.7	176	21.6	-258
$s_2^{2,s}$	98.6	-76.7	-13.8	91.6	-5.6	249.3	17.3	-377.7	176	21.6	-258
$s_2^{3,s}$	98.6	-76.7	-13.8	91.6	-5.6	249.3	17.3	-377.7	176	21.6	-258

Table 1: After 3 pivots from original, x_3 is the entering variable

Note the repetition of the rows of the shared constraints, leading to ties in choosing the leaving variable.

	q	w_1	w_2	w_3	$s_1^{1,s}$	$\lambda_1^{1,s}$	$\lambda_1^{2,s}$	$\lambda_1^{3,s}$	$\lambda_2^{1,s}$	$\lambda_2^{2,s}$	$\lambda_2^{3,s}$
z_0	2.17	0.40	-0.01	0.60	0.01	-1.31	0.02	-2.48	-0.93	0.02	-1.69
x_1	14	19.3	-1.5	-17.7	-0.13	**-62.9**	1.8	73	-44	2.3	50
x_2	3.7	-3.8	8.7	-4.8	-0.04	12.4	-10.9	20	8.7	-13.6	13.6
x_3	12.5	-14	-1.5	15.6	-0.13	45.5	1.8	-64.3	32.1	2.3	-44
$s_1^{2,s}$	0	0	0	0	1	0	0	0	0	0	0
$s_1^{3,s}$	0	0	0	0	1	0	0	0	0	0	0
$s_2^{1,s}$	29	1.2	-5.6	4.7	0.7	-3.9	7	-19.5	-2.7	8.8	-13.3
$s_2^{2,s}$	29	1.2	-5.6	4.7	0.7	-3.9	7	-19.5	-2.7	8.8	-13.3
$s_2^{3,s}$	29	1.2	-5.6	4.7	0.7	-3.9	7	-19.5	-2.7	8.8	-13.3

Table 2: Next pivot is on the distinguished shared multiplier $\lambda_{1,1}^{s}$

Note the two rows of the first shared constraints one of whose slack variables ($s_1^{1,s}$) is nonbasic.

	q	w_1	w_2	w_3	$s_{1,1}$	x_1	$\lambda_1^{2,s}$	$\lambda_1^{3,s}$	$\lambda_2^{1,s}$	$\lambda_2^{2,s}$	$\lambda_2^{3,s}$
z_0	1.88	0	-0.02	0.97	0.01	0.02	-0.02	-4.01	0	-0.03	-2.74
$\lambda_{1,1}^s$	0.23	0.31	-0.02	-0.28	-0.002	-0.02	0.03	1.2	-0.71	0.04	0.8
x_2	6.5	0	8.4	-8.3	-0.06	-0.20	-10.5	34.4	0	-13.1	23.5
x_3	22.7	0	-2.5	2.8	-0.22	-0.72	3.18	-11.4	0	4	-7.8
$s_1^{2,s}$	0	0	0	0	1	0	0	0	0	0	0
$s_1^{3,s}$	0	0	0	0	1	0	0	0	0	0	0
$s_2^{1,s}$	28.1	0	-5.5	5.8	0.7	0.06	6.9	-24	0	8.7	-16.4
$s_2^{2,s}$	28.1	0	-5.5	5.8	0.7	0.06	6.9	-24	0	8.7	-16.4
$s_2^{3,s}$	28.1	0	-5.5	5.8	0.7	0.06	6.9	-24	0	8.7	-16.4

Table 3: Post pivot on $\lambda_1^{1,s}$; ray termination on w_1

Return to Table 1 and choose $s_1^{3,s}$ as the blocking variable; ray termination occurs after pivot. Return to Table 1 and make the last choice to break tie; i.e., choose $s_1^{2,s}$ as the blocking variable, leading also to ray termination.

Now we group the first shared constraints and the corresponding multipliers, obtaining the next table.

	q	w_1	w_2	w_3	x_3	λ_1^{s}	$\lambda_2^{1,\mathrm{s}}$	$\lambda_2^{2,\mathrm{s}}$	$\lambda_2^{3,\mathrm{s}}$
z_0	2.83	-0.33	-0.09	1.42	-0.05	-4.68	0.76	0.14	-4.01
x_1	1.67	33.33	0	-33.33	1	29.17	-76.38	0	93.75
x_2	0.27	0	9.09	-9.09	0.27	26.14	0	-14.20	25.57
s_1^{s}	97.07	-108.67	-11.45	121.12	**-7.77**	-132.14	249.01	17.90	-340.65
$s_2^{1,\mathrm{s}}$	98.60	-76.72	-13.84	91.56	-5.57	-111.04	175.80	21.63	-257.51
$s_2^{2,\mathrm{s}}$	98.60	-76.72	-13.84	91.56	-5.57	-111.04	175.80	21.63	-257.51
$s_2^{3,\mathrm{s}}$	98.60	-76.72	-13.84	91.56	-5.57	-111.04	175.80	21.63	-257.51

Table 4: After collapsing shared constraint 1 and its multipliers

	q	w_1	w_2	w_3	s_1	λ_1^{s}	$\lambda_2^{1,\mathrm{s}}$	$\lambda_2^{2,\mathrm{s}}$	$\lambda_2^{3,\mathrm{s}}$
z_0	2.17	0.40	-0.01	0.60	0.01	**-3.78**	-0.93	0.02	-1.69
x_1	14.16	19.35	-1.47	-17.74	-0.13	12.16	-44.33	2.30	49.90
x_2	3.68	-3.81	8.69	-4.84	-0.04	21.50	8.74	-13.58	13.61
x_3	12.50	-13.99	-1.47	15.59	-0.13	-17.01	32.05	2.30	-43.85
$s_2^{1,\mathrm{s}}$	29.01	1.19	-5.63	4.73	0.72	-16.32	-2.72	8.80	-13.30
$s_2^{2,\mathrm{s}}$	29.01	1.19	-5.63	4.73	0.72	-16.32	-2.72	8.80	-13.30
$s_2^{3,\mathrm{s}}$	29.01	1.19	-5.63	4.73	0.72	-16.32	-2.72	8.80	-13.30

Table 5: Solution found after pivot on λ_1^{s}

Regularization and generalized VE

$$M_i \triangleq M_{\mathsf{NE}} + \left[\begin{array}{ccc|ccc|ccc}
E_i^{1;x} & & & & & & & & \\
& \ddots & & & & & & & \\
& & E_i^{n;x} & & & & & & \\
\hline
& & & E_i^{1;\mathbf{p}} & & & & & \\
& & & & \ddots & & & & \\
& & & & & E_i^{n;\mathbf{p}} & & & \\
\hline
& & & & & & E_i^{1;\mathbf{s}} & & \\
& & & & & & & \ddots & \\
& & & & & & & & E_i^{n;\mathbf{p}}
\end{array}\right]$$

where $E_i^{\nu;x,\mathbf{p},\mathbf{s}}$ is a positive diagonal matrix with $\lim_{i\to\infty} E_i^{\nu;x,\mathbf{p},\mathbf{s}} = 0$ and for all ν, $\nu' = 1, \cdots, n$ and all $\ell = 1, \cdots, m_s$,

$$\lim_{i\to\infty} \frac{\left(E_i^{\nu;\mathbf{s}}\right)_{\ell\ell}}{\left(E_i^{\nu';\mathbf{s}}\right)_{\ell\ell}} = e_\ell^{\nu\nu'} > 0;$$

Generalized VE

For any m_s positive matrices $\mathcal{E}_\ell^{\rm s} \triangleq \left[e_\ell^{\nu\nu'} \right]_{\nu,\nu'=1}^n \in \mathbf{R}^{n \times n}$ satisfying $e_\ell^{\nu\nu'} e_\ell^{\nu'\nu} = 1$ for all ν and ν', let $\mathbf{\Lambda}(\mathcal{E}^{\rm s})$ be defined as follows:

$$\mathbf{\Lambda}(\mathcal{E}^{\rm s}) \triangleq \left\{ \boldsymbol{\lambda}^{\rm s} \mid \lambda_\ell^{\nu,{\rm s}} = e_\ell^{\nu\nu'} \lambda_\ell^{\nu',{\rm s}} \text{ for all } \ell = 1, \cdots, m_s, \text{ and all } \nu, \nu' = 1, \cdots, n \right\}.$$

Rosen's normalized Nash equilibria correspond to the case where each matrix $\mathcal{E}_\ell^{\rm s}$ is the same with entries given by $e_\ell^{\nu\nu'} = e_{\nu'}/e_\nu$ for a positive vector $e \in \mathbb{R}^n$.

A variational equilibrium is a special kind of normalized NE where e is the vector of all ones.

Proposition. If $\widehat{z} = \lim_{i \to \infty} z^i$ with $z^i \in \text{SOL}(q_{\text{NE}}, M_i)$, then $\widehat{z} \triangleq \left(\widehat{x}, \widehat{\lambda}^{\rm p}, \widehat{\lambda}^{\rm s} \right)$ is a solution of the LCP $(q_{\text{NE}}, M_{\text{NE}})$ and $\widehat{\lambda}^{\rm s} \in \mathbf{\Lambda}(\mathcal{E}^{\rm s})$. □