SS 2020 Marc Kegel

Topology of 3-Manifolds

Exercise sheet 7

Exercise 1.

(a) Let $g, h \geq 2$. Then there exists a (unbranched) covering $\Sigma_g \to \Sigma_h$ if and only if $(h-1)$ divides $(g-1)$.

(b) Verify the Rieman–Hurwitz formula for the branched covers $F \to S^2$ constructed in both proofs of Theorem 6.1 from the lecture.

Exercise 2.

For integers $p, q \in \mathbb{Z}$ we define the (p,q)-torus link $T_{p,q}$ to be the link given by the curve $p\mu + q\lambda$ on the boundary of a tubular neighborhood of an unknot, where (μ, λ) denote meridian and Seifert longitude of the unknot.

(a) Draw diagrams of torus links and identify familiar links as torus links.

(b) For any permutation (p, q, r) of $(5, 2, 3)$ there exists a p-fold branched covering from the Poincaré homology sphere to S^3 branched along the (r, s)-torus knot.

Exercise 3.

A link L in S^3 is called **strongly-invertible** if there exists an orientation-preserving involution of S^3 inducing an involution with two fixed points on each component of L.

Let M be a 3-manifold presented by a surgery diagram along a strongly-invertible n-component link L. Show that M is a 2-fold covering of S^3 branched over a link of at most $(n+1)$ components and that conversely, any 2-fold branched covering of S^3 can be obtained by surgery along a strongly-invertible link.

Hint: See J. Montesinos: Surgery on links and double branched covers of S^3, in: *Knots, Groups and 3-Manifolds (AM-84)* **84**, 227–260.

Exercise 4.

Let K be a knot on the Heegaard surface of the standard genus-2 Heegaard splitting of S^3 representing a free generator of the fundamental group of both genus-2 Heegaard handlebodies. Show that there exists an integer surgery on K yielding a lens space.

Hint: See J. Berge: Some knots with surgeries yielding lens spaces, `arXiv:1802.09722`.

Remark: A knot K as in the exercise is called Berge knot. The Berge conjecture, a famous unsolved conjecture in 3-manifold topology, says that any knot in S^3 admitting a surgery to a lens space is a Berge knot.

p.t.o.

Bonus exercise.
Which 3-manifold is presented in the following surgery diagram?
Hint: Download and use the program KLO (Knot-Like Objects) from
`https://community.middlebury.edu/~mathanimations/klo/`
Verify that Haken's knot is an unknot using KLO (see Challenge from Sheet 1).

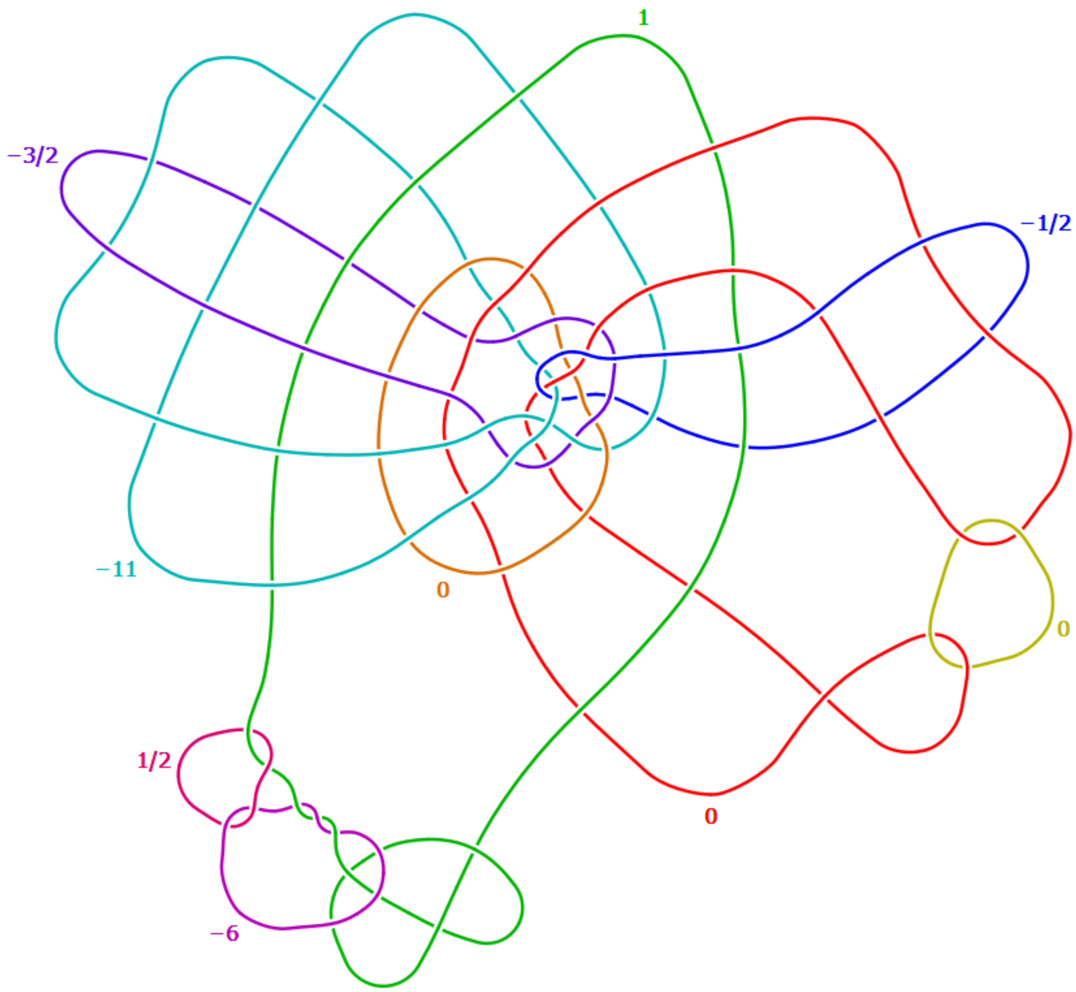

Abbildung 1: The attaching disks of the 1-handles are pairwise identified via a reflection along the horizontal middle line in this planar Heegaard diagram.

Bonus exercise. Understand the proof in the book by Prasolov and Sossinsky of the theorem stating that the Borromean rings are universal, see Section 24 and 25.

This sheet will not be discussed in an exercise session, but may be used to prepare for the oral exam. You may solve the tasks of course also for pure pleasure. At Wednesday 22.07.20 at 9:15 I will offer an office hour for discussing further questions about the lecture in which we can also speak about the exercises from this sheet.