SS 2021 Marc Kegel

4-Manifolds and Kirby calculus

Exercise sheet 3

Exercise 1.
Figure 1 shows framed knots in $S^1 \times S^2$.

(a) Which of these knots are (as framed knots) isotopic?

(b) Classify the isotopy classes of framed knots in $S^1 \times S^2$, that intersect the S^2-factor transversely exactly once.
Hint: What can you say about such knots without framings?

(c) Which manifolds are described by the Kirby diagrams in Figure 1?

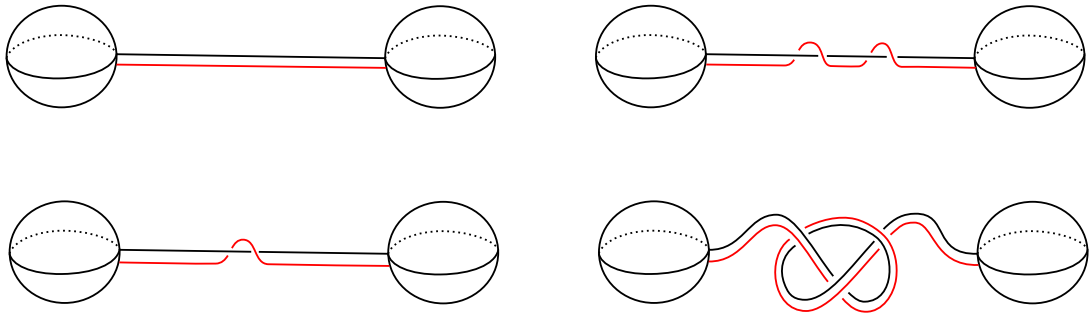

Abbildung 1: Four diagrams of framed knots in $S^1 \times S^2$.

Exercise 2.

(a) Show that the S^1-bundle over S^2 with Euler number $e \in \mathbb{Z}$ is diffeomorphic to the lens space $L(-e,1) = -L(e,1)$.
Hint: Consider the Kirby diagram of the D^2-bundle over S^2 with Euler number $e \in \mathbb{Z}$ from the lecture and compare their boundaries with the Heegaard diagrams of the lens spaces.

(b) Describe maps $S^1 \to L(e,1) \to S^2$ that induce the structure of an S^1-bundle over S^2 on the lens space $L(e,1)$.

(c) Draw a Kirby diagram of the complex projective plane $\mathbb{C}P^2$.

Exercise 3.

(a) Let F be a closed oriented surface with a handle decomposition with exactly one 0-handle and exactly one 2-handle.
Describe handle decompositions of $I \times F$ and $S^1 \times F$ that are induced by the given handle decomposition of F. Use this to draw Heegaard diagrams of $S^1 \times F$.

(b) Construct from a Heegaard diagram of a 3-manifold M a Kirby diagram of $S^1 \times M$.
Hint: It might be helpful to first describe a Kirby diagram of $I \times M$.

(c) Describe the 4-torus $T^4 = S^1 \times S^1 \times S^1 \times S^1$ as a Kirby diagram.

p.t.o.

Exercise 4.
An orientable manifold M is called **reversible** if it admits a diffeomorphism that reverses the orientation.

(a) Show that S^n and $S^1 \times M^{n-1}$ are reversible.

(b) How do we get from a planar Heegaard diagram of a 3-manifold M a planar Heegaard diagram of the same manifold with opposite orientation $-M$?

(c) How do we get from a Kirby diagram of a 4-manifold W a Kirby diagram of the same manifold with opposite orientation $-W$?

(d) Show via the Kirby diagrams of S^4, $S^1 \times S^3$ and T^4, that these 4-manifolds are reversible.

(e) Describe a Kirby diagram of $-\mathbb{C}P^2$.

(f) Is $\mathbb{C}P^2$ reversible? *Hint:* Consider the intersection form of $\mathbb{C}P^2$.

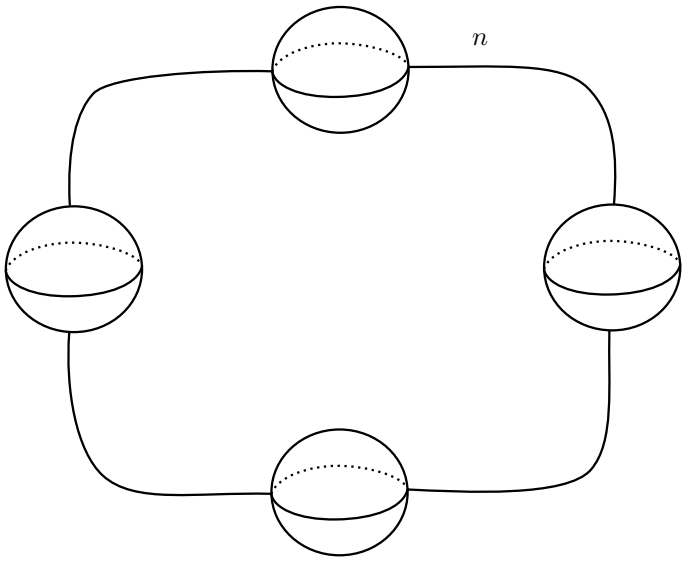

Abbildung 2: A D^2-bundle over T^2 with Euler number n.

Exercise 5.

(a) Show that the attaching knot of the 2-handle in the Kirby diagram of Figure 2 bounds a genus 1 surface. Explain how we get a well-defined (isotopy-invariant) 0-framing from this information and why we can describe a framing of this knot then via an integer $n \in \mathbb{Z}$.

(b) Show that the Kirby diagram in Figure 2 describes a D^2-bundle over T^2 with Euler number n.

(c) Draw a Kirby diagram of a D^2-bundle over a general genus g surface Σ_g of genus g with Euler number n.

This sheet will be discussed on Friday 21.5. and should be solved by then.