SS 2021 Marc Kegel

4-Manifolds and Kirby calculus

Exercise sheet 5

Exercise 1.
Which manifolds are described by the Kirby diagram from Figure 1?

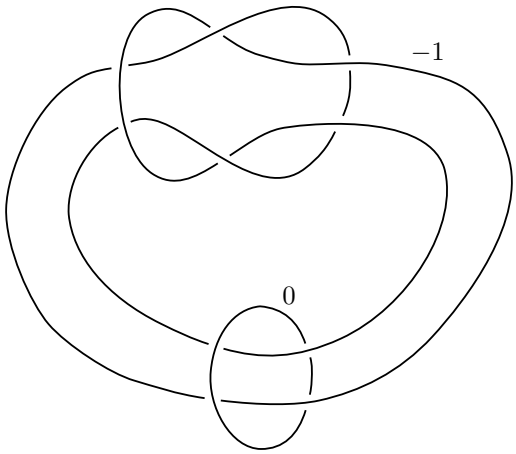

Abbildung 1: A Kirby diagram of a closed 4 manifold.

Exercise 2.
We denote the compact 4-manifold with boundary given by the left Kirby diagram in Figure 2 by P. Further let Q be the compact 4-manifold with boundary given by the right Kirby diagram in Figure 2.
Show that $P\#\mathbb{C}P^2$ is diffeomorphic to $Q\#\mathbb{C}P^2\#_7(-\mathbb{C}P^2)$.

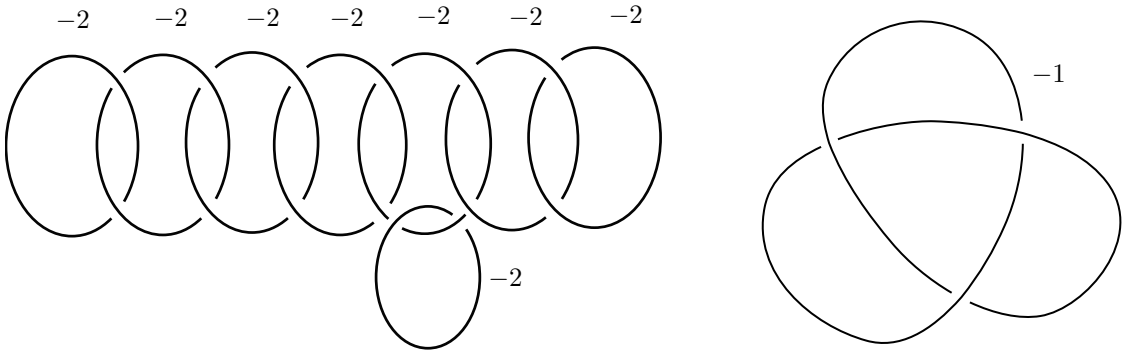

Abbildung 2: Kirby diagrams of compact 4-manifolds P (left) and Q (right).

p.t.o.

Exercise 3.

(a) How do you perform 2-handle slides of multiple parallel strands of attaching knots of 2-handles simultaneously? Describe examples and prove a general result.

(b) Describe a version of Lemma 5.1 from the lecture in which the attaching knots of the 2-handles are also allowed to pass through the (± 1)-framed unknot more than once.

Exercise 4.
Discuss S^2-bundles over general surfaces of genus g. Proceed analogously to the proof of Theorem 5.3 from the lecture.

Exercise 5.

(a) Describe a way to compute the fundamental group of a manifold with a given handle decomposition.

(b) The fundamental group of a compact smooth manifold is finitely presented. Conversely, we can get for any $n \geq 4$ any finitely presented group as the fundamental group of a closed oriented n-manifold.

(c) On the other hand, not every finitely presented group occurs as the fundamental group of a closed orientable 3-manifold. Groups arising as the fundamental group of a closed orientable 3-manifolds are called 3-**manifold groups**.
Hint: Let $\langle g_1, \dots g_n | r_1, \dots r_k \rangle$ be a finite presentation of a group G. We call $n - k$ the deficiency of this presentation. The **deficiency** of a finitely presented group G is the maximum deficiency of a finite presentation for G. Then you need to show that any 3-manifold group has non-negative deficiency and find a group with negative deficiency.

Bonus exercise.
Describe the effects of Kirby moves on the intersection form. Start with the case of a Kirby diagram without 1 and 3 handles.

Bonus exercise.
Prove the formula for the change of framing coefficients for a 2-handle slide by representing framings as parallel knots.

This sheet will be discussed on Friday 18.6. and should be solved by then.