EXERCISE
(JUNE 04 2019, TO BE DISCUSSED JUNE 25 2019)

DYSON SCHWINGER EQUATIONS (KREIMER, SUMMER '19)

All combinatorial Green functions

$$G^{(i)}(g) \in H[[g]],\; G^{(i)}(g) = 1 + \sum_{k=1}^{\infty} g^k c_k^{(i)},$$

are infinite series with coefficients in the Hopf algebra of decorated rooted tress. For all, please work the $c_k^{(i)}$ out up to $k = 3$ at least and check if they are closed under *Delta*.

- 1.

  $$G^{(1)}(g) = 1 + gB_+^a((G^{(1)})^3(g)),$$

  and

  $$G^{(1)}(g) = 1 + gB_+^a((G^{(1)})^3(g)) + g^2 B_+^b((G^{(1)})^5(g)).$$

- 2.

  $$G^{(1)}(g) = 1 + gB_+^a(G^{(1)}(g)G^{(2)}(g)),$$
  $$G^{(2)}(g) = 1 + gB_+^b((G^{(2)})^3(g)).$$

  Determine a condition for $G^{(2)}(g)$ so that you get a sub-Hopf algebra. Is there a corresponding co-ideal?

- 3.

  $$G^{(1)}(g) = 1 + gB_+^a((G^{(1)})^2(g)G^{(2)}(g)),$$
  $$G^{(2)}(g) = 1 + gB_+^b((G^{(2)})^4(g)).$$

  Determine a condition for $G^{(1)}(g), G^{(2)}(g)$ so that you get a sub-Hopf algebra. Is there a corresponding co-ideal?