DYSON–SCHWINGER EQUATIONS AND QUANTIZATION OF GAUGE THEORIES (SUMMER 21)

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1. Introduction

1.1. **Prerequisites.** Helpful: Some familiarity with Qantum Field Theory.
Script: `http://www2.mathematik.hu-berlin.de/~kreimer/wp-content/uploads/SkriptRGE.pdf` DSE

This is an advanced course on the actual state of research. You will be considered as a budding scientist. No more no less.

Course homepage (will be updated regularly):
`http://www2.mathematik.hu-berlin.de/ kreimer/wp-content/uploads/SkriptDSE.pdf`
On that homepage is also the moodle link to the course.

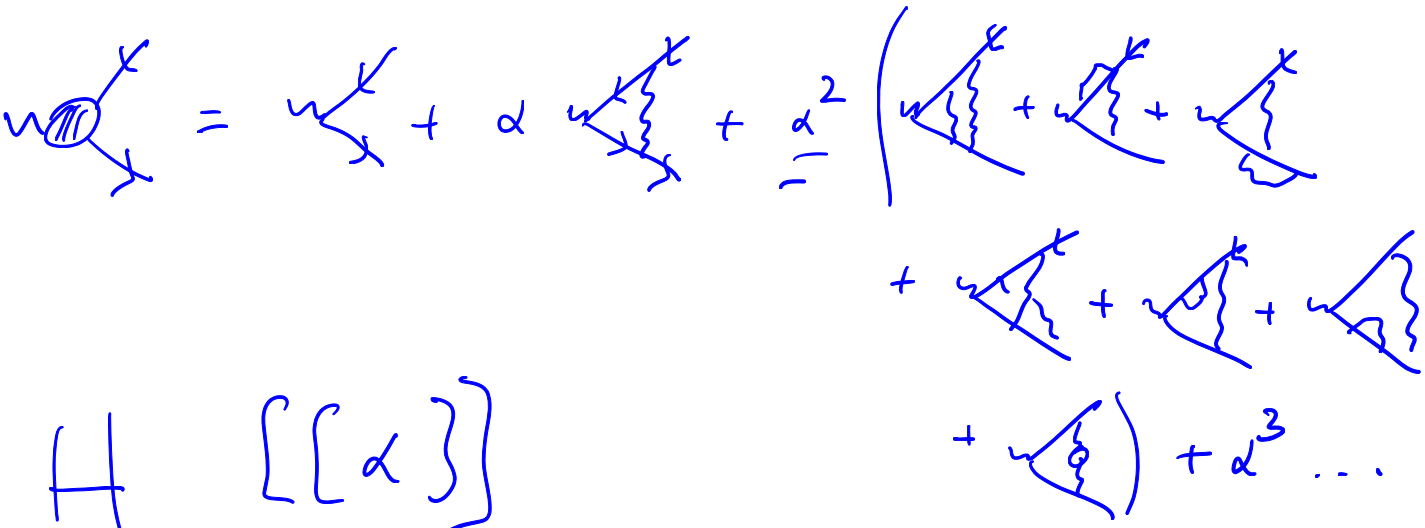
Exercises: as needed. Could be reading exercises, demand to give little presentations on chosen topics, or request to check computations, or whatever.
(Check moodle instructions how to submit material if needed).

Final exam: oral, probably online.
DO NOT FORGET TO REGISTER FOR THE FINAL EXAM WITH THE EXAMINATION OFFICE (Prüfungsbüro Mathematik oder Physik) IN TIME (do this say by mid June)

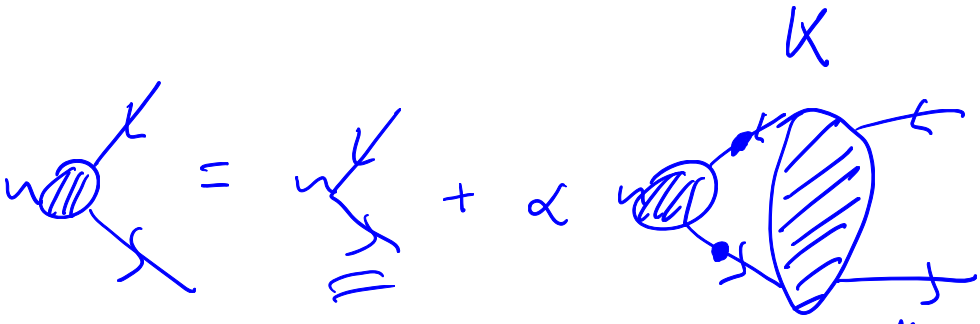
2. Topics

- (April 12) Overview
- (April 13) Recap Hopf algebras
- (April 19) fixed point equations from Hopf algebras
- (April 20) Exercise class
- (April 26) Combinatorial DSE
- (April 27) Massless Yukawa example
- (May 03) sub Hopf algebras
- (May 04) Exercise class
- (May 10) rooted tree examples
- (May 11) Foissy's classification
- (May 17) QED Ward ids
- (May 18) Exercise class
- (May 25) Anatomy of a gauge theory
- (May 31) cont'd
- (June 01) Exercise class
- (June 07) Graph homology and gauge theory
- (June 08) ODEs and gauge theory
- (June 14) nonperturbative Yukawa example
- (June 15) Exercise class
- (June 21) field diffeomorphisms
- (June 22) quantum gravity
- (June 28) cont'd
- (June 29) Exercise class
- (July 05) Hodge structures
- (July 06) cointeracting bialgebras and DSE
- (July 12) recap

Go back to an old textbook on QFT. Bjorken Drell:

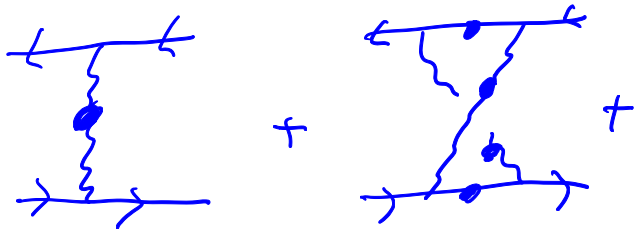


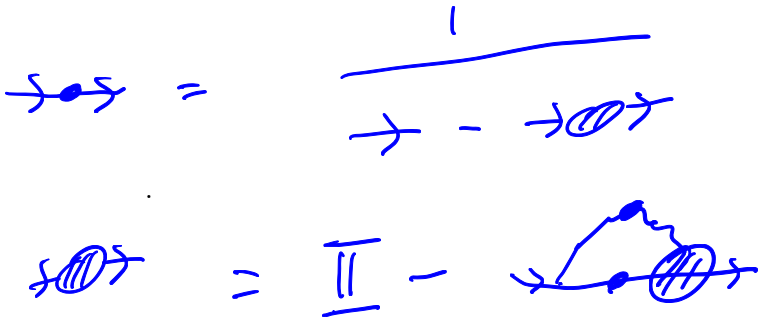
$H_{FG}[[\alpha]]$



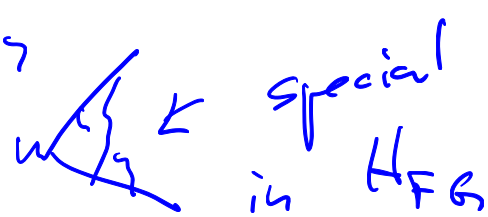
"2PI" in the s-channel

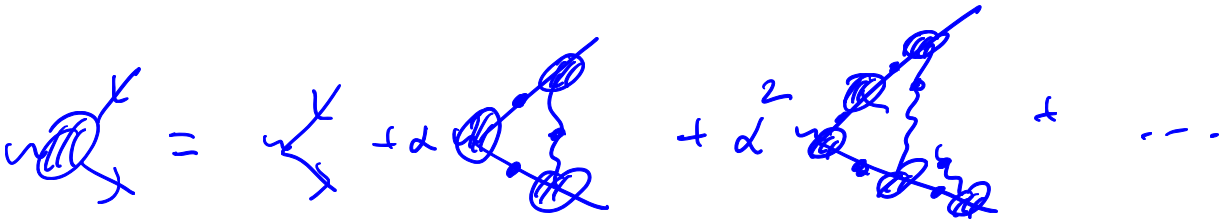
$K =$



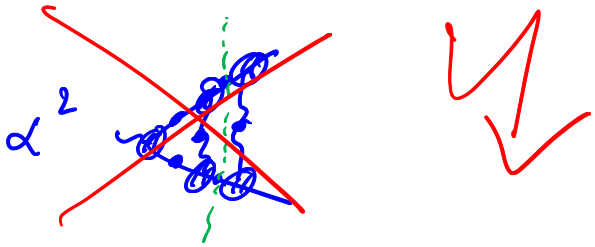


This can be rewritten as





but not



$\Rightarrow$ Such equations are fixed point eq's in the Hochschild cohomology of the Hopf algebra of Feynman graphs.

Next: gauge theories.

In QED say, you have Ward identities.

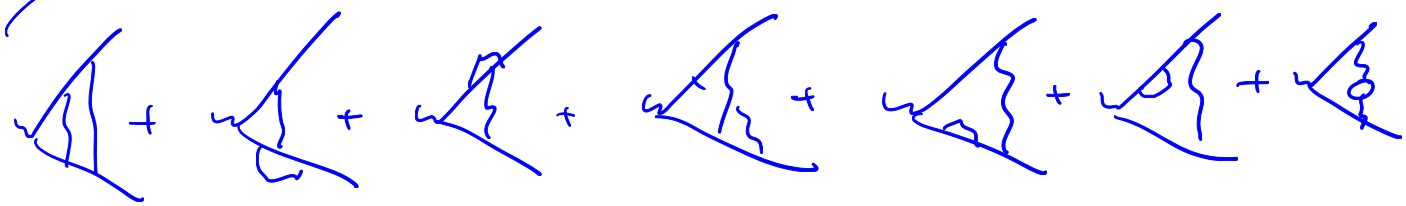
ϕ_R $= G_R^{\mu\nu}(q, p_1, p_2; \alpha; \mu)_\mu$

$$q^\mu \cdot G_\mu = \sum (p_1; \alpha; \mu) - \sum (p_2; \alpha; \mu)$$

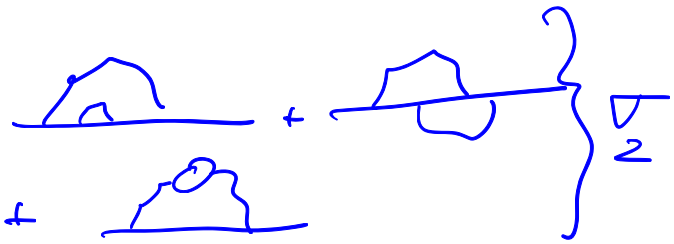
↑ inverse propagator for fermion

Ward - Takahashi id's.

g_2



$G_\mu^{\mu\nu}$ to two loops

Σ to two loops: 

$g_2, \sigma_2 \in H_{FG}$

$\Delta : H_{FG} \to H_{FG} \otimes H_{FG}$

$$\Delta(\underbrace{g_2 + \sigma_2}_{i_2})$$

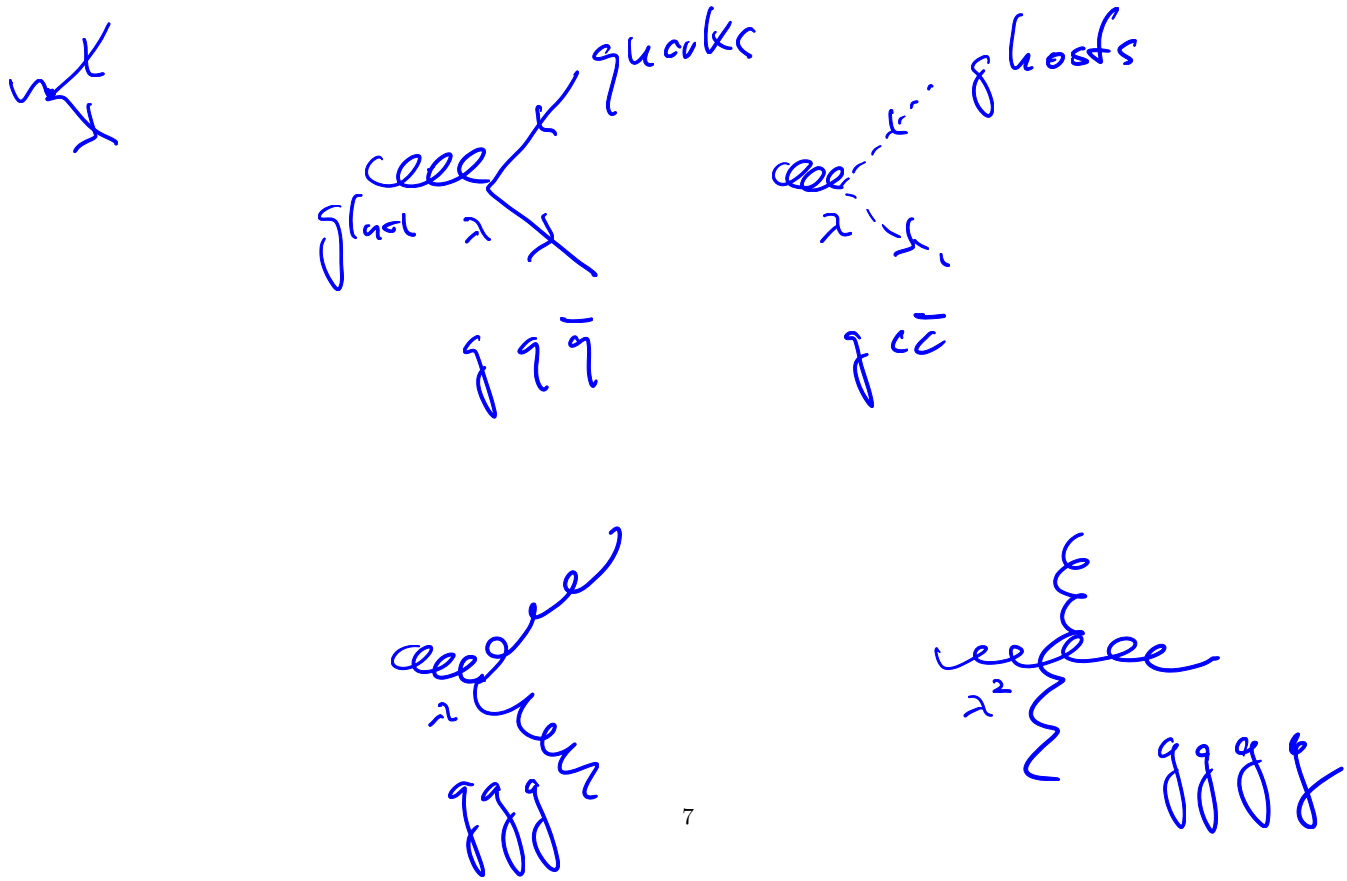
$$\Delta(i_n) \subseteq H_{FG} \otimes I + I \otimes H_{FG}$$

$$I = \mathrm{span}_n \langle i_n \rangle$$

$$\phi_R(i_n) = 0 \iff \text{W-T ids.}$$

Next step: Slavnov Taylor id's in non-abelian gauge th.

But again, there exists maximal ideals and co-ideals $I_1, \dots, I_4$ s.t. they give the S-T ids. expressing that after quantization & renormalization you have just one charge in an $SU(N)$ gauge theory.



Co-ideals reflect internal symmetry. (holds also after spontaneous symmetry breaking, and in susy theories).

This gives you ideas about more general theories.

i) field diffeomorphisms of free theories.

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2$$

$$\phi = \rho + c_1 \rho^2 + c_2 \rho^3 + \dots$$

gives an interacting theory in ρ-fields which must be unitarily equivalent to free

theory at the level of S-matrix.

This is true for certain renormalization schemes, those which allow the definition of S-matrix.

Again, co-ideals. (for infinitely many).

Quantum gravity:

has vertices of any valence

one can make about co-ideals and Ward id's in QG.

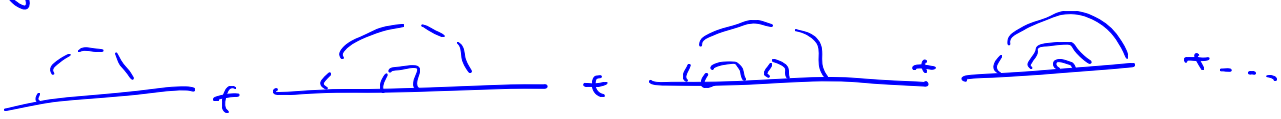
Non perturbative physics:

DSE → ODE's

typically, non-linear

anom. dimension

$$\gamma(\alpha) = \gamma(\alpha)\,(1 + \alpha\,\partial_\alpha)\,\gamma(\alpha)$$



Solutions involve series

$$y(x) = c_1 x + \cdots$$

$$c(x) + d(x) e^{-\frac{1}{x}} + e(x) e^{-\frac{2}{x}} + f(x) e^{-\frac{3}{x}} + \cdots$$

$d(x)$ and all the others are functions of $c(x)$.

Hodge theory and number theory of Feynman graphs and how it connects to DSE.

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Humboldt U. Berlin