DYSON–SCHWINGER EQUATIONS AND QUANTIZATION OF GAUGE THEORIES (SUMMER '21)

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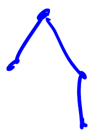
1. Hopf algebras and DSE

1.1. Hopf algebra Examples. .

$H_{\text{rooted trees}}$



$$X(\alpha) = \mathbb{I} + \alpha B_+(X^n(\alpha))$$

$$X(\alpha) = \mathbb{I} + \alpha B_+(F(X(\alpha)))$$

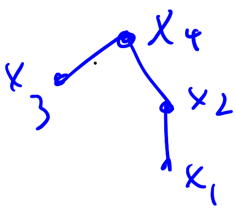
which F are allowed?

every F needed in physics is allowed...

X^4, $\left(\frac{1}{X}\right)^n$, ...

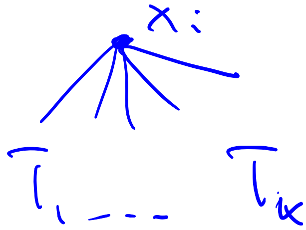
More interesting Hopf algebras

"decorated rooted trees"



$\tilde{\Delta}\left(\,{}^{\bullet x_2}_{\,\bullet x_1}\right) = \bullet_{x_1} \otimes \bullet_{x_2}$

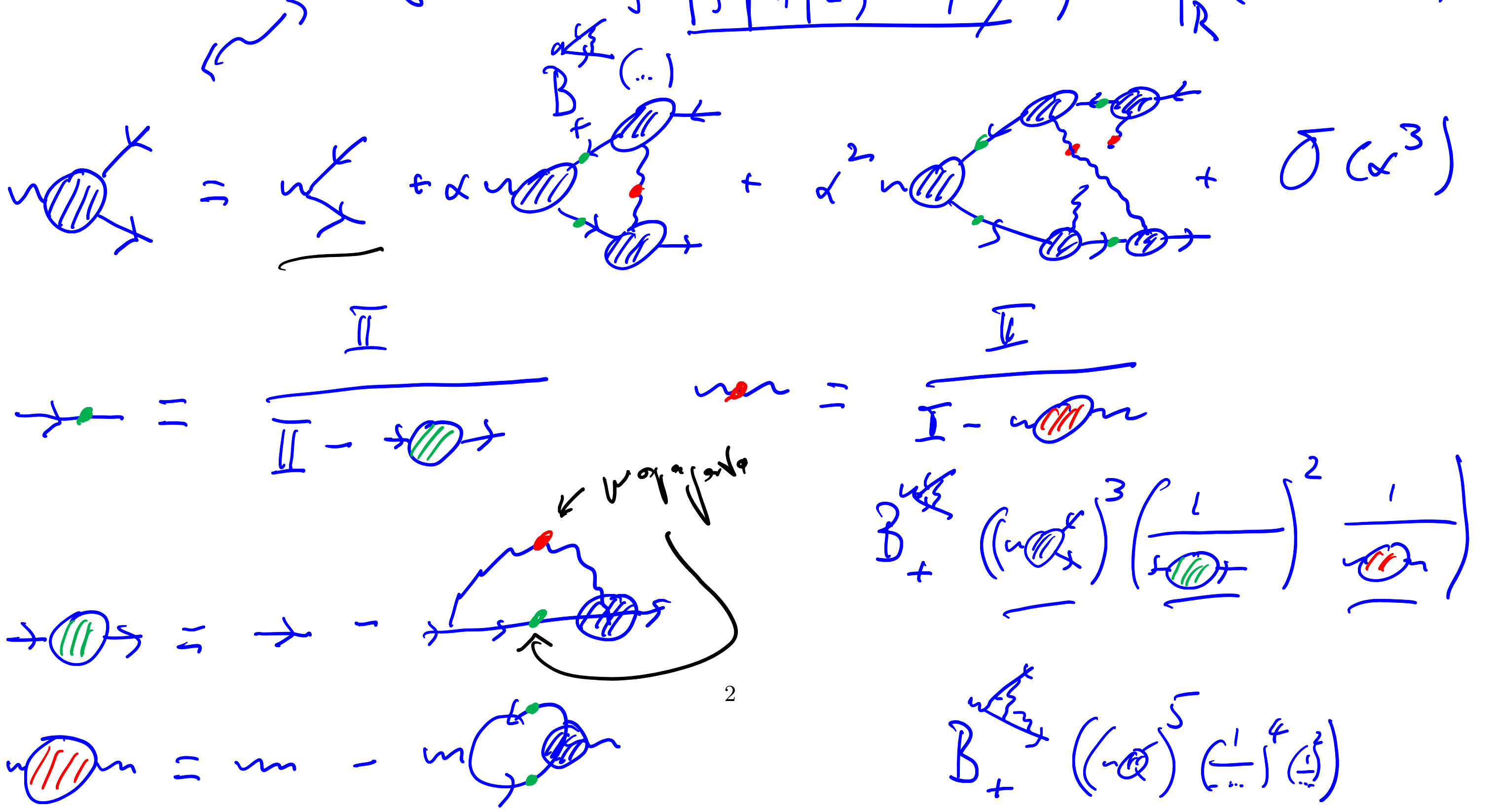
$B_+^{x_i}(T_1 \ldots T_k) =$

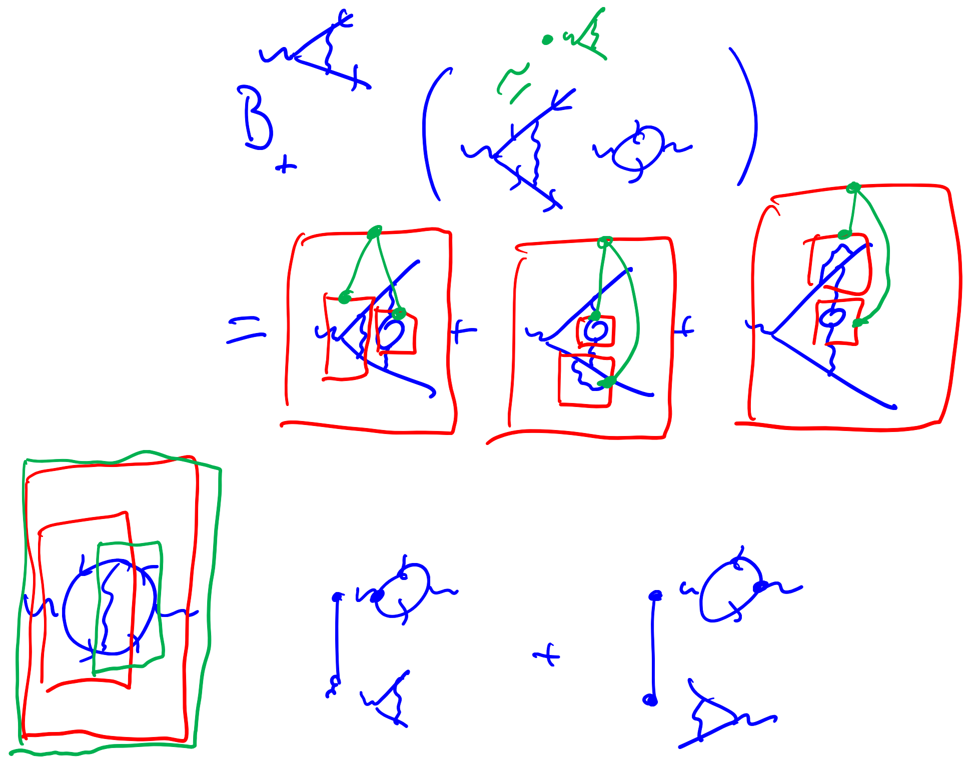


→ systems of Dyson Schwinger equations.

This is needed in physics:

QED: $G^{\ldots}(\alpha;\, q, p_1, p_2, m, \mu) = \Phi_R(X^{\ldots}(\alpha))(\ldots)$





Basically, decorated rooted trees are good to describe all QFT.

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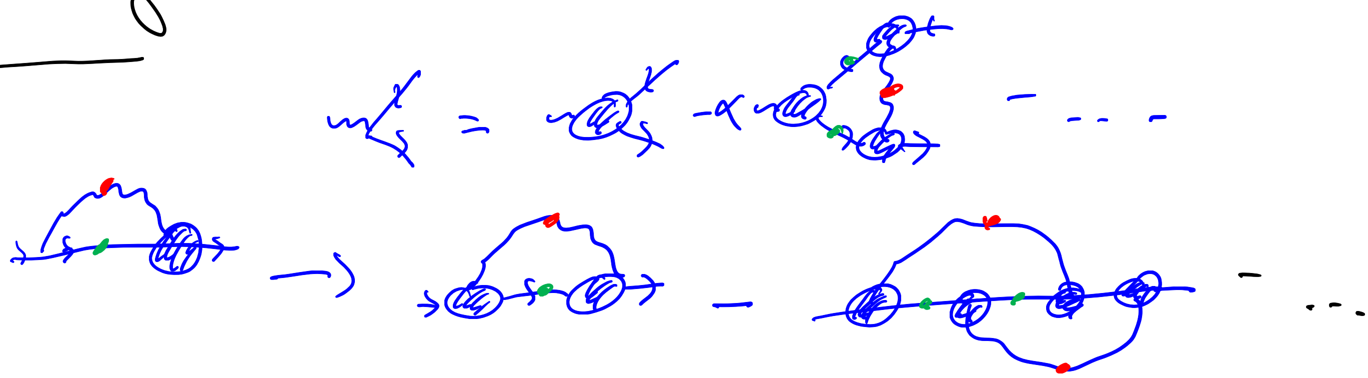
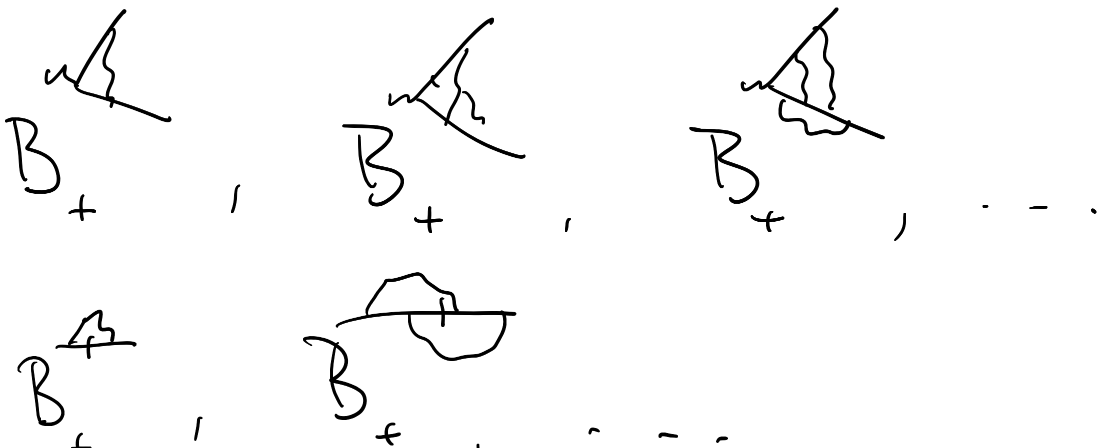
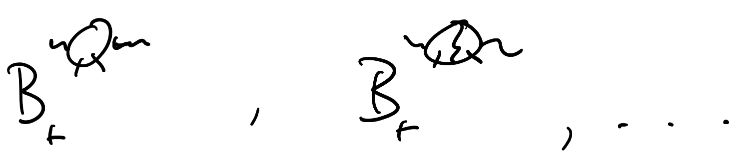
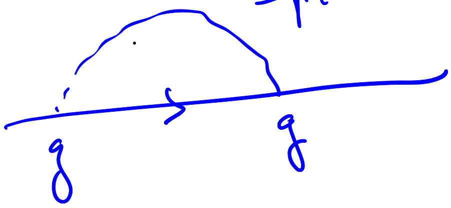


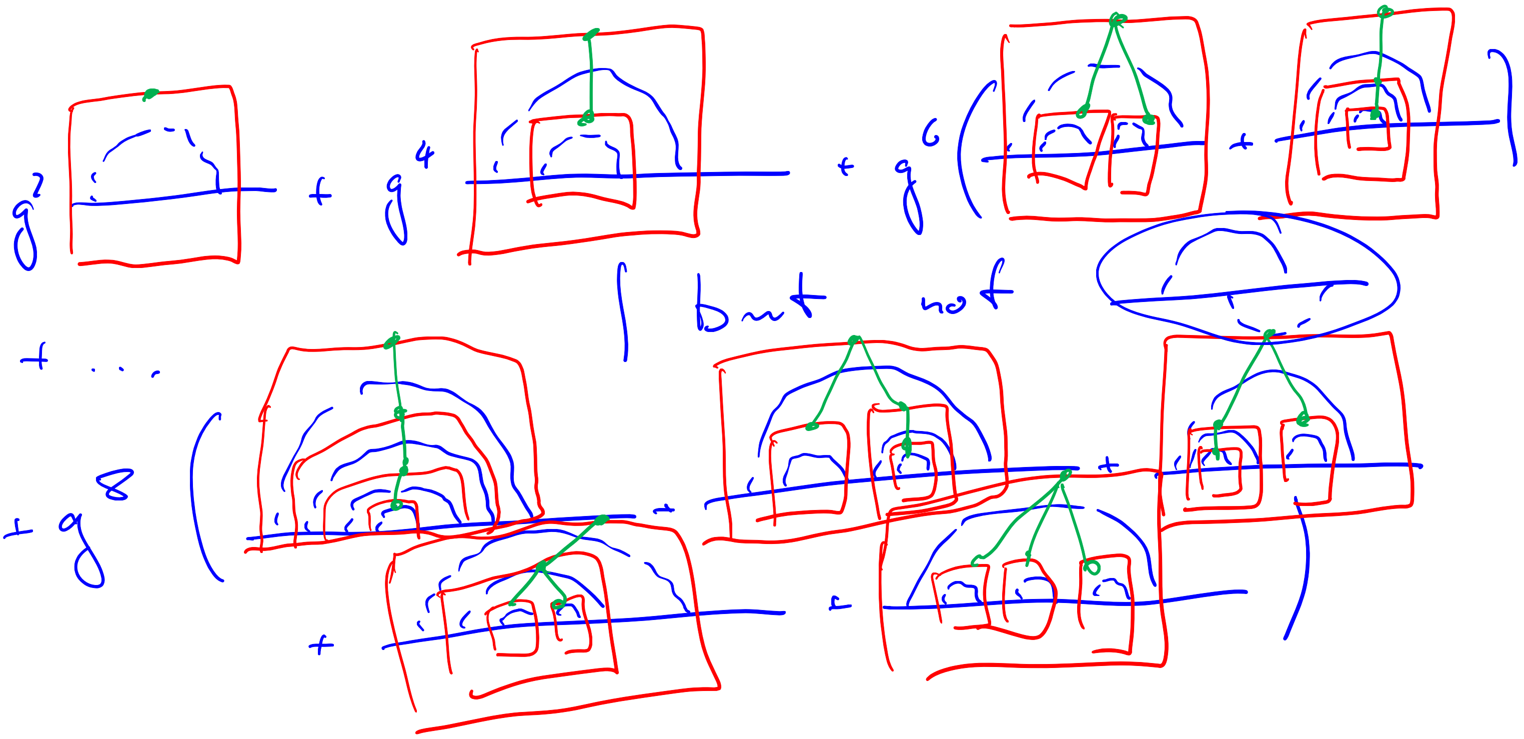


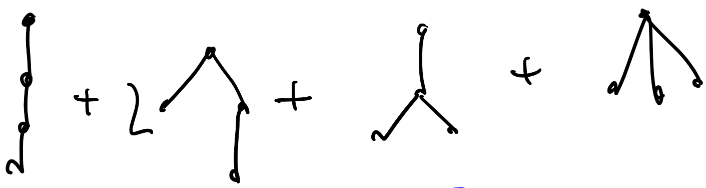


Yukawa theory, massless spin 0



sum up the "Kilroy diagrams"





$$X(g^2) = \mathbb{I} - g^2 B_+ \left(\frac{1}{X(g^2)} \right)$$

$$= \sum_{j=0}^{\infty} (g^2)^j \, c_j \,, \quad c_0 = \mathbb{I} .$$

The c_j form a sub Hopf alg.

$$\Phi_R(X(q^2)) \equiv q\!\!\!/\, G_R\left(g^2, \ln \frac{q^2}{\mu^2}\right)$$

$$G_R\left(g^2, \ln\frac{q^2}{\mu^2}\right) = q\!\!\!/ - g^2\left\{\int d^4k \frac{\operatorname{Tr}(q\!\!\!/ k\!\!\!/)}{(k+q)^2\, G_R\left(g^2, \ln\frac{k^2}{\mu^2}\right)} - \int \cdots \Big|_{q^2=\mu^2}\right\}$$

What are the $\Phi_R(c_j)$?

What can I learn from the fact that the c_j form a sub-Hopf algebra?

What can we learn beyond perturbation theory?

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which Hopf algebras?

You need decorated rooted trees (and administer symmetry factors)

Which DSE ?

Basically, rational functions F of all $X^i(g^i)$.

1.2. **Fixed point equations: Examples.** .

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1.3. **Linear DSE.** .

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Humboldt U. Berlin