DYSON–SCHWINGER EQUATIONS AND QUANTIZATION OF GAUGE THEORIES (SUMMER '21)

DIRK KREIMER (LECT.-EXERC. APRIL 26, 2021)

1. General Structure of Combinatorial DSE

1.1. DSE and RGE. .

Dyson-Schwinger eq's

→ Renormalization group eq.

Let us look at a linear DSE.

Take massless Yukawa theory.

$\not{q}\, \underline{F(q^2)}$

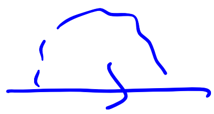
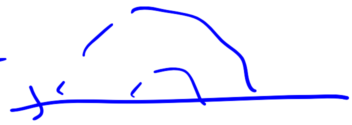
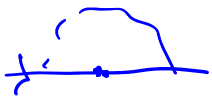
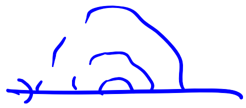
↓

$= \not{q}$

$$\Phi_R(q) = \frac{1}{q^2} \int d^4k \, \frac{\mathrm{Tr}\, \not{k}\not{q}}{k^2 (k+q)^2}$$

$$\Phi_R(q)(\rho) = \frac{1}{q^2}\int d^4k \, \frac{k\cdot q}{(k^2)^{1+\rho}\,((k+q)^2)}$$

$$= (q^2)^{-\rho}\int d^4k \, \underbrace{\frac{k\cdot \hat q}{(k^2)^{1+\rho}\,((k+\hat q)^2)}}_{-\frac{1}{\rho(2-\rho)}}$$

$\mathbb{I} + \alpha$  $+ \, \alpha^2$   $+ \, \alpha^3$  $+ \ldots$

$$X(\alpha) = \mathbb{I} + \alpha \, B_+\big(X(\alpha)\big)$$

↑ linear comb. DSE

Make an Ansatz $L = \log \frac{q^2}{\mu^2}$

$$\Phi_R\big(X(\alpha)\big) = G(L,\alpha),$$

with $G(1, \alpha) = 1$

Ansatz:

$$G(L,\alpha) = \left(\frac{q^2}{\mu^2}\right)^{-\gamma(\alpha)}$$

$$= \exp(-\gamma(\alpha) L).$$

$$\left(\frac{q^2}{\mu^2}\right)^{-\gamma(\alpha)} = 1 + \alpha \frac{1}{q^2}\left\{\int d^4k \; G\left(\ln\frac{k^2}{\mu^2},\alpha\right) \frac{q\cdot k}{k^2 (k+q)^2} - \dots\right\}$$

$$= 1 + \alpha \frac{1}{q^2}\left\{\int d^4k \left(\frac{k^2}{\mu^2}\right)^{-\gamma(\alpha)} \frac{q\cdot k}{k^2 (k+q)^2} - \dots \; q^2=\mu^2\right\}$$

$$\left(\frac{q^2}{\mu^2}\right)^{-\gamma(\alpha)} = 1 + \alpha \left(\frac{-1}{\gamma(\alpha)(2-\gamma(\alpha))}\left(\left(\frac{q^2}{\mu^2}\right)^{-\gamma(\alpha)} - 1\right)\right)$$

$$\left(\left(\frac{q^2}{\mu^2}\right)^{-\gamma(\alpha)} - 1\right) = \alpha \left(\frac{-1}{\gamma(\alpha)(2-\gamma(\alpha))}\right)\left(\left(\frac{q^2}{\mu^2}\right)^{-\gamma(\alpha)} - 1\right)$$

$$1 = -\alpha \frac{1}{\gamma(\alpha)(2-\gamma(\alpha))}$$

This is a quadratic eq for $\gamma(\alpha)$

Take the solution which has $\gamma(\alpha)$: $\gamma(0) = 0$.

$\hookrightarrow \gamma(\alpha)$.

$\hookrightarrow G(L, \alpha) = \left(\frac{q^2}{\mu^2}\right)^{-\gamma(\alpha)}$

This way you can solve any linear DSE, for systems, read H. Kissler.

" Systems of lin. DSE's. "

Usually, physics is not scale invariant.

all allowed

$$X(\alpha) = \mathbb{I} + \alpha B_+(X(\alpha)) \quad \text{linear}$$

by

$$X(\alpha) = \mathbb{I} - \alpha B_+\left(\frac{1}{X(\alpha)}\right) \quad \text{non-linear}$$

.

.

.

.

1.2. **Yukawa Example.** .

.

.

.

.

.

.

.

Humboldt U. Berlin