Dyson Schwinger Eq's and Quantization of gauge theories (Summer '21)

Dirk Kreimer lect. June 21 2021

$$\Phi_R\left(q \overset{m_1}{\underset{m_2}{-\!\bigcirc\!-}} q\right) = \frac{\sqrt{\lambda(q^2,m_1^2,m_2^2)}}{2q^2}\left\{\ln\left(\frac{q^2-m_1^2-m_2^2+\sqrt{}}{q^2-m_1^2-m_2^2-\sqrt{}}\right) + \dots\right\}$$

$$\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$$

$$\Im\left(\Phi_R(q^2)\right) = \Theta\left(q^2-(m_1+m_2)^2\right)\frac{\sqrt{\lambda}}{2q^2}$$

Wightman axioms:

monodromy in full Green fct.

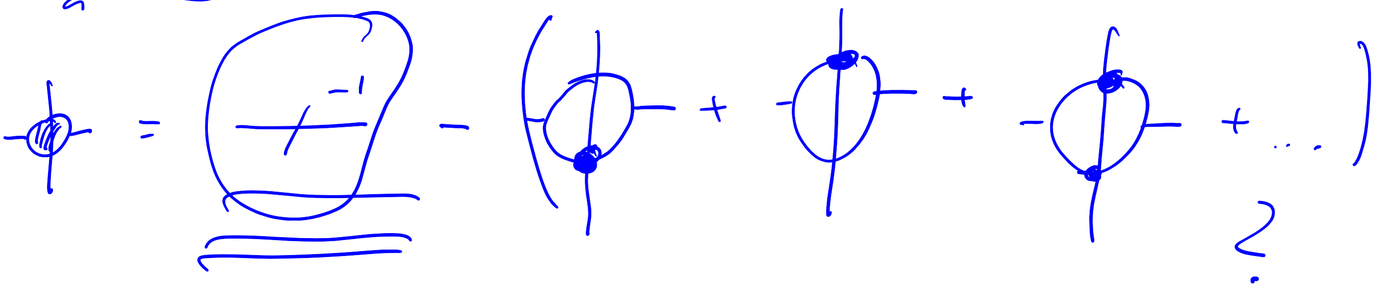
So far $X^- = \mathbb{I} - \sum_{j=1}^{\infty}\alpha^j B_+^{j,2}\left(X^- Q^{2j}\right)$

$$\Phi_R(X^-)(q^2,\dots)$$

$$\Im\,\Phi_R(X^-) = \delta(q^2-m^2) - \int_{(m_1+m_2)^2}^{\infty}\frac{\rho(x)}{x(x-m^2)}\,dx$$

Källen-representation

Wanted: $\Im\,\Phi_R(X^r)$ as a solution to a fixed pt equation, so as a solution to a DSE.



To solve that problem:

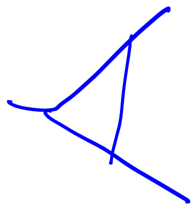
i) need core Hopf algebra

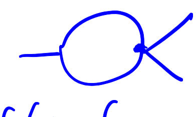
H_{core} $\qquad \Delta_{core}(\Gamma) = \Gamma \otimes \mathbb{I} + \mathbb{I} \otimes \Gamma + \sum_{\gamma \subsetneq \Gamma} \gamma \otimes \Gamma/\gamma$

$\gamma = \coprod \gamma_i$

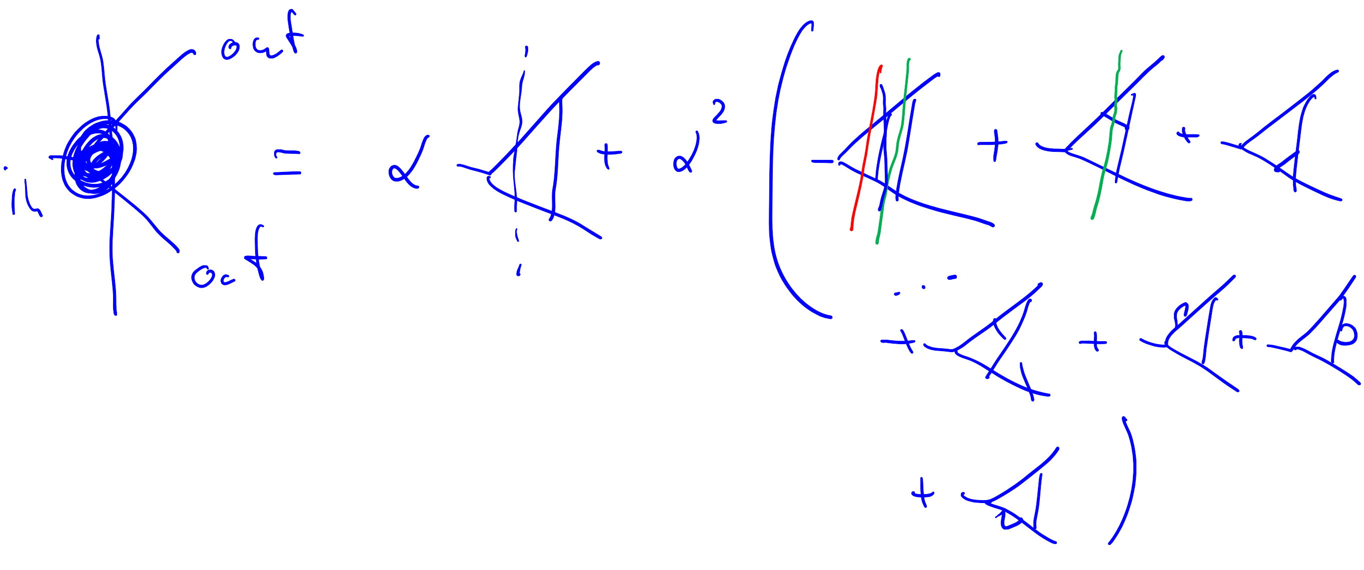
as quotient Hopf algebras, you have renormalization Hopf algebras as needed (ϕ^3_6 - graph)

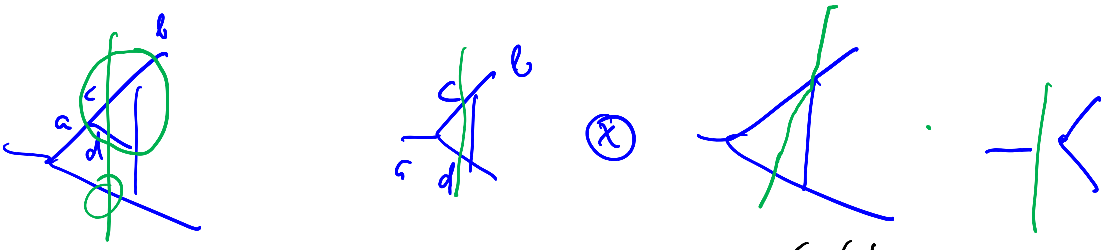
monodromy of



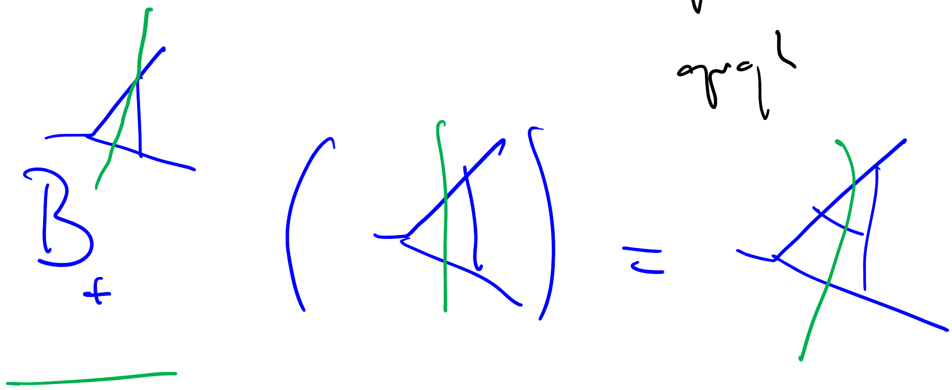


ii) Hopf algebra of pre-Cutkosky graphs.





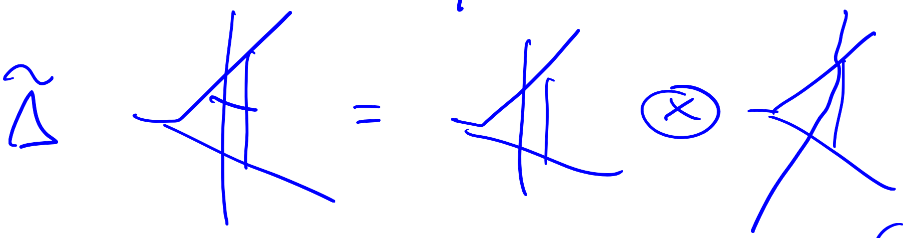
pre-Cutkosky graph



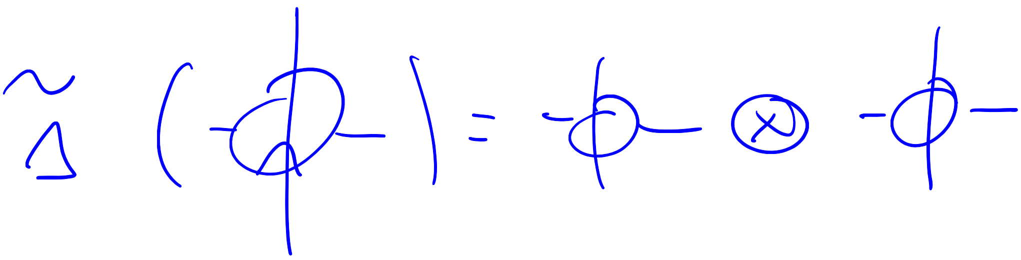
1-cocycle

fl$_{pc}$?

iii) H_c vector space of Cutkosky



Cutkosky pre-Cutkosky



iv) Co-actions

$H \xrightarrow[\Delta]{} H \otimes H$

Start with us V, find a

map:

Hopf alg. Δ

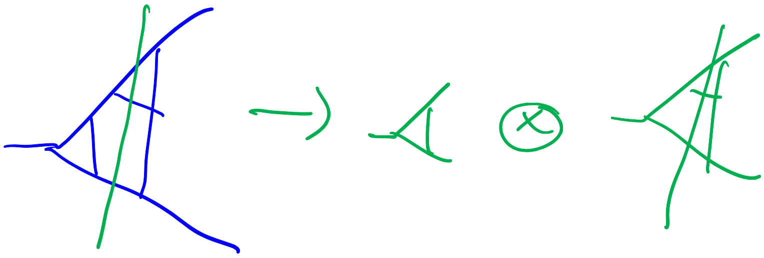
$$\bar{\Delta} : V \to H \otimes V$$

such that

$$(id \otimes \bar{\Delta})\,\bar{\Delta} : V \to H \otimes H \otimes V$$
$$\shortparallel$$
$$(\Delta \otimes id)\,\bar{\Delta} : V \to H \otimes H \otimes V$$

this gives a co-action.

$\tilde{\bar{\Delta}}_{core}$



$\bar{\Delta}_{ren}$ ....

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monodromy co-action S.

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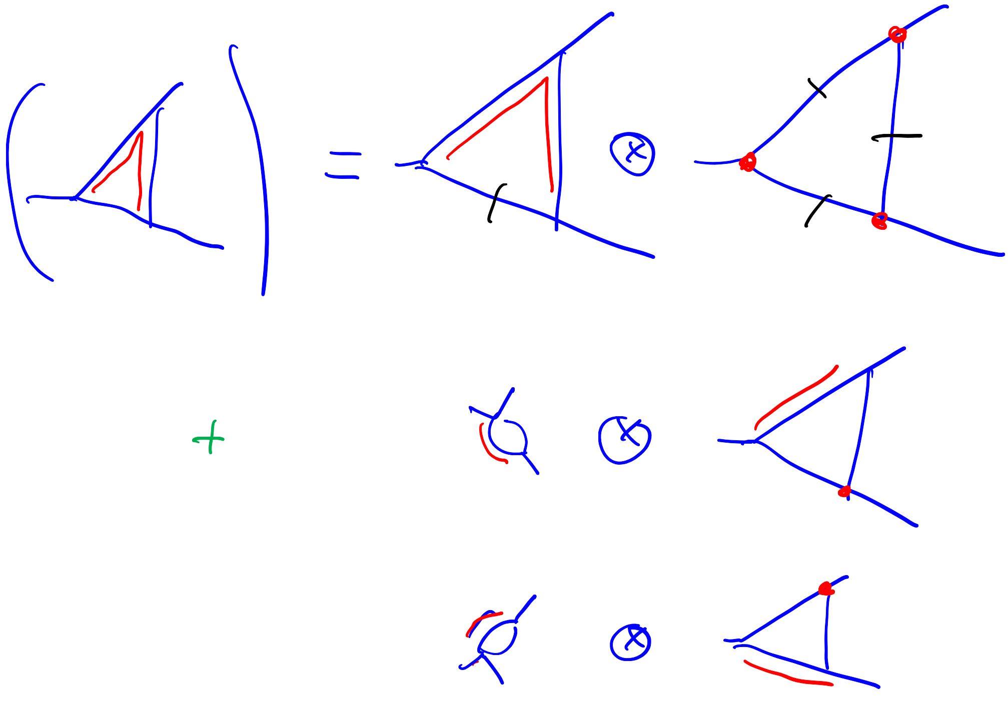
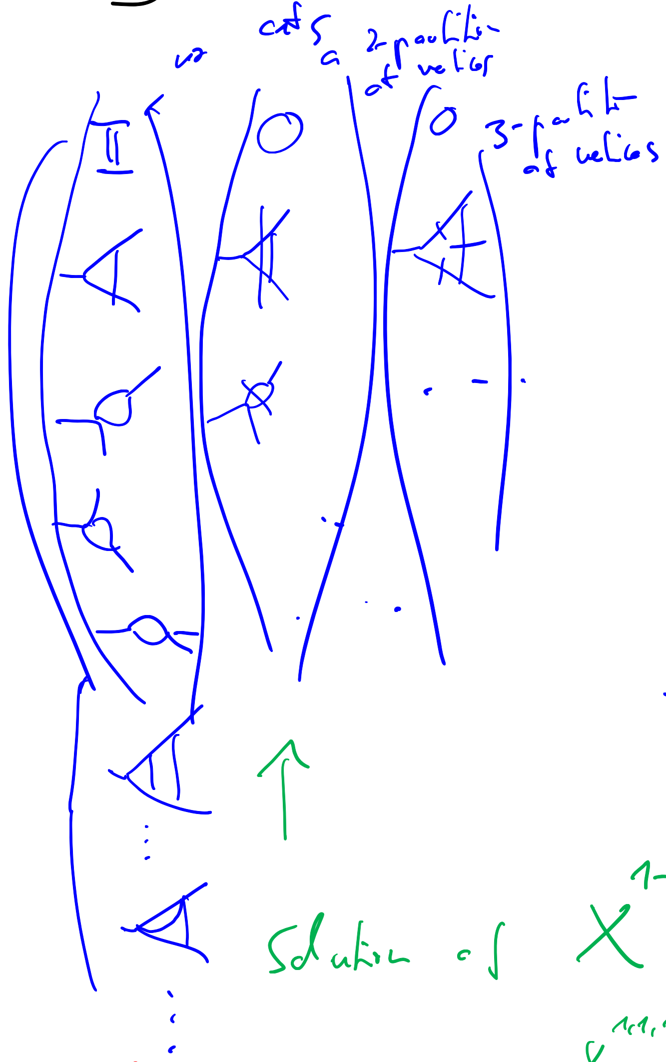


this coaction co-exists "nicely" with the one above to give a "Co-interacting Bialgebras" (work of Manchon, Foissy, ...).

↳ $\exists$ a matrix M^T



these are Hodge matrices

Solution of $X^{1\to 2} = \sum B_+^{1\to 2, j}(X^{\dots})$

$X^{1,1,1}$

$\underline{Q}$

left most column is solution of $X = \mathbb{I} \pm \sum_j \alpha^j B_+^{r,j}(\dots)$

Lower triangular matrices which are Hodge,
being Hodge $\Longleftrightarrow$ the $C^{(i+1)}$ column fixes
the monodromy of the column $C^{(i)}$, row by row.

Famous example: the polylog

$$Li_k(z) = \int_1^z \frac{Li_{k-1}(\tilde z)}{\tilde z}\, d\tilde z \qquad \forall\ k \geq 2$$

$$Li_1(z) = -\log(1-z)$$

$$Li_2(z) = \int_1^z \frac{Li_1(\tilde z)}{\tilde z}\, d\tilde z$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ Li_1(z) & 2\pi i & 0 & 0 & \cdots \\ Li_2(z) & 2\pi i \log z & (2\pi i)^2 & 0 & \cdots \\ Li_3(z) & 2\pi i \frac{1}{2}\log^2 z & (2\pi i)^2 \log z & (2\pi i)^3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \\ & 2\pi i \frac{1}{n!}\log^n z & (2\pi i)^2 \log^{n-1} z & (2\pi i)^3 \log^{n-2} z & \cdots \\ & \vdots & \vdots & \vdots & \end{pmatrix}$$

$$z \frac{\partial Li_k(z)}{\partial z} = Li_{k-1}(z)$$

Want to do the same for Feynman graphs