## DYSON–SCHWINGER EQUATIONS AND QUANTIZATION OF GAUGE THEORIES (SUMMER '21)

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# 1. General Structure of Combinatorial DSE

## 1.1. **Non-perturbative aspects.**

1.1.1. *combinatorial aspects.* Reference: Hopf subalgebras of the Hopf algebra of rooted trees coming from Dyson-Schwinger equations and Faà di Bruno Lie algebras. Motives, QFT and PsDO, Clay Math. Proc. 12 (2010), 189-210. (`http://loic.foissy.free.fr/pageperso/p18.pdf`)
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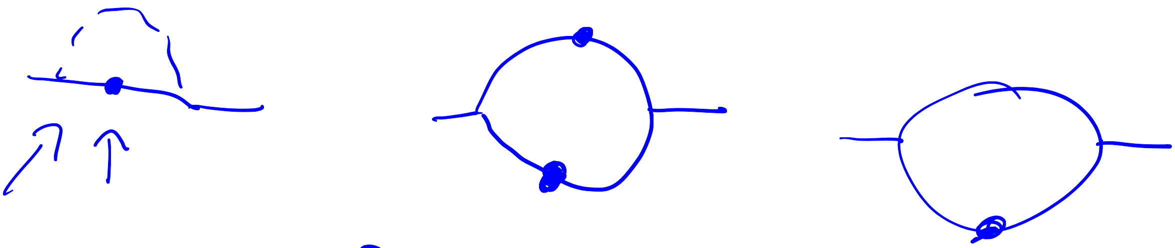


$\phi^3_6$ is renormalizable

Today: wrap combinatorial aspects of DSE up, from tomorrow: gauge theory.

$$X(\alpha) = 1 - \alpha\, B_+\left(\frac{1}{X(\alpha)}\right)$$

$$\frac{1}{X} = X \frac{1}{X^{2}} \simeq Q$$

invariant

$$\dots \qquad B_+(X\,Q)$$

$$w^{(m)} = \mathbb{I} - \alpha\, B_+^{k;n}\left(X^{n} \frac{(X^{k})^2}{\underbrace{X^{k\,2} X^{m}}}\right)$$

$\parallel$

$X^{m}$

by Ward:

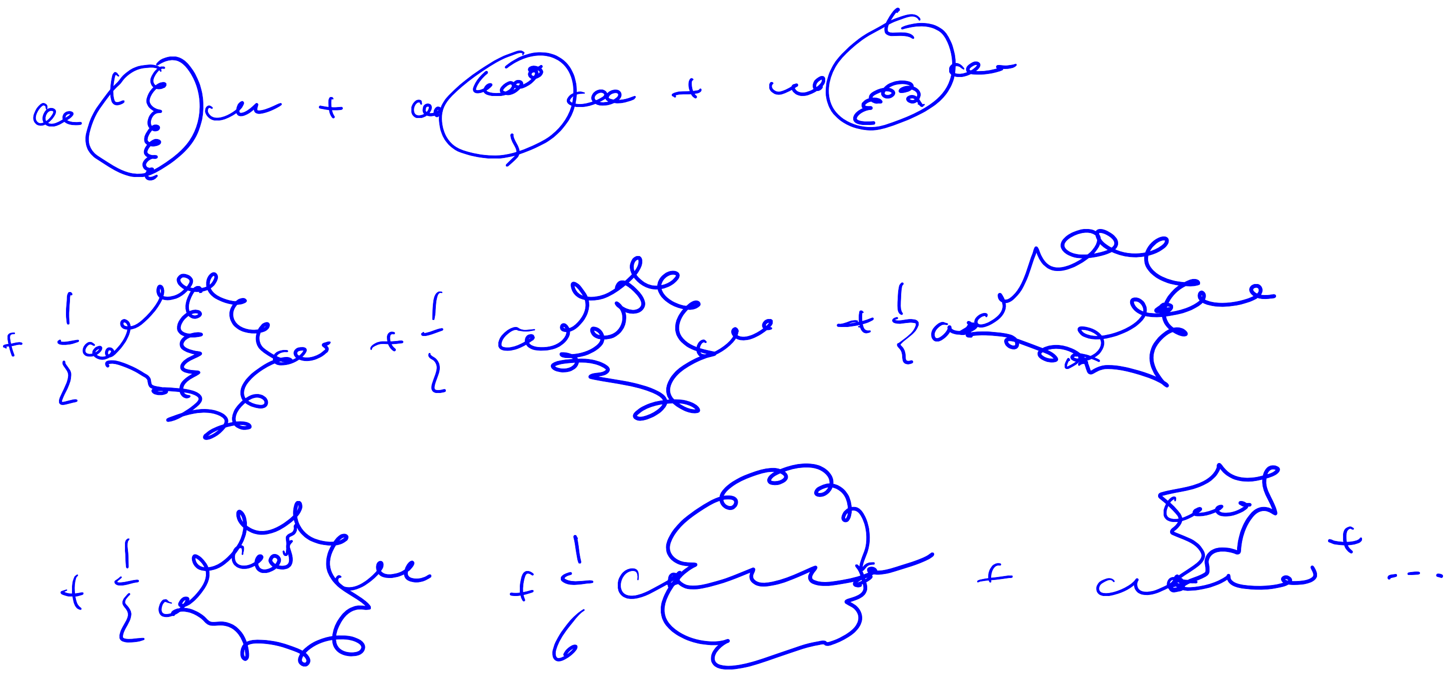
$$X^{nk} \simeq X^{k}\, Q^2$$

$$Q^2 = \frac{1}{X^{m}}$$

"corresponds to" "$\underline{s=1}$"

gauge theory:

For non-abelian gauge theory, it looks *much* harder:



+ ghost terms.

1.1.2. *Analytic aspects: the work of Dunne et.al.* Gerald Dunne: *Resurgent Asymptotics of Hopf Algebraic Dyson-Schwinger Equations*, Talk at ESI, Vienna,October 2020, Yukawa
https://www.esi.ac.at//uploads/48d56e70-6463-4a59-aeb7-50b6c7df2044.pdf

M. Borinsky and G. V. Dunne, *Non-Perturbative Completion of Hopf-Algebraic Dyson-Schwinger Equations,* Nucl. Phys. B **957** (2020), 115096 doi:10.1016/j.nuclphysb.2020.115096 [arXiv:2005.04265 [hep-th]].

M. Borinsky, G. V. Dunne and M. Meynig, *Semiclassical Trans-Series from the Perturbative Hopf-Algebraic Dyson-Schwinger Equations: $\phi^3$ QFT in 6 Dimensions,* [arXiv:2104.00593 [hep-th]].

$\uparrow \phi^3_6$

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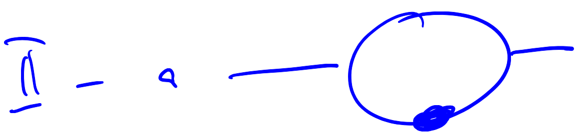
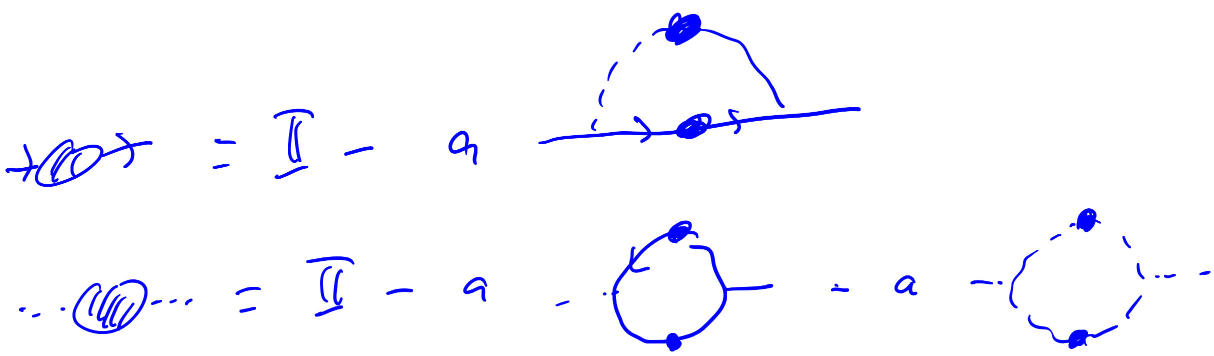
Interesting:

Yukawa:

$$X(a) = \mathbb{I} - a\, B_+\left(\frac{1}{X(a)}\right)$$

$\phi^3$:

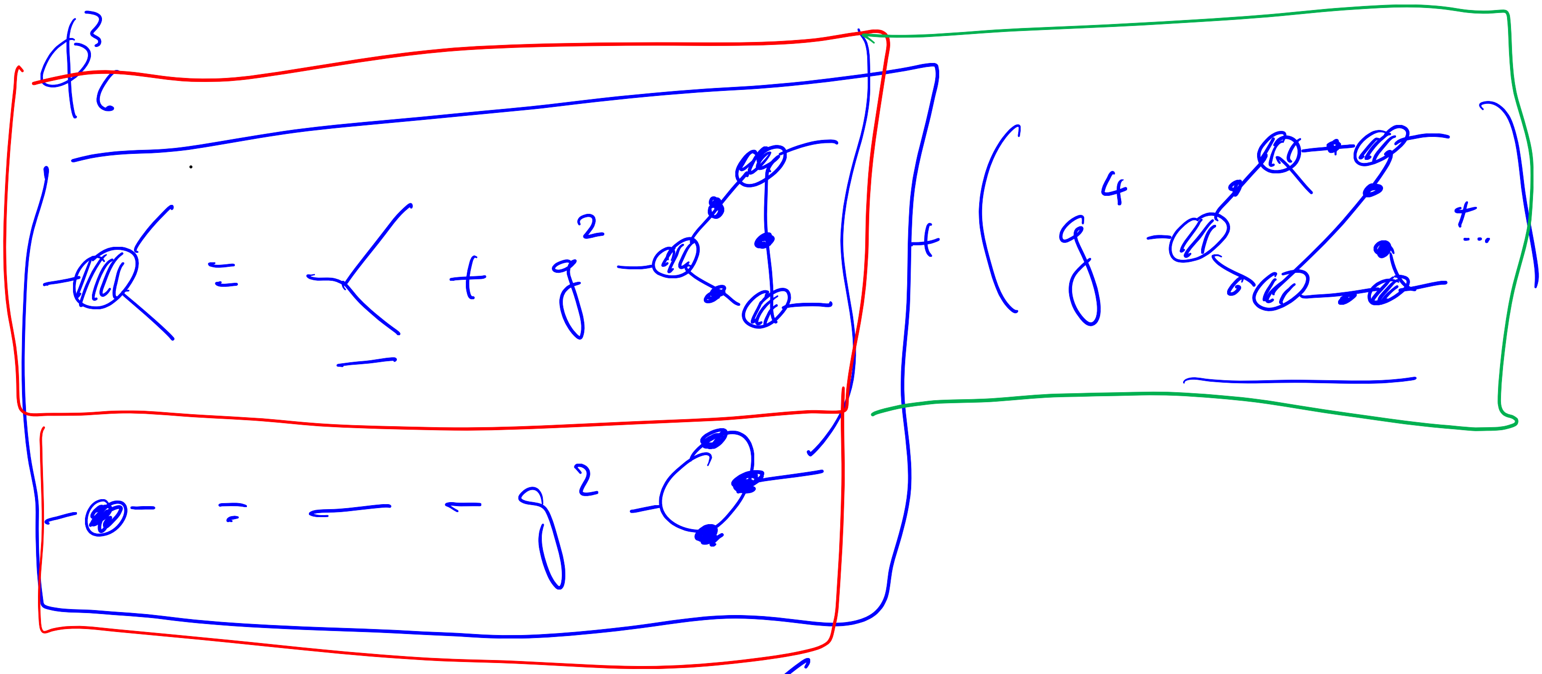


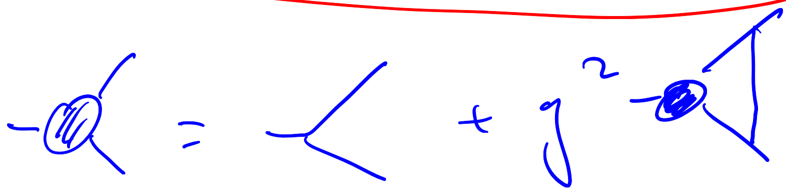




Next: involve full vertex functions.

$\phi^3_6$

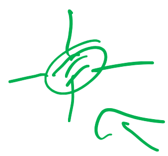




Eventually, this could answer: is $\phi^4_4$ trivial or not?

Fröhlich et. al. gave arguments for triviality. $\phi^4_4$ exists only when coupling $= 0$.

$p^2 Z_1 - m^2 Z_2$



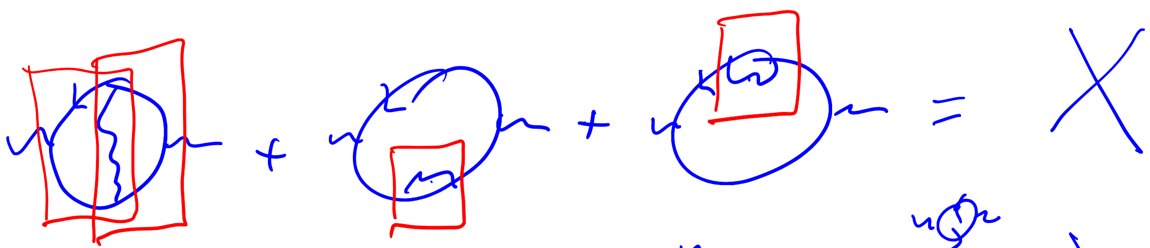
The other big thing: Gauge theory.

Humboldt U. Berlin

First of all: Ward identities can reduce a system of Eq's to a single eq.

(relates to the old conjec~~ture by Steven Adler: "QED is a finite theory"~~) wrong!

But most interestingly wrong:



looks very divergent.

$$X^n = 1 - \alpha B_+(\mathbb{I}) - \alpha^2 B_+^{X}\left(\frac{1}{X^n}\right) - \alpha^3 B_+^{/}\left(\frac{1}{(X^n)^3}\right)$$

$$\tilde{\Delta} X = \underbrace{\left[\,2\quad + 2\quad\right] \otimes \quad}_{=\,0 \text{ by Ward Id.}}$$

$\Rightarrow \tilde{\Delta} X = 0$ $X$ is primitive.