EXERCISE
(DEC 18 2018, TO BE DISCUSSED JAN 15 2018)

INTRODUCTION TO THE RENORMALIZATION GROUP EQUATION (KREIMER, WS 18/19)

- 1.
Let $B_+ : H \to \text{Aug}$ as in the lecture. With $\Delta(h_1 h_2) = \Delta(h_1)\Delta(h_2)$ and $\Delta(1) = 1 \otimes 1$, let
$$\Delta B_+(X) = B_+(X) \otimes 1 + (\text{id} \otimes B_+)\Delta(X),$$
where X is a rooted tree.
Show: $(\Delta \otimes \text{id})\Delta = (\text{id} \otimes \Delta)\Delta$.

- 2.
Set $p_k := \frac{1}{k} S \star Y(x_k)$, where $x_k := \underbrace{B_+ \circ B_+ \cdots B_+(1)}_{k-\text{times}}$.
Show: $\Delta(p_k) = p_k \otimes 1 + 1 \otimes p_k$.

- 3.
Let $X(a) \in H[[a]]$ be given by
$$X(a) = 1 + \sum_{j=1}^{\infty} a^j x_j.$$
Show: $\Delta(X(a)) = X(a) \otimes X(a)$.

- 4.
Show: $X(a) = e^{\sum_{j=1}^{\infty} a^j p_j}$.
- 5.
Let $S_R^\phi := -R(m_V(S_R^\phi \otimes \phi P)\Delta)$, with $\phi \in G_V^H$, $R : V \to V$ Rota–Baxter $(R(vw) + R(v)R(w) = R(vR(w)) + R(R(v)w))$. Show: $S_R^\phi \in G_V^H$.

- 6.
Consider $f(z) \in \mathbb{C}[z^{-1}, z]]$, $f(z) := \sum_{j:=-k}^{\infty} c_j z^j$. Show: $R[f](z) := \sum_{j:=-k}^{-1} c_j z^j$ is Rota–Baxter.

- 7.
  Read *An Etude in non-linear Dyson-Schwinger Equations* (Kreimer, Yeats), Nucl.Phys.Proc.Suppl. 160 (2006) 116-121, hep-th/0605096.