EXERCISE
(JUNE 22 2016, TO BE HANDED IN JULY 06 2016)

FEYNMAN DIGRAMS AND THE S-MATRIX (KREIMER, SUMMER '16)

- 1.
  Let Γ_n, $n \geq 2$, be the graph on two vertices a and b, connected by n edges with masses m_i, $1 \leq 1 \leq n$. The external momentum entering at a say is q.

  i) determine the first and second Symanik polynomials,

  ii) confirm a physical Landau singularity at
  $$q^2 = (m1 + \cdots + m_n)^2,\ A_i m_i = A_j m_j.$$
- 2.
  Consider the one-loop graphs G_n consisting of a circle with n dots on it, representing vertices $v_1, \ldots, v_n$. The external momenta are $q_1, \ldots, q_n$ at these vertices. We orient the circle counterclockwise, with edge e_1 between v_1, v_2, so edge e_n is between v_n, v_1. Edge e_i has mass m_i.

  i) For the second Symanzik polynomial Φ, show (for $n \geq 3$)
  $$\Phi(G_n) = \Phi(G_{n-1}) + A_n Y - A_n^2 m_n^2.$$
  ii) Find an expression for Y in terms of graph polynomials.

  iii) Find all normal thresholds for G_n.

  iv) Solve $\Phi(G_3) = 0$ for A_3. Consider the discriminant of this quadratic equation. How do the zeroes of $\Phi(G_3)$ regarded as a function of A_2 depend on the normal threshold of the reduced graph G_2?