

EXERCISE I
(NOV 14 2012, TO BE HANDED IN NOV 28 2012)

INTRODUCTION TO THE RENORMALIZATION GROUP EQUATION (KREIMER, WS 12/13)

- 1.
Let $B_+ : H \rightarrow \text{Aug}$ as in the lecture. With $\Delta(h_1 h_2) = \Delta(h_1)\Delta(h_2)$ and $\Delta(1) = 1 \otimes 1$, let

$$\Delta B_+(X) = B_+(X) \otimes 1 + (\text{id} \otimes B_+)\Delta(X),$$

where X is a rooted tree.

Show: $(\Delta \otimes \text{id})\Delta = (\text{id} \otimes \Delta)\Delta$.

- 2.
Set $p_k := \frac{1}{k} S \star Y(x_k)$, where $x_k := \underbrace{B_+ \circ B_+ \cdots B_+}_{k\text{-times}}(1)$.

Show: $\Delta(p_k) = p_k \otimes 1 + 1 \otimes p_k$.

- 3.
Let $X(a) \in H[[a]]$ be given by

$$X(a) = 1 + \sum_{j=1}^{\infty} a^j x_j.$$

Show: $\Delta(X(a)) = X(a) \otimes X(a)$.

- 4.
Show: $X(a) = e^{\sum_{j=1}^{\infty} a^j p_j}$.