

**EXERCISE IV**  
**(JAN 23 2013, TO BE HANDED IN FEB 06 2013)**

INTRODUCTION TO THE RENORMALIZATION GROUP EQUATION (KREIMER, WS 12/13)

- 1.

Set  $Q(g) = \prod_{r \in \mathcal{R}} X^r(g)^{-s_r}$  and

$$X^r(g) = 1 - \text{sign}(s_r) \sum_{k \geq 1} \sum_{i=0}^{t_k^r} g^k B_+^{k,i;r} (X^r(g) Q^k(g)),$$

where  $B_+^{k,i;r}$  is a Hochschild closed 1-cocycle by assumption.

Show:

$$\Delta X^r(g)|_k = \sum_{j=0}^k (X^r(g) Q^{k-j}(g))|_j \otimes X^r(g)|_{k-j},$$

and

$$\Delta(X^r(g) Q^l(g))|_k = \sum_{j=0}^k (X^r(g) Q^{k+l-j}(g))|_j \otimes (X^r(g) Q^l(g))|_{k-j}.$$

Here, a subscript  $|_n$  denotes a term of order  $n$  in  $g$ . Hint: use arXiv:0810.2249 (Yeats).

- 2.

Read *Decomposing Feynman Rules* (Brown, Kreimer), hep-th/1212.3923.