

Off-shell Diagrammatics for Quantum Gravity

Henry Kießler

Department of Mathematics, Humboldt-Universität zu Berlin, Rudower Chaussee 25, D-12489 Berlin, Germany

Abstract

This article reports on how diagrammatic identities of Yang–Mills theory translate to diagrammatics for pure gravity. For this, we consider the Einstein–Hilbert action and follow the approach of Capper, Leibbrandt, and Medrano and expand the inverse metric density around the Minkowski metric. By analogy to Yang–Mills theory, cancellation identities are constructed for the graviton as well as the ghost vertices up to the valency of six.

1. Introduction

Ideas and methods from algebraic geometry have been shown to successfully inform our understanding of perturbative quantum field theory [1, 2, 3, 4, 5, 6, 7, 8, 9]. Most of these mathematical methods utilize the parametric representation of Feynman rules that is based on two graph polynomials, usually called Kirchhoff or Symanzik polynomials. In [10] a third graph polynomial has been introduced in order to obtain a manifest representation of tensorial amplitudes in Yang–Mills theory. This polynomial is called corolla polynomial for its parameters are indexed by the half-edges of a 3-valent graph G . By acting with the corolla polynomial $C(G)$ as a differential operator on the scalar integrand of G , all contributions of Yang–Mills diagrams are obtained that can be constructed from G by collapsing some edges that connect two 3-valent vertices or by turning cycles into ghost cycles (in this case both orientations for each ghost cycle are considered). These two diagrammatic operations, edge-collapsing and generating ghost cycles, turn out to be useful, because they constitute two cochain complexes, which are called the gauge and ghost cycle complexes [10, 11]. Crucially, the physical amplitudes lie in the kernel of these operations, which is similar to the BRST complex. The exact relation between BRST and these gauge complexes as well as concrete physical constraints are an interesting topic of ongoing research.

Aside from that, it has been shown that the representation in terms of graph polynomials can be extended to the full Standard Model of elementary particle physics [12]. The natural question whether this approach extends to a quantum theory of gravity remains to be answered. This question, however, is much more intricate due to the infinite number of gravitational vertices and their algebraic complexity. As a first step into this direction, the momentum space Feynman rules of gravity have been derived combinatorially for any valence in [13, 14, 15]. We are approaching this topic by a different angle, namely we are pointing out how the diagrammatic cancellation identities of Yang–Mills theory translate to quantum gravity when quantized following Capper, Leibbrandt, and Medrano [16]. The high degree of similarity between Yang–Mills and gravity ought to allow for a deduction of appropriate cochain maps and their complexes for gravity. We believe that both such a construction and the constraints, that result from identities reported here, inform the construction of a corolla polynomial for gravity.

2. Yang–Mills theory

This section reviews the cancellation identities underlying Yang–Mills theory, because the subsequent construction of gravitational identities will depend on them.

Email address: kissler@physik.hu-berlin.de (Henry Kießler)

2.1. Model and conventions

Consider the standard Yang–Mills Lagrangian with a linear covariant gauge fixing

$$\mathcal{L} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{gf}} + \frac{1}{\xi} \mathcal{L}_{\text{gh}} \quad (1)$$

$$\mathcal{L}_{\text{Dirac}} = i\bar{\psi} \not{D} \psi, \quad \text{where } (D_\mu)_{jk} = \delta_{jk} \partial_\mu - ig T_{jk}^a A_\mu^a \quad (2)$$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a}, \quad \text{where } F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c \quad (3)$$

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\xi} (\partial^\mu A_\mu^a)(\partial^\nu A_\nu^a), \quad \mathcal{L}_{\text{gh}} = -(\partial^\mu \bar{c}^a) [\delta_{ac} \partial_\mu + gf^{abc} A_\mu^b] c^c. \quad (4)$$

Here we mainly follow the conventions of [17] with the exception of an additional factor of $1/\xi$ in front of the ghost Lagrangian \mathcal{L}_{gh} . By means of standard quantization procedures and perturbative expansion, the well-known Feynman rules of this model are readily derived. For the subsequent discussion, we are mainly concerned with the diagrammatic expressions. The residues (i.e. edges and vertices; see [10, 18]) of this model read

$$\mathcal{R}_{\text{YM}} = \left\{ \begin{array}{c} \leftarrow, \quad \text{wavy}, \quad \text{dashed} \leftarrow, \quad \begin{array}{c} \text{wavy} \\ \swarrow \quad \searrow \\ \text{wavy} \end{array}, \quad \begin{array}{c} \text{wavy} \\ \swarrow \quad \searrow \\ \text{wavy} \end{array}, \quad \begin{array}{c} \text{wavy} \\ \swarrow \quad \searrow \\ \text{wavy} \end{array}, \quad \begin{array}{c} \text{wavy} \\ \swarrow \quad \searrow \\ \text{dashed} \end{array} \end{array} \right\}. \quad (5)$$

As usual, quarks are represented by straight directed lines, the wavy lines represent gluons and the dashed directed lines represent the ghosts. For reasons that will be clear later on, we will restrict our attention to residues without quarks in the following.

2.2. Slavnov–Taylor identities

The enormous phenomenological success of Yang–Mills theories in the high-energy domain is crucially founded on what is called renormalizability. With an appropriate treatment of singularities, which are based in quantum corrections, renormalizability translates into the famous Slavnov–Taylor identities [19, 20]. For instance, the respective identity for the 1-particle irreducible 3-point function at first loop order is

$$\begin{array}{c} \text{blob} \\ \uparrow \\ \text{wavy} \end{array} = \begin{array}{c} \text{blob} \\ \uparrow \\ \text{wavy} \end{array} + \begin{array}{c} \text{blob} \\ \uparrow \\ \text{wavy} \end{array} - \begin{array}{c} \text{blob} \\ \uparrow \\ \text{wavy} \end{array} - \begin{array}{c} \text{blob} \\ \uparrow \\ \text{wavy} \end{array} - \begin{array}{c} \text{blob} \\ \uparrow \\ \text{wavy} \end{array}, \quad (6)$$

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where the blob represents all 1-particle irreducible diagrams. Furthermore, the slashed edges, which pair the straight with the wavy half-edges, have been introduced to represent cancelled gluon propagators and the triangle on the left-hand side and letters T represent certain projectors acting on the external gluons. For further details, the interested reader is referred to [21] where this identity has not only been derived by both Hopf-algebraic and diagrammatic methods, but has also been perturbatively verified in dimensional regularization in the most general momentum setting. Here, it is worth emphasizing that the diagrammatic technique solely relies on a set of cancellation identities at tree-level, which will be discussed in the following section.

2.3. Cancellation identities

Most of the cancellation identities shown in this sections go back to the early work of Lautrup and Cvitanović [22, 23]. Since these will establish the foundation for our subsequent construction of gravity cancellations, we need to review them in some detail. Here, we will mainly focus on the diagrammatics. A reader interested in the analytic details might want to consider [21] to complement the original work of Lautrup and Cvitanović. The first identity relates the gluon propagator to the ghost propagator

$$\blacktriangleright \text{wavy} = \text{---} \blacktriangleleft \text{---} \blacktriangleright \quad (7)$$

Here, the black triangle represents a longitudinal contraction of the gluon propagator. To be more precise, abbreviating the gluon propagator by $D_{ab}^{\mu\nu}(p)$ (again we follow the conventions of [17], but drop the additional superscript 0), the left hand side represents the longitudinal contraction, that is $p_\mu D_{ab}^{\mu\nu}(p)$. An easy derivation shows that contracted propagator equals the ghost propagator times the longitudinal contraction with respect to the remaining Lorentz index ν , explaining the black triangle on the right. Similar identities hold for the vertices. A longitudinal contraction of a 3-valent gluon vertex yields

$$\text{3-valent vertex} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \quad (8)$$

Here, we needed to introduce an auxiliary vertex that is connected by a wavy gluon line, a straight line and an outgoing ghost line. Labelling the straight and wavy lines with Lorentz indices μ and ν respectively, the Feynman rule of this auxiliary vertex equals (up to a conventional complex constant) to Minkowski metric $\eta^{\mu\nu}$ and a factor of the structure constants. Also note the slashed propagators in the first and third diagrams that connect a straight to the wavy gluon line. These slashed propagators represent the inverse of the gluon propagator and result in contractions as utilized in [10]. The other diagrams incorporate the usual ghost–gluon vertex followed by an ongoing longitudinal contraction. Similarly, one shows the following identities for the 4-valent vertex.

$$\text{4-valent vertex} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} \quad (9)$$

$$0 = \text{diagram 4} + \text{diagram 5} + \text{diagram 6} + \text{diagram 7} \quad (10)$$

Their similarity to Lie algebra cohomology, namely the IHX relation, is not by accident, but due to a Jacobi identity for the structure constants for the underlying Lie algebra of the gauge group. Finally, the cancellation of ghost loops is explained by the identity

$$0 = \text{[diagram 1]} - \text{[diagram 2]} + \text{[diagram 3]} - \text{[diagram 4]} \quad (11)$$

3. Pure gravity

Gravity was shown to be not renormalizable in the traditional sense in the background field gauge [24, 25, 26] and heat-kernel method [27]. Nonetheless, renormalization is possible when understood as an effective field theory [28, 29] by adding higher derivative terms to the Lagrangian. This situation raises the question what kind of structure these additional terms obey and how they can be characterized. In the following, we first introduce the model in a tensor-density-based approach and secondly report on cancellation relations for all vertices that might be encountered in propagator corrections at two loops, which is known to be the lowest order to require subtraction of higher derivative terms.

3.1. The model

The most established way to quantize the Einstein–Hilbert action utilizes the background field technique. Crucially, the metric tensor is decomposed into a combination of a classical background metric and a quantum field. The caveat of the approach is that the expansion of the Lagrangian yields a term which only depends linearly on the quantum metric. In perturbation theory, this term corresponds to a non-interacting source of the graviton field that results in superfluous complications in Feynman diagrammatic computations. Therefore, these linear terms are usually removed by somehow artificial subtraction.

Here, we follow a complementary approach that consists in quantizing the so-called tensor density instead of the metric tensor which goes back to [16] and is also known as the Landau-Lifshitz approach in the context of classical field theory [30]. First, the tensor density \tilde{g} is defined to be the metric tensor g scaled by the inverse of local coordinate function of the Riemannian volume form. In the following, the Einstein–Hilbert action is understood as a function of the tensor density rather than the metric. Also, we choose to quantize the inverse tensor density rather than just the tensor in the customary quantization procedure. Therefore, the action is perturbatively expanded in terms of the inverse tensor density

$$\tilde{g}^{\mu\nu} = \sqrt{-|g|} g^{\mu\nu} = \eta^{\mu\nu} + \kappa \Phi^{\mu\nu} \quad (12)$$

which decomposes into the Minkowski metric η and the quantum, also called graviton, field Φ . As it was observed in [16] this procedure avoids the tedious source terms of the graviton field; the authors defined a simplified version of the gauge harmonic gauge fixing and derived an appropriate ghost Lagrangian. In this work, we will mainly follow their conventions throughout:

$$iS = i \int d^4x \left(\mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{gf}} + \frac{1}{\xi} \mathcal{L}_{\text{gh}} \right) \quad (13)$$

$$\mathcal{L}_{\text{EH}} = -\frac{2}{\kappa^2} R \sqrt{-|g|}, \quad \mathcal{L}_{\text{gf}} = -\frac{1}{2\xi} \eta_{\nu\sigma} (\partial_\mu \Phi^{\mu\nu}) (\partial_\rho \Phi^{\rho\sigma}), \quad \mathcal{L}_{\text{gh}} = \bar{c}_\mu \partial_\nu D^{\mu\nu\lambda} c_\lambda, \quad (14)$$

$$D^{\mu\nu\lambda} = \eta^{\mu\lambda} \partial^\nu + \kappa [-(\partial^\lambda \Phi^{\mu\nu}) - \Phi^{\mu\nu} \partial^\lambda + \eta^{\mu\lambda} \Phi^{\nu\rho} \partial_\rho + \eta^{\nu\lambda} \Phi^{\mu\rho} \partial_\rho]. \quad (15)$$

For a detailed account on the construction of the ghost Lagrangian and derivation of Feynman rules the reader is referred to [15]. A major advantage of this approach is that the perturbative expansion of the Einstein–Hilbert Lagrangian is free of linear graviton terms. In addition to that, the Feynman rules of the propagators and the 3-valent vertex are available in the literature [31] allowing for cross checks of our results.

In this conventions, the action gives rise to the residues

where it is worth noting that the expansion of the inverse metric (12) in combination with the simple gauge fixing ensures that the interaction of graviton and their ghosts is due to a single 3-valent vertex. We use FORM [32, 33] to expand the action and derive Feynman rules up to the 6-valent graviton vertex. Technically, our implementation allows for higher orders as well, but we stop at the 6-valent vertex for practical reasons—these vertices are sufficient to study the two-point function at two loops, which is expected to be the lowest order that requires higher derivative counterterms and hence reveals the non-renormalizability aspect of gravity.

3.2. Cancellation identities

Subsequently, we take Kreimer’s analogy [34] of gravity and non-abelian gauge theories seriously and try to identify some underlying diagrammatic mechanism. Here, the above QCD cancellation identities serve as a guiding principle.

3.2.1. First observation: comparison of the propagators

First, a propagator identity is constructed by following the identity (7). Indeed, we find

Here, the diagrammatics require some explanation. Whereas the gluon propagator has a single Lorentz index and the black triangle represents the longitudinal contraction of this index with the in-going momentum, the graviton propagator has two indices. The black triangle is meant to contract the two indices, say μ and ν , and possesses an out-going ghost line that has a single index, say λ . In these conventions the black triangle represents $(\delta_\mu^\lambda p_\nu + \delta_\nu^\lambda p_\mu)/2$, where p denotes the in-going momentum. On the right hand side of the equation above, the dashed line represents the (graviton-) ghost propagator in straight analogy to (7) for Yang–Mills. However, due to the more complicated numerator structure of the graviton propagator, the symmetrized longitudinal contraction gets modified by propagating through the propagator. The white triangle denotes this modified contraction and represents the expression $(\delta_\mu^\lambda p_\nu + \delta_\nu^\lambda p_\mu - \eta_{\mu\nu} p^\lambda)$. This needs to be carefully taken into account with dealing with Feynman diagrams with amputated external propagators.

3.2.2. Enforcing the philosophy: the 3-graviton identity

Since the graviton propagator allows for a propagation of the symmetrized longitudinal contraction, it is natural to ask whether this propagation also aligns with the graviton vertices. Indeed, by implementing a cancellation identity for each of the graviton vertices, relations between different diagrams and their divergences are established.

Hence, the next step is to derive a cancellation identity beginning with the 3-valent graviton vertex. It turns out, that the corresponding cancellation identity of Yang–Mills (9) can be seen as the blueprint for quantum gravity. After replacement of the gluon by gravitons (and their ghosts respectively), the only remaining unknown is the auxiliary vertex. It can however be constructed such that the cancellation identity

holds, again in straight analogy to Yang–Mills diagrammatics

$$\begin{array}{c}
 \text{Diagram 1} \\
 = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} \quad .
 \end{array} \quad (18)$$

Here, it is worth remarking that the left hand side is a 3-graviton vertex with propagators attached to it, the white triangle is in accordance with (17), and the slashed edges that consist of a graviton and a straight half-edge indicate a cancelled graviton propagator. The actual expression for the auxiliary vertex is lengthy and will be discussed in more detail in future work. Nonetheless, it is worth remarking that as a function of the kinematic data, it reads $V_{\text{aux}}(p_1, \mu_1, \nu_1; p_2, \mu_2, \nu_2; p_3, \mu_3)$, where the semicolons separate the momentum and Lorentz indices of the graviton, straight, and ghost edges in this order. Whereas V_{aux} inherits the symmetry under permutations of μ_i and ν_i for both the graviton and straight edge (i.e. for $i = 1, 2$), it is not symmetric under permutation of these edges.

3.2.3. An assessment: the 4-graviton identity

At this stage, higher order cancellation identities will provide further non-trivial conditions to the auxiliary vertex constructed above. The 4-valent identity (9) serves as a first test and it translates to

$$\begin{array}{c}
 \text{Diagram 1} \\
 = - \text{Diagram 2} - \text{Diagram 3} - \text{Diagram 4} \quad ,
 \end{array} \quad (19)$$

where the auxiliary vertex is determined through (18).

3.2.4. Exercise fine judgment: higher order identities

In contrast to the preceding identities, the cancellation of (10) does not directly translate to gravity. However, its interpretation that Yang–Mills does not require a 5-valent vertex is also not correct for gravity. Hence it is reasonable to insert the longitudinal contraction of the 5-valent vertex on the left hand side of the equation and it turns out that corrects the equation resulting in the identity

$$\begin{array}{c}
 \text{Diagram 1} \\
 = - \text{Diagram 2} - \text{Diagram 3} - \text{Diagram 4} - \text{Diagram 5} \quad .
 \end{array} \quad (20)$$

From this identity and (19), it is easy to deduce the pattern for vertices of higher valence: the symmetrized longitudinal contraction of a graviton vertex of valence $(n + 1)$ yields all terms obtained by attaching the auxiliary vertex to a graviton vertex of valence n . We explicitly checked this for the 6-valent vertex and verified

$$= - \dots - \dots - \dots - \dots - \dots \quad (21)$$

Finally, we also verified that the ghost cancellation identity of Yang–Mills (11) translates to the gravitation identity

$$0 = \dots + \dots - \dots \quad (22)$$

where all propagators of loose half-edges are considered to be amputated (as indicated due to the white triangles). We close this section with a table that lists the terms to be cancelled when the right hand side of an identity is subtracted from its left hand side. Here it is worth remarking that we always amputate propagators of external (i.e. loose) half-edges to avoid a tremendous increase in the number of terms generated by FORM.

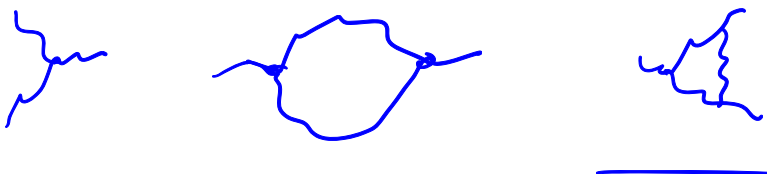
identity	#cancelled terms
(17)	35
(18)	3,168
(19)	30,888
(20)	607,488
(21)	12,105,600
(22)	1,536

Note that over 12 million terms have to match up for (21) and checking explicitly that this is the case non-trivially substantiates our initial approach.

4. Discussion

$\tilde{g}_{\mu\nu}$ $g_{\mu\nu}$

The cancellation identities of Yang–Mills theory can be translated to identities for pure quantum gravity in the classical approach by Capper, Leibbrandt, and Medrano [16]. The reason for choosing this setting is due to its relatively simple gauge (14) that induces only a single ghost-vertex. This allowed us to establish the identities and success of this approach will serve as a basis for future extensions. In close analogy to Yang–Mills, the cancellation identity for the 3-graviton vertex determines an auxiliary vertex (18). However,



the generalization of the longitudinal contraction of a graviton propagator requires the introduction of an additional contraction, which we represented by a white triangle (17). We demonstrated that the cancellation identities for the 4-graviton as well as the ghost vertex exactly match their Yang–Mills analogues. Also higher order identities have been constructed by considering longitudinal contracted graviton vertices of that order (which do not have analogues in Yang–Mills). Notably, the cancellation identities (19–21) demonstrate that a gravitational vertex of a certain valence relates to vertices of lower valence. This observation parallels to Yang–Mills theory, where the relation for 4-valent vertex (9) can be understood as strong evidence to the existence of a corolla differential, that produces 4-valent vertices from purely 3-valent graphs. Our findings support the conjecture that a full generation of gravitational amplitudes from 3-valent graphs by an appropriate corolla differential is possible.

Intuitively, it might come as a surprise that exactly those diagrammatic identities which guarantee the renormalizability of Yang–Mills theory do generalize to gravity. However, the existence of diagrammatic cancellations seems plausible in the light of [35] and the presence of a BRST invariance and Ward identities [36, 37, 38]. We do consider the exact mathematical connection of diagrammatics and the BRST cohomology an exciting topic for further investigation. The cancellation identities derived here open up the possibility to study the gauge dependence by diagrammatic techniques [39, 40], where it was also empirically observed that this mechanism applies to more general situations [41, 42, 43]. For gravity, this has potential to determine the divergences that are of genuine physical significance or to diagrammatically construct generalized Slavnov–Taylor identities for gravity and to study their self-consistency off-shell as was recently accomplished for Yang–Mills [21].

Another significance of the demonstrated cancellations stems from the diagrams with cancelled propagators. In the sense of graph theory, these define an operation on graphs by edge-collapsing. In combination with the ghost identity, this informs the definition of a gravitational graph and cycle homology as in the Yang–Mills case [10, 11] and possibly enables the construction of a corolla polynomial for gravity as extension of [44, 10, 12, 45]. For this, it worth stressing that on the graphical level, the identity for the gravitational ghosts (22) matches exactly its analogue (11) of Yang–Mills theory.

In the future, our approach might benefit from the vast range of double-copy techniques [46, 47, 48, 49], which excitingly have been employed in the off-shell regime in [50, 51], to mention but a few. So far, it has been our objectives to consider a Lagrangian as close as possible to Einstein–Hilbert theory and concentrate on a simple gauge-fixing, which minimizes the number of graviton-ghost interactions for the sake of simple cancellation identities. Within this class of gauges, we have been studying the most general parametrization in order to allow the gauge parameter ξ to absorb off-shell singularities that inhere in loop diagrams. In this tensor-density-based expansion, the observed diagrammatic relations encourage interpretation of a gravitational vertex as a certain convolution of gluonic kinematics. However, from the analytic form of the graviton propagator as displayed in [31], it seems reasonable to expect such an endeavour to require the introduction of supplementary fields or even a change of gauge.

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References

- [1] S. Bloch, H. Esnault, D. Kreimer, On Motives associated to graph polynomials, *Commun. Math. Phys.* 267 (2006) 181–225. [arXiv:math/0510011](#), [doi:10.1007/s00220-006-0040-2](#).
- [2] E. Panzer, Feynman integrals and hyperlogarithms, Ph.D. thesis, Humboldt U. (2015). [arXiv:1506.07243](#), [doi:10.18452/17157](#).
- [3] S. Bloch, D. Kreimer, Cutkosky Rules and Outer Space [arXiv:1512.01705](#).
- [4] F. Brown, Feynman amplitudes, coaction principle, and cosmic Galois group, *Commun. Num. Theor. Phys.* 11 (2017) 453–556. [arXiv:1512.06409](#), [doi:10.4310/CNTP.2017.v11.n3.a1](#).

- [5] O. Schnetz, The Galois coaction on the electron anomalous magnetic moment, *Commun. Num. Theor. Phys.* 12 (2018) 335–354. [arXiv:1711.05118](#), [doi:10.4310/CNTP.2018.v12.n2.a4](#).
- [6] O. Schnetz, Geometries in perturbative quantum field theory [arXiv:1905.08083](#).
- [7] E. Panzer, Hepp's bound for Feynman graphs and matroids [arXiv:1908.09820](#).
- [8] R. P. Klausen, Hypergeometric Series Representations of Feynman Integrals by GKZ Hypergeometric Systems, *JHEP* 04 (2020) 121. [arXiv:1910.08651](#), [doi:10.1007/JHEP04\(2020\)121](#).
- [9] M. Tapušković, Motivic Galois coaction and one-loop Feynman graphs [arXiv:1911.01540](#).
- [10] D. Kreimer, M. Sars, W. van Suijlekom, Quantization of gauge fields, graph polynomials and graph homology, *Annals Phys.* 336 (2013) 180–222. [arXiv:1208.6477](#), [doi:10.1016/j.aop.2013.04.019](#).
- [11] M. Berghoff, A. Knispel, Complexes of marked graphs in gauge theory, *Letters in Mathematical Physics* [arXiv:1908.06640](#), [doi:10.1007/s11005-020-01301-0](#).
- [12] D. Prinz, The Corolla Polynomial for spontaneously broken Gauge Theories, *Math. Phys. Anal. Geom.* 19 (3) (2016) 18. [arXiv:1603.03321](#), [doi:10.1007/s11040-016-9222-0](#).
- [13] C. Cheung, G. N. Remmen, Hidden Simplicity of the Gravity Action, *JHEP* 09 (2017) 002. [arXiv:1705.00626](#), [doi:10.1007/JHEP09\(2017\)002](#).
- [14] D. Prinz, Algebraic Structures in the Coupling of Gravity to Gauge Theorie [arXiv:1812.09919](#).
- [15] D. Prinz, Gravity-Matter Feynman Rules for any Valence [arXiv:2004.09543](#).
- [16] D. Capper, G. Leibbrandt, M. Ramon Medrano, Calculation of the graviton selfenergy using dimensional regularization, *Phys. Rev. D* 8 (1973) 4320–4331. [doi:10.1103/PhysRevD.8.4320](#).
- [17] P. Pascual, R. Tarrach, QCD: Renormalization for the Practitioner, *Lect. Notes Phys.* 194 (1984) 1–277.
- [18] D. Kreimer, Anatomy of a gauge theory, *Annals Phys.* 321 (2006) 2757–2781. [arXiv:hep-th/0509135](#), [doi:10.1016/j.aop.2006.01.004](#).
- [19] A. Slavnov, Ward Identities in Gauge Theories, *Theor. Math. Phys.* 10 (1972) 99–107. [doi:10.1007/BF01090719](#).
- [20] J. C. Taylor, Ward Identities and Charge Renormalization of the Yang-Mills Field, *Nucl. Phys. B* 33 (1971) 436–444. [doi:10.1016/0550-3213\(71\)90297-5](#).
- [21] J. A. Gracey, H. Kifler, D. Kreimer, Self-consistency of off-shell Slavnov-Taylor identities in QCD, *Phys. Rev. D* 100 (8) (2019) 085001. [arXiv:1906.07996](#), [doi:10.1103/PhysRevD.100.085001](#).
- [22] B. Lautrup, Of Ghoulies and Ghosties: An Introduction to QCD.
- [23] P. Cvitanović, Quantum Chromodynamics on the Mass Shell, *Nucl. Phys. B* 130 (1977) 114–144. [doi:10.1016/0550-3213\(77\)90396-0](#).
- [24] G. 't Hooft, M. Veltman, One loop divergencies in the theory of gravitation, *Ann. Inst. H. Poincaré* 20 (1974) 69–94.
- [25] M. H. Goroff, A. Sagnotti, Quantum Gravity at Two Loops, *Phys. Lett. B* 160 (1985) 81–86. [doi:10.1016/0370-2693\(85\)91470-4](#).
- [26] M. H. Goroff, A. Sagnotti, The Ultraviolet Behavior of Einstein Gravity, *Nucl. Phys. B* 266 (1986) 709–736. [doi:10.1016/0550-3213\(86\)90193-8](#).
- [27] A. van de Ven, Two loop quantum gravity, *Nucl. Phys. B* 378 (1992) 309–366. [doi:10.1016/0550-3213\(92\)90011-Y](#).
- [28] S. Weinberg, Effective Field Theory, Past and Future, *PoS CD09* (2009) 001. [arXiv:0908.1964](#), [doi:10.22323/1.086.0001](#).
- [29] S. Weinberg, Effective field theory, past and future, *Int. J. Mod. Phys. A* 31 (06) (2016) 1630007. [doi:10.1142/S0217751X16300076](#).
- [30] L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields*, Pergamon Press, Oxford, 1951.
- [31] D. Capper, M. Namazi, A General Gauge Calculation of the Graviton Selfenergy, *Nucl. Phys. B* 142 (1978) 535–547. [doi:10.1016/0550-3213\(78\)90229-8](#).
- [32] J. A. M. Vermaseren, New features of FORM [arXiv:math-ph/0010025](#).
- [33] B. Ruijl, T. Ueda, J. A. M. Vermaseren, FORM version 4.2 [arXiv:1707.06453](#).
- [34] D. Kreimer, Not so non-renormalizable gravity, in: *Conference on Recent Developments in Quantum Field Theory, 2008*, pp. 155–162. [arXiv:0805.4545](#), [doi:10.1007/978-3-7643-8736-5_9](#).
- [35] B. S. DeWitt, Quantum Theory of Gravity. 2. The Manifestly Covariant Theory, *Phys. Rev.* 162 (1967) 1195–1239. [doi:10.1103/PhysRev.162.1195](#).
- [36] D. Capper, M. Medrano, Gravitational slavnov-ward identities, *Phys. Rev. D* 9 (1974) 1641–1647. [doi:10.1103/PhysRevD.9.1641](#).
- [37] R. Delbourgo, M. Ramon Medrano, Becchi-Rouet-Stora Gauge Identities for Gravity, *Nucl. Phys. B* 110 (1976) 467–472. [doi:10.1016/0550-3213\(76\)90235-2](#).
- [38] R. Delbourgo, T. Matsuki, Gravitational counterterms and Becchi-Rouet-Stora symmetry, *Phys. Rev. D* 32 (1985) 2579. [doi:10.1103/PhysRevD.32.2579](#).
- [39] H. Kifler, D. Kreimer, Diagrammatic Cancellations and the Gauge Dependence of QED, *Phys. Lett. B* 764 (2017) 318–321. [arXiv:1607.05729](#), [doi:10.1016/j.physletb.2016.11.052](#).
- [40] H. Kifler, On the gauge dependence of Quantum Electrodynamics, *PoS LL2018* (2018) 032. [arXiv:1808.00291](#), [doi:10.22323/1.303.0032](#).
- [41] N. Zerf, P. Marquard, R. Boyack, J. Maciejko, Critical behavior of the QED₃-Gross-Neveu-Yukawa model at four loops, *Phys. Rev. B* 98 (16) (2018) 165125. [arXiv:1808.00549](#), [doi:10.1103/PhysRevB.98.165125](#).
- [42] B. Ihrig, N. Zerf, P. Marquard, I. F. Herbut, M. M. Scherer, Abelian Higgs model at four loops, fixed-point collision and deconfined criticality, *Phys. Rev. B* 100 (13) (2019) 134507. [arXiv:1907.08140](#), [doi:10.1103/PhysRevB.100.134507](#).
- [43] J. A. Gracey, Six dimensional ultraviolet completion of the CP(N) σ model at two loops [arXiv:2003.06618](#).
- [44] D. Kreimer, K. Yeats, Properties of the corolla polynomial of a 3-regular graph [arXiv:1207.5460](#).
- [45] D. Kreimer, The corolla polynomial: a graph polynomial on half-edges, *PoS LL2018* (2018) 068. [arXiv:1807.02385](#),

doi:10.22323/1.303.0068.

- [46] Z. Bern, J. J. M. Carrasco, H. Johansson, New Relations for Gauge-Theory Amplitudes, Phys. Rev. D 78 (2008) 085011. [arXiv:0805.3993](#), [doi:10.1103/PhysRevD.78.085011](#).
- [47] Z. Bern, J. J. M. Carrasco, H. Johansson, Perturbative Quantum Gravity as a Double Copy of Gauge Theory, Phys. Rev. Lett. 105 (2010) 061602. [arXiv:1004.0476](#), [doi:10.1103/PhysRevLett.105.061602](#).
- [48] Z. Bern, T. Dennen, Y.-t. Huang, M. Kiermaier, Gravity as the Square of Gauge Theory, Phys. Rev. D 82 (2010) 065003. [arXiv:1004.0693](#), [doi:10.1103/PhysRevD.82.065003](#).
- [49] Z. Bern, J. J. Carrasco, M. Chiodaroli, H. Johansson, R. Roiban, The Duality Between Color and Kinematics and its Applications [arXiv:1909.01358](#).
- [50] A. Anastasiou, L. Borsten, M. J. Duff, S. Nagy, M. Zoccali, Gravity as Gauge Theory Squared: A Ghost Story, Phys. Rev. Lett. 121 (21) (2018) 211601. [arXiv:1807.02486](#), [doi:10.1103/PhysRevLett.121.211601](#).
- [51] L. Borsten, S. Nagy, The pure BRST Einstein-Hilbert Lagrangian from the double-copy to cubic order, JHEP 07 (2020) 093. [arXiv:2004.14945](#), [doi:10.1007/JHEP07\(2020\)093](#).
- [52] J. C. Collins, J. A. M. Vermaseren, Axodraw Version 2 [arXiv:1606.01177](#).

