Graphs, Zinn-Justin facts, parametric renormalization

What is a graph?

$G$ is a triple $(V_G, E_G, E_0)$ where

$V_G$ is the set of half-edges

$E_G$ is a partition into points of cardinality $\geq 3$,

$E_0$ is a partition of half-edges into points of cardinality $< 3$,

Cardinality $1$ = an external edge
Cardinality $2$ = an internal edge
$V^*_G = \{1, 2, 3, 4, 5, 6\}$

$\mathcal{V} = \{\{1, 2, 4\}, \{3, 5, 6\}\}$

$a, 2$ vertices

$E_G = \{2, 3, 4, 5, 6\}$

$C_v$ is the corolla at $v$.

$C_a = \{1, 2, 6\}$, $C_b = \{3, 5, 6\}$

Graphically:

\[ \text{\includegraphics[width=0.2\textwidth]{graph.png}} \]

$E$ is the set of cuts of cood.

$\mathcal{E}_G$ is $2$ in $E_G$

$|E_G| = \mathcal{E}_G$

$|V_G| = \mathcal{V}_G$

Typically, we consider bridge-free 2-connected 1-PSP graphs.
There is a valuation of $D^7 q^6$,
\[ \omega_3^D(6) = D \cdot |6| - \sum_{c \in E_6} \omega_c - \sum_{v \in V_6} \omega_v \]
loop number

For a renormalizable theory, our $\omega_3^D(6)$
\[ \omega_3^D(6_1) = \omega_3^D(6_2) \neq 6_1, 6_2 \]
such that $l = l_6$ , regardless of
$|6_1|, |6_2|$. 

$l_6$ is number of points of
codimension 1 in $E_6$

$= \# \text{ of ext. edges}$

$|6| = E_6 - V_6 + 1 \neq \text{ connected}$

\[ \begin{array}{c}
\begin{array}{c}
\text{loops}
\end{array}
\end{array} \]

\[ \omega_6(A) = 6 \cdot 2 - 2 \cdot 6 + 0 \cdot 5 = 0 \]

\[ \omega_6(A) = 6 \cdot 1 - 2 \cdot 3 + 0 \cdot 3 = 0 \]
For a 1PI superficially divergent graph $\Gamma$, we define $\Delta_D(\Gamma) \geq 0$.

A forest $\mathcal{F}$ of proper superficially divergent subgraphs $\Gamma \subseteq \Gamma$, $i \in I^\Phi$ on an index set $I^\Phi$, is

called $\Gamma_i \cap \Gamma_j = \emptyset$ for $i, j \in I^\Phi$, or

$\Gamma_i \subseteq \Gamma_j \Rightarrow \Gamma_j \not\subseteq \Gamma_i$.

In particular, a forest is a product, hence a disjoint union of graphs $\bigcup_{i \in I^\Phi} \Gamma_i$.

By $\Gamma / \mathcal{F}$ we denote the graph obtained by contracting all $\Gamma_i$, no points (shrink internal edges of $\Gamma_i$).
\[ \phi^2 \]

\[ f = \begin{cases} 1 \\ 3 \\ 2 \end{cases} \]

\[ \frac{1}{3} \]

\[ \frac{4}{6} \]

So \( f \) is maximal.

(Also see Ex. 1 in paper.)

If you have a maximal forest \( F \), we call \( \Gamma \) maximal for \( F \).

For \( f \geq F_i \subset \Gamma \), each index set \( I_f \) defines a dedicated index set \( I_{f_i} \) for all forests \( \Gamma \) strictly contained in \( F_i \), s.t.
\( \Gamma \subset \Gamma_i \quad \forall \ i \in I^f \).

We call a forest complete, if \( \Gamma_i \) is maximal for \( \Gamma \) and \( \Gamma_i \) maximal for each proper subgraph \( \Gamma_i \) of \( \Gamma \).

**Ex.** Complete maximal forests.

Each graph \( \Gamma \) has only a finite number of complete maximal forests.

3 complete max. forests.

Complete max. forests are in 1:1 correspondence with decorated rooted trees. The set of decorations \( \mathcal{F} \) at vertices \( v \) of a tree is given by...
Suppose the graph is free of subdiv. divs.

\[ p_u = \frac{\Gamma_i}{\bigcup_{j \in \Gamma_i} \Gamma_j} \]

\( \Gamma_i \) is the maximal forest.

\text{decorated rooted tree}

Next: the powerset \( P_E(T) \) of edges \( E(T) \) of such a tree \( T \) gives all possible cuts of \( T \).

\[
\begin{align*}
1 \sim & 2 \\
1 \sim & 3 \\
2 \sim & 1 \cup 3 \\
& \vdots \\
\end{align*}
\]

\( R^c \), \( P^c \) as before, and \( C \in P_E(T) \).
One more example, $\phi^f$

\[ \omega_4(\Gamma) = 0 \]

\[ 3^4 \equiv \delta_{34} \]

\[ 3^4 \sim \delta \]

\[ \delta_{34} \equiv \delta \equiv \Gamma \]

Forests:

\[ \mathcal{F}_0 = \emptyset \]

\[ \mathcal{F}_1 = \delta_{34} \quad \text{useful, max. non-complete} \]

\[ \mathcal{F}_2 = \delta \quad \text{both maximal and not complete} \]

\[ \mathcal{F}_3 = \delta \quad \text{both maximal and complete} \]

\[ \mathcal{F}_4 = \{ \delta_{34}, \mathcal{F}_3 \} \quad \text{maximal and complete} \]

\[ \mathcal{F}_5 = \{ \delta_{34}, \delta \} \quad \text{maximal and complete} \]
On Wednesday, we map back and forth between the world of decorated rooted trees and $H_r$ for Potts.

$p : H_r \to H_{\text{Dec}}$

\[(S \otimes S) \Delta g \equiv \Delta g \cdot S \quad \]

And then we introduce the map

$\tau = S \ast P$

$\tau : p : H_r \to A g H_r \equiv A_r$

And then there's an identity

$id : A g H_r \to A g H_r$

$id_{A g} = \sum_{j=1}^{\infty} g^j = \sum_{j=1}^{\infty} g^j.$

This is the co-radical filtration alluded to last time.

So that $H_r$ lives in $g r.$
This will give us the R6B.