Today, we start parametrically, and in momentum space renormalization.

parametrically: check all sectors ← Zimmermann

in momentum space: convenient for Bogoliubov - Parasiuk approach

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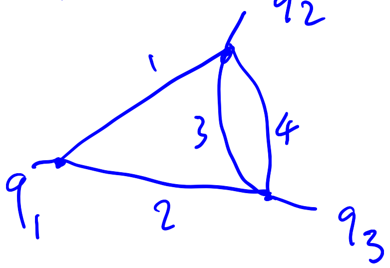


$$\psi = (A_1 + A_2)(A_3 + A_4) + A_3 A_4$$

$$\varphi = -\left(q_1^2 A_1 A_2 (A_3 + A_4) + q_2^2 A_1 A_3 A_4 + q_3^2 A_2 A_3 A_4\right)$$

$$M \cdot A = \sum_{j=1}^{4} A_j m_j^2$$

$$\Phi = \varphi + M \cdot A \, \psi$$

$$\int_0^\infty \cdots \int_0^\infty \frac{e^{-\frac{\Phi}{\psi}}}{\psi^2} \, dA_1 \ldots dA_4$$

$$A_1 = A_1$$

$$a_j = A_j / A_1 \qquad j = 2, \ldots, 4$$

$$\varphi \to \psi = A_1^2\left((1+a_2)(a_3+a_4)+a_3a_4\right)$$

φ similarly

$$\int\ldots\int_0^\infty dA_1 \frac{e^{-A_1 \frac{\varphi(a_2,\ldots a_4)}{\psi(a_2\ldots a_4)}} A_1^3\, da_2\ldots da_3}{\left(A_1^2\left((1+a_2)(a_3+a_4)+a_3a_4\right)\right)^2}$$

$$\Rightarrow \int_0^\infty \frac{dA_1}{A_1} \frac{e^{-A_1\ldots}}{\left((1+a_2)(\ldots)+a_3a_4\right)^2}$$

$$\lim_{c\to 0} \int_c^\infty \frac{dA_1}{A_1} \frac{e^{-A_1 \ldots}}{\ldots}$$

$$\Leftrightarrow \int_0^\infty da_2\ldots da_4 \frac{\ln\left(\frac{\varphi(a_2\ldots a_4)}{\varphi_0(a_2\ldots a_4)}\right)}{\left((1+a_2)(a_3+a_4)+a_3a_4\right)^2}$$

φ_0: empty forest

still ill-defined

but: contribution of 2 forest

$$\int da_2\ldots da_4 \frac{\ln\left(\frac{\varphi(\alpha)\psi(\gamma)+\varphi_0(\gamma)\psi(\alpha)}{\varphi_0(\alpha)\psi(\gamma)+\varphi_0(\gamma)\psi(\alpha)}\right)}{(1+a_2)^2(a_3+a_4)^2}$$

φ_0 is φ evaluated at renormalization point.

$q_1^2, q_2^2, q_3^2 \to (q_1^0)^2, (q_2^0)^2, (q_2^0)^2$.

renormalization.

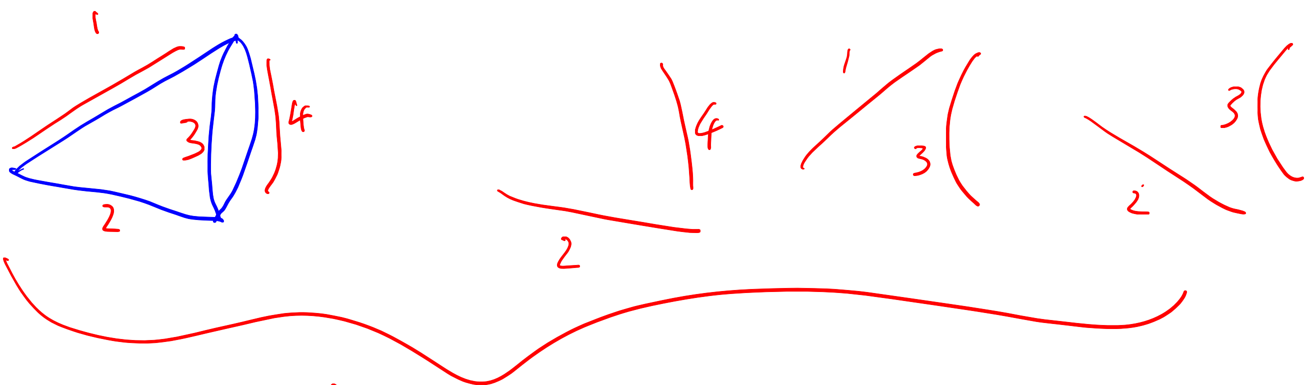
$$\sum_f (-1)^{|f|} \frac{\ln\left(\frac{\varphi_{G/f}\psi_f + \varphi_{0f}\psi_{G/f}}{\varphi_{0G/f}\psi_f + \varphi_{0f}\psi_{G/f}}\right)}{\psi_{G/f}^2\,\psi_f^2} \cdots$$

$$\left\{ \frac{\ln\left(\frac{a_2(q_1^2)(a_3+a_4) + q_2^2 a_3 a_4 + q_3^2 a_2 a_3 a_4 \cdots}{a_2(a_3+a_4)\mu^2 + \mu^2 a_3 a_4 + \mu^2 a_2 a_3 a_4}\right)}{\left((1+a_2)(a_3+a_4) + a_3 a_4\right)^2} - \frac{\ln\left(\frac{(q_1^2 a_2)(a_3+a_4) + \mu^2 a_3 a_4 (1+a_2)}{(\mu^2 a_2)(a_3+a_4) + \mu^2 a_3 a_4 (1+a_2)}\right)}{(1+a_2)^2 (a_3+a_4)^2} \right\}$$

$da_2\, da_3\, da_4$

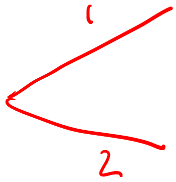
$$\to \ln\frac{q_1^2}{\mu^2} \int da_3\, da_4 \frac{1}{(a_3+a_4)^2} \quad ↯$$

$a_3, a_4 \to 0$ exists.



4 sp. trees

4 sectors, edges in sp. trees $\to 0$ well defined as the above.



$$\frac{\ln\left(\frac{\phi(a_1,a_2,a_4)}{\phi_0(a_1,a_2,a_4)}\right) da_1\, da_2\, da_4}{\left((a_1+a_2)(1+a_4)+a_4\right)^2}$$

$$-\ \frac{\ln \frac{\phi(a)\psi(\gamma)+\phi_0(\gamma)\psi(a)}{\phi_0(a)\ \cdots} \quad da_1\, da_2\, d_4}{(a_1+a_2)^2\,(1+a_4)^2}$$

$$\ln\left(\frac{a_1 a_2 \cancel{a_1^2}(1+a_4) + \overbrace{(a_1 \cancel{+} a_2)\cancel{\mu^2 a_4}}^{\psi(a)\,=}}{a_1 a_2 \cancel{\mu^2}(1+a_4) + (a_1+a_2)\cancel{\mu^2 a_4}}\right)$$

$$\Rightarrow \ln(1) \equiv 0$$

in all sectors, divergences vanishes because poles cancel as the arg in the numerator is convergent to 1. (

This works as

$$\int_0^\infty \frac{dx}{x}\, e^{-xW} \sim \log W + \cdots$$

log universal.

$$\int_{\mathbb{P}_G} \sum_f (-1)^{|f|} \frac{\ln\left(\frac{\phi(G/f)\psi(f)+\phi_0(f)\psi(G/f)}{\phi^0(G/f)\psi(f)+\phi_0(f)\psi(G/f)}\right)}{\psi^2_{G/f}\,\psi^2_f}\,\Omega_G$$

the rest follows from $\ln(1) \equiv 0$.

For sector decomposition: Gudrun Heinrich

For Hopf algebras in IR sector decomp.
read Herzog, Borinsky et. al. (recently)
on R^* operation and IR rearrangement.

Let us start again from momentum space.

$$\Phi_R\Big(\underbrace{B_+^\gamma(X)}_{G}\Big)\,, \quad \text{where } \Delta B_+^\gamma = (B_+^\gamma \otimes \mathbb{I}) + (\mathrm{id} \otimes B_+^\gamma)\Delta$$

$$\Phi_R(G) = m_C\left(S_R^\Phi \otimes \Phi\right)\Delta B_+^\gamma(X) \qquad H \to A_H$$

$$= [\mathrm{id} - R]\, m_C\left(S_R^\Phi \otimes \Phi P\right)\Delta B_+^\gamma(X)$$

$$= [\mathrm{id} - R]\, m_C\left(S_R^\Phi \otimes \Phi\right)\left(X' \otimes B_+^\gamma(X'')\right)$$

Assume $\Phi(B_+^\gamma(X)) = \int d\mu_\gamma\, \Phi(X)$ $\quad R: q_i \to q_i^0$ ren. pt.

$$= [\mathrm{id} - R]\, m_C\left(S_R^\Phi(X') \otimes \int d\mu_\gamma\, \Phi(X'')\right)$$

$$= (id - R)\left(\underline{S_R^{\bar\Phi}(X')} \int d\mu_\gamma \, \bar\Phi(X'') \right)$$

$$= (id - R) \int d\mu_\gamma \, \underline{\Phi_R(X)}$$

This is Bogoliubov Parasiuk recursion which gives the renormalization of a graph from the renormalization of its subgraphs.

$$\Delta G = G \otimes \mathbb{I} + \mathbb{I} \otimes G + \sum_{\gamma \subsetneq G} \gamma \otimes G/\gamma$$

$$S_R^{\bar\Phi} * \bar\Phi \equiv \bar\Phi_R$$

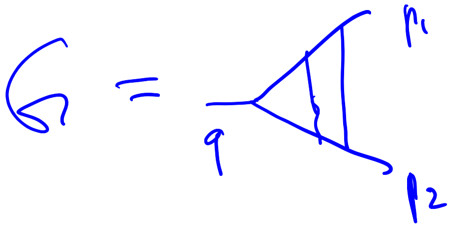
$$S_R^{\bar\Phi} * \bar\Phi(G) = S_R^{\bar\Phi}(G) + \underline{\bar\Phi(G)}$$

$$+ \underline{\sum_{\gamma \subsetneq G} S_R^{\bar\Phi}(\gamma)\, \bar\Phi(G/\gamma)}.$$

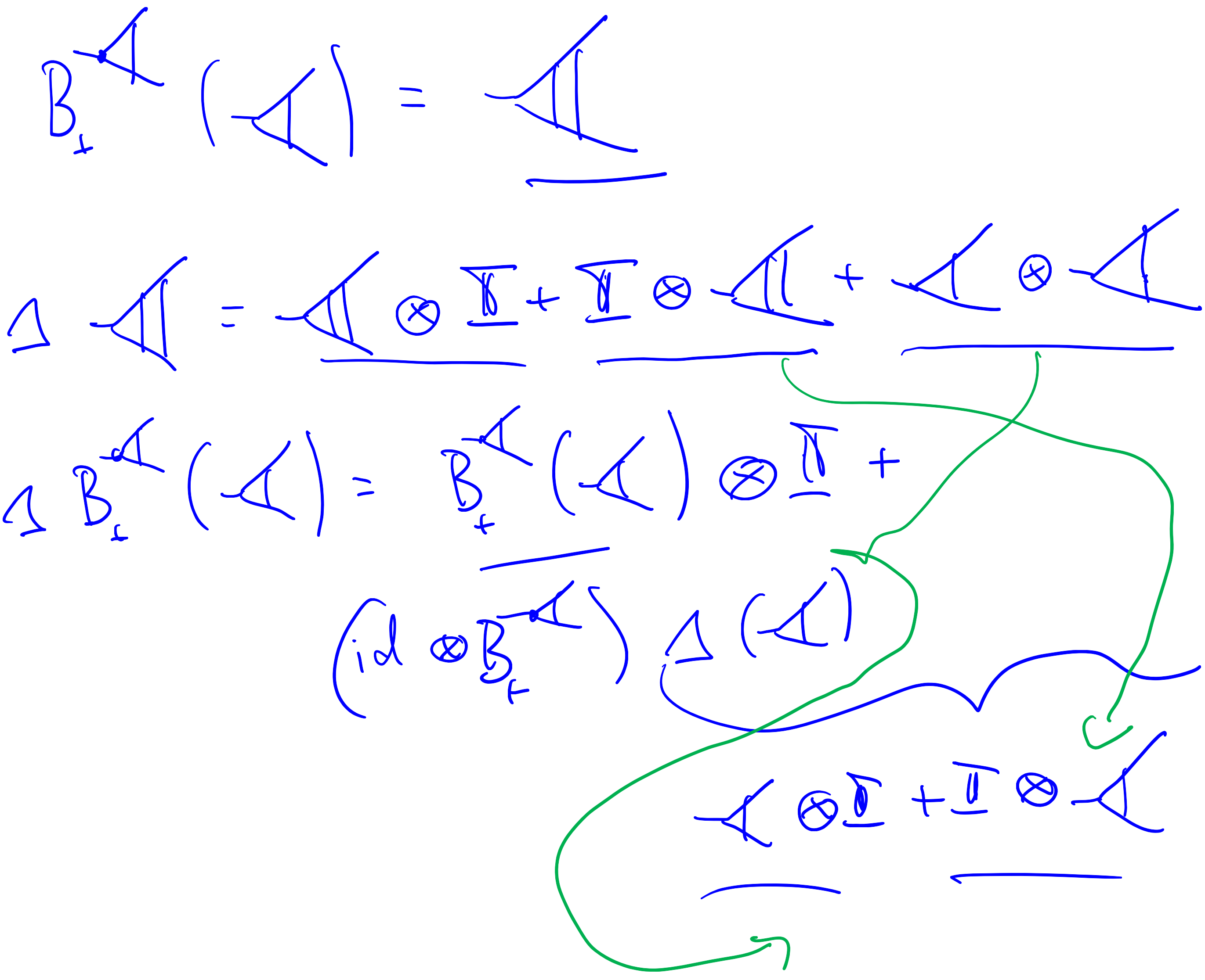
Def. of $S_R^{\bar\Phi}(G) = \underline{-R}\left[\bar\Phi(G) + \underline{\sum_{\gamma \subsetneq G} S_R^{\bar\Phi}(\gamma)\bar\Phi(G/\gamma)}\right].$

$$\bar\Phi_R(G) = [id - R]\, m_C \left(S_R^{\bar\Phi} \otimes \bar\Phi P \right) \Delta(G)$$

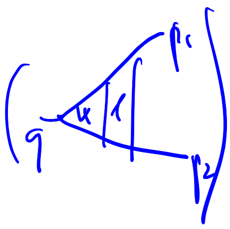
$$S(G) = - m_H (S \otimes P) \Delta(G)$$



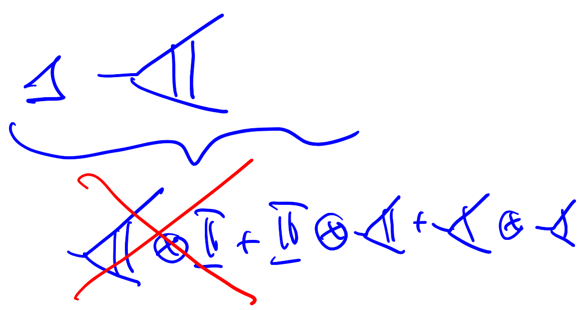
ϕ^3_6



$$(*)\ \Phi\left(\ \right) = \int d^6k\, d^6\ell \, \frac{1}{k^2 (k+q)^2 (\ell-k)^2 (\ell+q)^2 (\ell-p_1)^2 \ell^2}$$



$$\Rightarrow [\mathrm{id} - R]\, m_c \left(S_R^{\bar\Phi} \otimes \bar\Phi\, P \right) \Delta$$



$$[\mathrm{id} - R]\, m_F \Big(\underbrace{S_R^{\Phi}(\;)}_{1} \otimes \Phi(\;) + \underline{\underline{S_R^{\Phi}(\;) \otimes \Phi(\;)}} \Big)$$

$$\Phi(\;) = \int d^6\ell \int d^6 k \; \frac{1}{\cdots\cdots} \qquad \text{see } (*)$$

$$= \int d^6\ell \; \frac{1}{\ell^2 (\ell+q)^2 (\ell-p_1)^2} \; \underline{\Phi(\;)}\big(\ell^2, (\ell+q)^2, q^2\big)$$

subgraph

ϕ massless $\quad$ 2^{nd} Sym.

φ $\quad$ 1^{st} "

$$\underline{\Phi} = \underline{\phi} + \varphi \underbrace{\Big(\sum_e m_e^2 t_e\Big)}_{\text{M.A}}$$

In parametric,
always Φ, and φ.