Master Thesis:

The Corolla Polynomial for spontaneously broken Gauge Theories

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Structure of Local
Quantum Field Theories

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Contents

1 Introduction

Quantum Field Theories (QFT's) are still one of the most important tools for the description of nature in its smallest scale. The precision of the predictions of the Standard Model are astonishing, but also the amount of work needed to gain mathematical well-defined theories make QFT's one of the most fascinating topics in mathematical physics [1, 2, 3].

One of the curiosities of non-abelian gauge theories is the observation that gauge bosons in the quantized theory do not only possess the two experimentally verified transversal degrees of freedom, but also a longitudinal one, which is not cancelled with the timelike degree of freedom as in quantum electrodynamics by the virtue of the Ward identity. This problem was solved by the introduction of unphysical particles[1], existing only in closed loops, the so-called ghosts. Then, the Slavnov-Taylor identity ensures that all gauge boson amplitudes are purely transversal. Although it was shown that this could be done in a self-consistent way, it remained unsatisfying since there was no convenient argument despite adjusting the theory to experimental facts. However, this question is solved in the definition of the Corolla polynomial [4] with the introduction of cycle homology and allows, as such, a covariant quantization of gauge fields without the need of introducing ghost fields [1, 2, 3, 4].

Furthermore, the introduction of the Corolla polynomial also clarifies the relation between scalar field theory with cubic interaction and gauge theory. This is done using the parametric representation with its two Kirchhoff or Symanzik polynomials and the creation of a Corolla differential out of the Corolla polynomial making implicit use of graph homology. In particular, the parametric integrand for all gauge theory Feynman graphs, which can be created from a 3-regular scalar QFT Feynman graph by shrinking edges, can be obtained whilst acting with the Corolla differential $\mathcal{D}(\Gamma)$ on the parametric integrand $I(\Gamma)$ of the corresponding scalar QFT such that the gauge theory amplitude $\tilde{I}_F(\Gamma)$ reads[2] [4]

$$\tilde{I}_F(\Gamma) = \mathcal{D}(\Gamma) I(\Gamma) \,. \tag{1}$$

It is then possible to receive the renormalized gauge theory amplitude by replacing $I(\Gamma)$ by its renormalized analogue $I^R(\Gamma)$, i.e. the problem of renormalizing gauge theory gets translated back to the renormalization of scalar field theory with cubic interaction [4].

The aim of this thesis is now to generalize this approach to include the gauge bosons of the electroweak sector of the Standard Model (cf. Subsection 5.2) as well as it's scalar particles (cf. Subsection 5.3). This was done by first working out the combinatorics of labeling a 3-regular scalar QFT Feynman graph with labels of the gauge bosons of the electroweak sector of the Standard Model, and then by working out the additional tensor structures arising from the inclusion of the Feynman rules for the scalar particles of the electroweak sector of the Standard Model [5].

[1] In the sense of contradicting the Spin-Statistic theorem, i.e. being scalar particles (having spin 0) but obeying Fermi-Dirac-statistics [1, 2].

[2] Actually the different gauge theory graph contributions created by the Corolla differential are hidden in the parametric integrand $\tilde{I}_F(\Gamma)$ in it's regular part and residues along the Schwinger parameters, cf. Subsection 5.1 (mainly Theorem 5.6).

2 Notations and conventions

2.1 Metric tensor and Einstein summation convention

We denote the metric tensor by

$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

and it's components by $\eta^{\mu\nu}$.

Important in this thesis and also in [4, 6] is that Einstein summation convention is even assumed (if not indicated otherwise) whenever two indices are the same - independent if they correspond to the space of covariant or contravariant vectors:

$$\eta^{\mu\nu}\eta^{\nu\rho} := \eta^{\mu\nu_1}\eta_{\nu_1\nu_2}\eta^{\nu_2\rho} = \eta^{\mu\rho} \tag{3}$$

This at first sight inconvenient looking expression allows to define the Corolla polynomial in a more elegant way.

2.2 Feynman rules and chosen gauge

We use the definitions for the Feynman rules given in [5] with all appearing signs chosen positive, i.e. $\eta_G = \eta_s = \eta_e = \eta = \eta_Z = 1$. Furthermore we use the Feynman gauge throughout this thesis - this allows a more compact notation and avoids unnecessary applications of the Leibniz rule, i.e. $\xi_G = \xi_A = \xi_W = \xi_Z = 1$. The use of general gauges is explained in [4, 6]. All relevant Feynman rules are given explicitly in Appendix A.

2.3 Feynman graphs with oriented edges

In the Standard model some particle types have oriented edges (e.g. fermions, ghosts, $W^\pm$-particles, $\varphi^\pm$-particles). We work in this thesis with unoriented edges, where a graph with unoriented edges is understood as the sum of graphs with all possible orientations times the corresponding symmetry factor (cf. Definition 3.3). Nevertheless in Appendix A we give the Feynman rules with oriented edges for concreteness.

3 Graph theoretic notions

A QFT is characterized by it's Lagrangian density which dictates the set's $\mathcal{R}_V$ and $\mathcal{R}_E$ of all possible vertex- and edge-types, respectively. This states all allowed particle interactions and particle types out of which Feynman graphs can be built of. Now, we provide all necessary graph theoretic notions:

Definition 3.1 (Feynman graph, in parts literally quoted from [7, Definition 1])**.** A Feynman graph Γ is characterized by a set of vertices $\Gamma^{[0]}$ and a set of edges $\Gamma^{[1]} = \Gamma^{[1]}_{\text{ext}} \cup \Gamma^{[1]}_{\text{int}}$ whose elements are part of $\mathcal{R}_V$ and $\mathcal{R}_E$, respectively, and a set of maps

$$\partial_j : \quad \Gamma^{[1]} \mapsto \Gamma^{[0]} \cup \{ 1, 2, \dots, N \} , \qquad j \in \{ 0, 1 \} , \tag{4}$$

respecting the vertex and edge types given by $\mathcal{R}_V$ and $\mathcal{R}_E$. Furthermore the case ∂_0 and ∂_1 being both in $\{ 1, 2, \dots, N \}$ is excluded. The set $\{ 1, 2, \dots, N \}$ labels external lines of Γ, such that $\sum_{j=0}^{1} \operatorname{card} \partial_j^{-1}(v) = 1$ for all $v \in \{ 1, 2, \dots, N \}$. The set of external lines is therefore defined as $\Gamma^{[1]}_{\text{ext}} := \bigcup_{j=0}^{1} \partial_j \{ 1, 2, \dots, N \}$ and the set of internal lines as it's complement with respect to the set of edges of Γ, i.e. $\Gamma^{[1]}_{\text{int}} := \Gamma^{[1]} \setminus \Gamma^{[1]}_{\text{ext}}$. Therefore, external lines can be labeled by $e_1, e_2, \dots, e_N \in \Gamma^{[1]}_{\text{ext}}$, with $e_k := \bigcup_{j=0}^{1} \partial_j(k)$.

Furthermore we omit scalar graphs with edges which form self-loops (so-called tadpoles). This is possible without loss of generality since their amplitude vanishes during the renormalization process[3]. In the gauge theory amplitudes created by the Corolla polynomial, amplitudes from gauge theory graphs with tadpoles will show up again by the use of graph-homology, cf. Section 5 and [4].

The two Symanzik polynomials (Definition 4.3 and Definition 4.5) are polynomials in edge-variables $\{ A_e \}_{e \in \Gamma^{[1]}}$, whereas the Corolla polynomial (Definition 5.1) is a polynomial in half-edge-variables $\{ a_h \}_{h \in \Gamma^{[1/2]}}$ which are defined as follows:

Definition 3.2 (Half-edge [4])**.** Let Γ be a Feynman graph, $\Gamma^{[0]}$ the set of it's vertices, $\Gamma^{[1]}$ the set of it's edges and $n(v) \subset \Gamma^{[1]}$ the set of edges adjacent to the vertex v. Then the tuple

$$h := (v, e) , \qquad v \in \Gamma^{[0]} , \ e \in n(v) , \tag{5}$$

is called a half-edge of Γ. The set of all half-edges of Γ is denoted by $\Gamma^{[1/2]}$. Note that each internal edge defines two half-edges in a unique way, since we do not allow tadpoles (cf. Definition 3.1).

We move on by defining automorphisms of a Feynman graph Γ and it's symmetry factor $\operatorname{sym}(\Gamma)$:

Definition 3.3 (Automorphisms and symmetry factors of a Feynman graph, in parts literally quoted from [7, Definition 2])**.** Let Γ be a Feynman graph. An automorphism of Γ is given by an isomorphism $g^{[0]}$ from $\Gamma^{[0]}$ to itself and an isomorphism $g^{[1]}$ from $\Gamma^{[1]}$ to itself that is the identity on $\Gamma^{[1]}_{\text{ext}}$ and fulfilling for all $e \in \Gamma^{[1]}$

$$\bigcup_{j=0}^{1} g^{[0]} \left(\partial_j(e) \right) = \bigcup_{j=0}^{1} \partial_j \left(g^{[1]}(e) \right) . \tag{6}$$

[3] Speaking in Hopf-algebraic language, the graphs with tadpoles form a Hopf ideal I_{tad} in the Hopf-algebra H_{FG} of Feynman graphs and we can effectively work in the quotient space $H_{\text{FG}}/I_{\text{tad}}$ [4].

Additionally, we require $g^{[0]}$ and $g^{[1]}$ to respect the vertex and edge types given by the sets $\mathcal{R}_V$ and $\mathcal{R}_E$, respectively.

The automorphism group of Γ is denoted by $\operatorname{aut}(\Gamma)$ and consists of all such automorphisms of Γ. The order of the automorphism group of Γ is called the symmetry factor of Γ and denoted by $\operatorname{sym}(\Gamma)$, i.e.

$$\operatorname{sym}(\Gamma) := \# \operatorname{aut}(\Gamma) . \tag{7}$$

Definition 3.4 (Paths and cycles [8])**.** Let Γ be a graph, $\Gamma^{[0]}$ it's vertex set and $\Gamma^{[1]}$ it's edge set. Then:

1. Γ is called a path if it is non-empty with vertex set $\Gamma^{[0]} = \{ v_1, v_2, \dots, v_v \}$ and edge set $\Gamma^{[1]} = \{ v_1 v_2, v_2 v_3, \dots, v_{v-1} v_v \}$. In particular a path connects it's two endpoints in a unique way and every internal vertex has precisely two edges attatched to it. Paths are denoted by P, and sets of paths by $\mathscr{P}$.
2. Γ is called a cycle in mathematics or a loop in physics[4] if it is non-empty with vertex set $\Gamma^{[0]} = \{ v_1, v_2, \dots, v_v \}$ and edge set $\Gamma^{[1]} = \{ v_1 v_2, v_2 v_3, \dots, v_{v-1} v_v, v_v v_1 \}$ (where no repeated vertices are allowed, i.e. $v_i \neq v_j$ for $i \neq j$). In particular, a cycle can be created from the union of two disjoint paths having the same endpoints. Cycles are denoted by C, sets of cycles by $\mathscr{C}$ and bases of cycles by $\mathfrak{C}$.

Definition 3.5 (Trees, forests, spanning n-forests [8])**.** Let Γ be a graph. Then:

1. Γ is called a forest if it is non-empty and does not contain any cycles. Forests are denoted by F and sets of forests by $\mathscr{F}$. If F has n connected components, then F is also called an n-forest and denoted by F_n and the sets of all n-forests by $\mathscr{F}_n$.
2. If a forest F is connected (i.e. a 1-forest F_1) it also called a tree. Trees are denoted by T and sets of trees by $\mathscr{T}$.
3. If an (n-)forest or a tree covers all vertices of a graph Γ, then it is called a spanning (n-)forest of Γ or a spanning tree of Γ, respectively. The sets of all spanning (n-)forests F (F_n) and spanning trees T are denoted by $\mathscr{F}(\Gamma)$ ($\mathscr{F}_n(\Gamma)$) and $\mathscr{T}(\Gamma)$, respectively.

Definition 3.6 (Incidence matrix, in parts literally quoted from [4, Page 7])**.** We define the incidence matrix $\varepsilon(\Gamma)$ of a graph Γ componentwise as

$$\varepsilon_{ve}(\Gamma) = \begin{cases} +1 & \text{if the vertex } v \text{ is the endpoint of the edge } e \\ -1 & \text{if the vertex } v \text{ is the starting point of the edge } e \\ 0 & \text{if } e \text{ is not incident to the vertex } v \end{cases} . \tag{8}$$

Definition 3.7 (Assigning 4-vectors to a Feynman graph, in parts literally quoted from [4, Page 7])**.** We assign a 4-vector $\xi_e'^{\mu}$ to each edge e of a Feynman graph Γ in the following way: First we choose a basis of loops $\mathfrak{C}_\Gamma \subset \mathscr{C}_\Gamma$ of $|\mathscr{C}_\Gamma|$ independent loops of Γ and we choose for each $C \in \mathfrak{C}_\Gamma$ an orientation ε_{ve}^C (where ε_{ve}^C is defined in such a way that $\varepsilon_{ve_1}^C = -\varepsilon_{ve_2}^C$ with e_1 and e_2 being two edges adjacent to the vertex v and inside the loop C). Then we assign to each half-edge $h \equiv (v, e)$ the 4-vector[5]

$$\varepsilon_{ve} \xi_e'^{\mu} := \varepsilon_{ve} \xi_e^{\mu} + \sum_{C \in \mathfrak{C}_\Gamma} \sum_{e \in C^{[1]}} \varepsilon_{ve}^C k_C^{\mu} , \tag{9}$$

[4] Be aware that a loop in mathematics is what is called a self-loop or a tadpole in physics. We use the terms cycle and loop interchangeably in the above defined sense, depending if the context is more motivated from a mathematical or a physical point of view.

[5] The notion of the half-edge h is here only important to clarify the orientation of the 4-vector ξ_e^{μ}.

where the ξ_e^μ are independent in the sense that momentum conservation is not assumed until the end of all calculations, and the k_C^μ are the loop-momenta which are to be integrated out.

Definition 3.8 (Genus of a graph, in parts literally quoted from [4, Page 4])**.** Let Γ be a graph and $\mathcal{M}_k$ an oriented Riemannean manifold of genus k. Then Γ is said to be k-compatible, if it can be drawn on $\mathcal{M}_k$ without self-intersections. Furthermore Γ is said to be of genus k, if it can be drawn on $\mathcal{M}_k$ without self intersections, but not on $\mathcal{M}_l$ with $l < k$. Planar graphs are of genus 0.

Definition 3.9 (Orientation of a 3-regular graph, in parts literally quoted from [4, Page 4])**.** Let Γ be a 3-regular k-compatible Feynman graph, drawn on an oriented Riemannean manifold $\mathcal{M}_k$ of genus k. Then Γ inherits an orientation by $\mathcal{M}_k$ in the sense that every half-edge h incident to a vertex v has a well-defined successor h_+ and predecessor h_-.

4 Parametric representation of scalar quantum field theories

The parametric representation for scalar QFT's can be obtained by the use of the so-called Schwinger trick[6] [1, 3, 4, 6]:

$$\frac{1}{x} = \int_{\mathbb{R}_+} \mathrm{d}A e^{-Ax} \tag{10}$$

Using this trick, the product of propagators in the numerator from the standard Feynman rules (where the p_e^μ correspond to physical momentum 4-vectors, i.e. momentum conservation is assumed) can be turned into a sum of an exponential function (where an euclidean spacetime is assumed[7] and all appearing constants are collected in α) [1, 3, 6]:

$$\begin{aligned} \alpha \prod_{e \in \Gamma^{[1]}_{\text{int}}} \frac{1}{(p_e^2 + m_e^2)} =& \alpha \prod_{e \in \Gamma^{[1]}_{\text{int}}} \int_{\mathbb{R}_+} \mathrm{d}A_e e^{-A_e \left(p_e^2 + m_e^2\right)} \\ =& \alpha \int_{\mathbb{R}_+^{\left|\Gamma^{[1]}_{\text{int}}\right|}} \left(\prod_{e \in \Gamma^{[1]}_{\text{int}}} \mathrm{d}A_e \right) e^{-\left(\sum_{e \in \Gamma^{[1]}_{\text{int}}} A_e \left(p_e^2 + m_e^2\right)\right)} \end{aligned} \tag{11}$$

Remark 4.1. For our purposes in defining the Corolla polynomial in Section 5 we alter the standard definition of the parametric representation of scalar QFT in two ways: First, we will also include Schwinger variables A_e for external half-edges and secondly we assign 4-vectors $\xi_e'^\mu$ to each edge e of Γ which we define to consist of the sum of independent variables ξ_e^μ and the corresponding loop momenta k_C^μ for C a loop in the basis of loops $\mathfrak{C}_\Gamma$ of the Feynman graph Γ (cf. Definition 3.7).

Therefore we define the following simplified notation:

Definition 4.2 (Abbreviations [4])**.** We denote:

1. The simplex of our parametric integration domain by σ, i.e.

$$\sigma := \mathbb{R}_+^{\left|\Gamma^{[1]}\right|} .$$

2. The measure of our extended parametric space by $\mathrm{d}\underline{A}_\Gamma$, i.e.

$$\mathrm{d}\underline{A}_\Gamma := \prod_{e \in \Gamma^{[1]}} \mathrm{d}A_e .$$

3. The space of all loop-momenta by ρ (recall $|\mathfrak{C}_\Gamma|$ to be the dimension of the basis of loops of Γ from Definition 3.4 and Definition 3.7), i.e.[8]

$$\rho := \mathbb{R}^{4|\mathfrak{C}_\Gamma|} .$$

4. The measure of the loop-momenta integral by $\mathrm{d}\underline{k}_\Gamma$ (recall that we choose a basis of loops $\mathfrak{C}_\Gamma \subset \mathscr{C}_\Gamma$ of $|\mathscr{C}_\Gamma|$ independent loops of Γ from Definition 3.7, and $\mathrm{d}^4 k_C$ being the usual

[6] A similar result can be obtained using the so-called Feynman trick [1].

[7] This can be obtained from the Minkowski spacetime using the so-called Wick rotation [1, 2, 3].

[8] Note that the dimension is here actually the dimension of the basis of loops of Γ times the dimension of spacetime, but we're working in a 4-dimensional spacetime throughout this thesis [1, 2, 3].

Lorentz invariant loop-momentum measure for the loop C [1, 2, 3]), i.e.[9]

$$\mathrm{d}\underline{k}_\Gamma := \prod_{C \in \mathfrak{C}_\Gamma} \mathrm{d}^4 k_C \,.$$

5. The extended universal quadric by $\underline{Q}_\Gamma$ (where the case $m_e = 0$ for some $e \in \Gamma^{[1]}$ is allowed), i.e.

$$\underline{Q}_\Gamma := \sum_{e \in \Gamma^{[1]}} \left(\xi_e'^2 + m_e^2 \right) A_e \,.$$

6. The reduced universal quadric by $\underline{q}_\Gamma$ (where again the case $m_e = 0$ for some $e \in \Gamma^{[1]}$ is allowed), i.e.

$$\underline{q}_\Gamma := \left(\sum_{e \in \Gamma^{[1]}_{\text{ext}}} \xi_e^2 A_e \right) + \left(\sum_{e \in \Gamma^{[1]}} m_e^2 A_e \right) \,.$$

7. The product over the inverse external propagators by P_Γ, i.e.

$$P_\Gamma := \prod_{e \in \Gamma^{[1]}_{\text{ext}}} \left(\xi_e^2 + m_e^2 \right) \,.$$

8. The differential form concerning the parametric space by $I(\Gamma)$, i.e.

$$I(\Gamma) := \mathrm{d}\underline{A}_\Gamma \left(\alpha P_\Gamma \int_\rho \mathrm{d}\underline{k}_\Gamma e^{-\underline{Q}_\Gamma} \prod_{v \in \Gamma^{[0]}} \delta^{(4)} \left(\sum_{e \in \Gamma^{[1]}} \varepsilon_{ve} k_e^\mu \right) \right) \,.$$

Then, Equation (11) reads

$$\alpha \prod_{e \in \Gamma^{[1]}_{\text{int}}} \frac{1}{\left(\xi_e'^2 + m_e^2 \right)} = \alpha P_\Gamma \int_\sigma \mathrm{d}\underline{A}_\Gamma e^{-\underline{Q}_\Gamma} \,. \tag{12}$$

One of the advantages of passing to the parametric space is that now the loop-momentum integrals can be carried out changing the order of integration[10]. In doing so, the two so-called Kirchhoff- or Symanzik-polynomials ψ_Γ and ϕ_Γ[11] come into play:

Definition 4.3 (First Symanzik polynomial [1, 4, 6])**.** Let Γ be a scalar QFT Feynman graph and $\mathscr{T}(\Gamma)$ the set of it's spanning trees T. Then we define the first Symanzik polynomial as (external half-edges are excluded from the product)

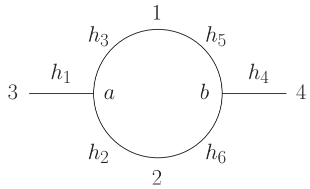
$$\psi_\Gamma := \sum_{T \in \mathscr{T}(\Gamma)} \prod_{e \notin T} A_e \,. \tag{13}$$

[9]The power 4 in the measure $\mathrm{d}^4 k_C$ is actually the dimension of spacetime, but again we're working in a 4-dimensional spacetime throughout this thesis [1, 2, 3].

[10]The change of the integration order for Minkowski spacetime or massless particles (i.e. ill-defined integral expressions) is formally only allowed if regulators $i\epsilon$ are introduced before in each such propagators and whose limits to 0 are understood to be taken after the integrations are carried out [1, 3].

[11]We will slightly alter the standard definition of the second Symanzik polynomial for our purposes and denote it by φ_Γ, cf. Definition 4.5.

Example 4.4. We consider the one-loop self-energy graph

$$\Gamma :=$$  . (14)

We have $\mathscr{T}(\Gamma) = \{1, 2\}$, and so

$$\psi_\Gamma = A_1 + A_2 \,. \tag{15}$$

Definition 4.5 (Second Symanzik polynomial, non-standard definition [1, 4, 6])**.** Let Γ be a scalar QFT Feynman graph, $\mathscr{F}_2$ the set of it's spanning two-forests F_2, which consist of the two components F_2^1 and F_2^2, and let $\varepsilon(\Gamma)$ be it's incidence matrix (cf. Definition 3.6). Then we define the second Symanzik polynomial as (again, external half-edges are excluded from the sum and the product)

$$\varphi_\Gamma := \sum_{F_2 \in \mathscr{F}_2(\Gamma)} \left(\sum_{e \notin F_2} \tau(e) \xi_e \right)^2 \prod_{e \notin F_2} A_e \,, \tag{16}$$

with[12]

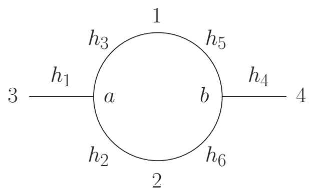
$$\tau(e) := \begin{cases} +1 & \text{if } e \text{ is oriented from } F_2^1 \text{ to } F_2^2 \\ -1 & \text{if } e \text{ is oriented from } F_2^2 \text{ to } F_2^1 \end{cases} . \tag{17}$$

Remark 4.6. Note that the usual expression ϕ_Γ for the second Symanzik polynomial of a Feynman graph Γ can be obtained by setting the ξ_e^μ in accordance with external momenta, i.e.

$$Q: \quad \xi_e \mapsto \xi_e + q_e, \qquad \forall e \in \Gamma^{[1]} \tag{18}$$

and setting all $\xi_e = 0$ afterwards, in parts literally quoted from [4, Page 9].

Example 4.7. Again, consider the one-loop self-energy graph with incoming external momenta p^{μ_3} and p^{μ_4}

$$\Gamma :=$$  . (19)

We have $\mathscr{F}_2 = \{\{a, b\}\}$, and so

$$\varphi_\Gamma = (\xi_1 - \xi_2)^2 A_1 A_2 \,. \tag{20}$$

Theorem 4.8 (Parametric integrand with non-standard second Symanzik polynomial [4])**.** *Integrating out loop-momenta gives rise to the two Symanzik polynomials*[13]*:*

$$\int_\rho \mathrm{d}\underline{k}_\Gamma e^{-\underline{Q}_\Gamma} \prod_{v \in \Gamma^{[0]}} \delta^{(4)} \left(\sum_{e \in \Gamma^{[1]}} \varepsilon_{ve} k_e^\mu \right) = \frac{e^{-\frac{\varphi_\Gamma}{\psi_\Gamma} - \underline{q}_\Gamma}}{\psi_\Gamma^2} \tag{21}$$

[12] Note that once we fix the two components F_2^1 and F_2^2 of a spanning 2-tree F_2 the second Symanzik polynomial is independent of that choice, since the expression $\sum_{e \notin F_2} \tau(e) \xi_e$ get's squared [6].

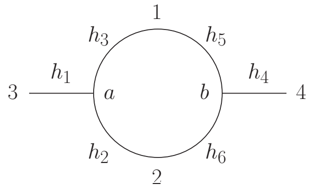
[13] The square of ψ_Γ in the numerator is actually a $d/2$, with d being the dimension of spacetime, but once more we're working in a 4-dimensional spacetime throughout this thesis [1, 6].

In particular, the parametric integrand with non-standard second Symanzik polynomial $I(\Gamma)$ can be written as

$$
\begin{aligned}
I(\Gamma) &= \mathrm{d}\underline{A}_\Gamma \left(\alpha P_\Gamma \int_\rho \mathrm{d}\underline{k}_\Gamma e^{-\underline{Q}_\Gamma} \right) \\
&= \mathrm{d}\underline{A}_\Gamma \left(\alpha P_\Gamma \frac{e^{-\frac{\varphi_\Gamma}{\psi_\Gamma} - \underline{q}_\Gamma}}{\psi_\Gamma^2} \right)
\end{aligned} \tag{22}
$$

Proof. We refer to [3, Pages 294 - 299] for a proof of the parametric integrand with standard second Symanzik polynomial and to [4, Page 10] for the notational difference concerning the non-standard second Symanzik polynomial. ∎

Example 4.9. Continuing Example 4.4 and Example 4.7 for the one-loop self-energy graph

$\Gamma :=$ 

, (23)

we have

$$
I(\Gamma) = \left(\xi_3^2 + m_3^2\right) \left(\xi_4^2 + m_4^2\right) \frac{e^{-\frac{(\xi_1 - \xi_2)^2 A_1 A_2}{A_1 + A_2} - \xi_3^2 A_3 - \xi_4^2 A_4 - \sum_{e=1}^4 m_e^2 A_e}}{(A_1 + A_2)^2} \,. \tag{24}
$$

Definition 4.10 (Parametric integrand for gauge theory amplitudes with non-standard second Symanzik polynomial, in parts literally quoted from [4, Page 10])**.** In the following we're in particular interested in gauge theory amplitudes. They can be represented in the parametric space by a slightly generalization of $I(\Gamma)$ via

$$
\tilde{I}_F(\Gamma) := F I(\Gamma)\,, \tag{25a}
$$

with

$$
F := \frac{F_N(\{\, A_e \,\}_{e \in \Gamma^{[1]}})}{F_D(\{\, A_e \,\}_{e \in \Gamma^{[1]}})}\,, \tag{25b}
$$

a rational function in the Schwinger parameters A_e and possible matrix structure. The rational function F can be created by acting with suitable differential operators on the parametric integrand $I(\Gamma)$ [4, 6].

Remark 4.11. Before $I(\Gamma)$ (or $\tilde{I}_F(\Gamma)$) can be integrated against the simplex σ to yield the Feynman amplitude $\mathcal{I}^R(\Gamma)$ (or $\tilde{\mathcal{I}}_F^R(\Gamma)$) of Γ it needs to be renormalized first (cf. [1, 2, 3, 4, 6, 7, 9] for the huge topic of renormalization). The corresponding renormalized differential form $I^R(\Gamma)$ (or $\tilde{I}_F^R(\Gamma)$) can be obtained from $I(\Gamma)$ (or $\tilde{I}_F(\Gamma)$) by using the Forest formula [1, 3, 4, 9].

5 Corolla polynomial and differential

5.1 Pure Yang-Mills theory

Now we define the Corolla polynomial for massless pure Yang-Mills theory:

Definition 5.1 (Corolla polynomial, in parts literally quoted from [4, Page 27] and [10, Definition 1], cf. [6])**.** Let Γ be a 3-regular scalar QFT Feynman graph. Then:

1. Associate to each half-edge h of Γ a variable a_h.
2. Recall from Definition 3.2: For a vertex v of Γ we denote the set of the three half-edges incident to v by $n(v)$.
3. Recall from Definition 3.9: For a vertex v of Γ and $h \in n(v)$ a half-edge of Γ, we denote according to the orientation of Γ it's successor by h_+ and it's predecessor by h_-.
4. We denote the edge e of which h is part of by $e(h)$.
5. Recall from Definition 3.4: We denote the set of all cycles of Γ by $\mathscr{C}_\Gamma$.
6. For $C \in \mathscr{C}_\Gamma$ a cycle and v a vertex in C, since Γ is 3-regular, there is a unique half-edge of Γ incident to v and not in C. We denote this half-edge by $h(C, v)$.
7. Assign to Γ a factor color(Γ).
8. We denote the combination of half-edge variables which will create in the Corolla differential the Feynman rules for the 3-valent gluon vertex v (and also the 4-valent gluon vertex as residues in Schwinger parameters if applied twice with special combinations of half-edge variables due to the Leibniz rule, cf. Theorem 5.6 and [4, 6]) as
$$\mathscr{V}_v := \sum_{h \in n(v)} \eta^{\mu_{e(h_+)} \mu_{e(h_-)}} \left(a_{h_+} - a_{h_-} \right) .$$
9. We denote the combination of half-edge variables which will create in the Corolla differential the Feynman rules for the closed ghost loop C_j as
$$\mathscr{G}_{C_j} := \sum_{k \in \{ +,- \}} \prod_{v \in C_j} a_{h(C_j, v)_k} .$$

Then we can define the various summands of the Corolla polynomial for pure Yang-Mills theory by
$$\mathcal{C}^0(\Gamma) := \prod_{v \in \Gamma^{[0]}} \mathscr{V}_v \tag{26a}$$
and for $i \geq 1$ by
$$\mathcal{C}^i(\Gamma) := \sum_{\substack{C_1, C_2, \dots, C_i \in \mathscr{C}_\Gamma, \\ C_j \text{ pairwise disjoint}}} \left[\left(\prod_{j=1}^{i} \mathscr{G}_{C_j} \right) \left(\prod_{\substack{v \in \Gamma^{[0]}, \\ v \notin \bigcup_{k=1}^{i} C_k}} \mathscr{V}_v \right) \right] , \tag{26b}$$
where we define $\prod_{v \in \emptyset} \mathscr{V}_v := 1$, i.e. if $\{ v \in \Gamma^{[0]} | v \notin \bigcup_{k=1}^{i} C_k \} = \emptyset$. Finally, we introduce the Corolla polynomial as the alternating sum over its summands:
$$\mathcal{C}(\Gamma) := \sum_{i=1}^{\infty} (-1)^i \mathcal{C}^i(\Gamma) \tag{26c}$$

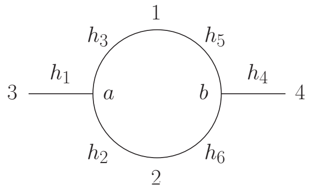
Furthermore, we define

$$\mathcal{C}_{\mathrm{QCD}}(\Gamma) := g_s^{\Gamma^{[0]}} \operatorname{color}(\Gamma)\mathcal{C}(\Gamma)\,. \tag{27}$$

Remark 5.2.

1. Note that our definition of the Corolla polynomial in the half-edge variables a_h differs from the ones given in [4, 6, 10], but we will also define a different differential operator $\mathscr{A}_h$ such that the Corolla differentials $\mathcal{D}(\Gamma)$ coincide again (despite from factors of $1/4^i$ which were missing in each summand $\mathcal{D}^i(\Gamma)$ in [10, 4], cf. [6] and the fact that our definition does not create 4-valent ghost vertex contributions from the start which need to be eliminated while working in a linear covariant gauge otherwise, cf. [4]). Furthermore, our definition is only valid for the Feynman gauge (which is in particular linear).
2. $\mathcal{C}(\Gamma)$ is a polynomial since $\mathcal{C}^i(\Gamma) = 0$ for all $i > |\mathscr{C}_\Gamma|$ [4, 10].
3. As in [4], the factors $\pm i$ for the vertex and propagator Feynman rules are not explicitly given and should be taken into account for concrete calculations.

Example 5.3. Consider the one-loop self-energy graph

$\Gamma :=$ 

(28)

with the six half-edges $\{\, h_1 := (a,3), h_2 = (a,2), h_3 = (a,1), h_4 = (b,4), h_5 = (b,1), h_6 = (b,2)\,\}$.

Then, we have

$$\begin{aligned}\mathcal{C}^0(\Gamma) = &\Big(\eta^{\mu_2\mu_1}\left(a_{h_{1+}} - a_{h_{1-}}\right) + \eta^{\mu_1\mu_3}\left(a_{h_{2+}} - a_{h_{2-}}\right) + \eta^{\mu_3\mu_2}\left(a_{h_{3+}} - a_{h_{3-}}\right)\Big)\\ &\times \Big(\eta^{\mu_1\mu_2}\left(a_{h_{4+}} - a_{h_{4-}}\right) + \eta^{\mu_2\mu_4}\left(a_{h_{5+}} - a_{h_{5-}}\right) + \eta^{\mu_4\mu_1}\left(a_{h_{6+}} - a_{h_{6-}}\right)\Big)\,,\end{aligned} \tag{29a}$$

$$\mathcal{C}^1(\Gamma) = a_{h_{1+}}a_{h_{4+}} + a_{h_{1-}}a_{h_{4-}} \tag{29b}$$

and

$$\mathcal{C}^i(\Gamma) = 0\,, \qquad \forall i > 1\,. \tag{29c}$$

So, in total we get:

$$\begin{aligned}\mathcal{C}(\Gamma) =&\mathcal{C}^0(\Gamma) - \mathcal{C}^1(\Gamma)\\ =&\Big(\eta^{\mu_2\mu_1}\left(a_{h_{1+}} - a_{h_{1-}}\right) + \eta^{\mu_1\mu_3}\left(a_{h_{2+}} - a_{h_{2-}}\right) + \eta^{\mu_3\mu_2}\left(a_{h_{3+}} - a_{h_{3-}}\right)\Big)\\ &\times \Big(\eta^{\mu_1\mu_2}\left(a_{h_{4+}} - a_{h_{4-}}\right) + \eta^{\mu_2\mu_4}\left(a_{h_{5+}} - a_{h_{5-}}\right) + \eta^{\mu_4\mu_1}\left(a_{h_{6+}} - a_{h_{6-}}\right)\Big)\\ &- a_{h_{1+}}a_{h_{4+}} - a_{h_{1-}}a_{h_{4-}}\end{aligned} \tag{29d}$$

Definition 5.4 (Corolla differential, in parts literally quoted from [4, Page 29])**.** Let Γ be a 3-regular scalar QFT Feynman graph. Then:

1. Assign to each external and internal edge a variable A_e and a 4-vector ξ_e^μ (as in Definition 3.7) and to each edge e a Lorentz index μ_e.
2. Define for each half-edge h_k the following differential operator (where $k \in \{\pm\}$):

$$\mathscr{A}_{h_k} := -k\varepsilon_{h_k}\frac{1}{2A_{e(h_k)}}\frac{\partial}{\partial \xi_{e(h_k)\mu_{e(h)}}}$$

Then, the summands of the Corolla differential $\mathcal{D}^i(\Gamma)$ are defined via the summands of the Corolla polynomial $\mathcal{C}^i(\Gamma)$ by replacing each half-edge variable a_{h_k} by the corresponding differential operator $\mathscr{A}_{h_k}$ (denoted by $a_{h_k} \mapsto \mathscr{A}_{h_k}$):

$$\mathcal{D}^i(\Gamma) := \mathcal{C}^i(\Gamma)\Big|_{a_{h_k}\mapsto\mathscr{A}_{h_k},\,\forall h_k\in\Gamma^{[1/2]}}\,, \tag{30a}$$

and similarly the Corolla differential $\mathcal{D}(\Gamma)$ is defined via the Corolla polynomial $\mathcal{C}(\Gamma)$ as

$$\mathcal{D}(\Gamma) := \mathcal{C}(\Gamma)\Big|_{a_{h_k}\mapsto\mathscr{A}_{h_k},\,\forall h_k\in\Gamma^{[1/2]}}\,. \tag{30b}$$

Likewise, the Corolla differentials $\mathcal{D}_{\mathrm{QCD}}(\Gamma)$ and $\mathcal{D}_{\mathrm{EW}}(\Gamma)$ are defined via the replacement of the half-edge variables a_{h_k} by the differential operators $\mathscr{A}_{h_k}$ in their corresponding Corolla polynomials $\mathcal{C}_{\mathrm{QCD}}(\Gamma)$ and $\mathcal{C}_{\mathrm{EW}}(\Gamma)$ (which will be defined in Definition 5.20), respectively.

Remark 5.5. 1. Expressions of the form $\mathcal{D}(\Gamma)I(\Gamma)$ are to be understood as

$$\mathcal{D}(\Gamma)I(\Gamma) := \big(\mathcal{D}(\Gamma)I(\Gamma)\big)\Big|_{\left.\begin{smallmatrix}\xi_e\mapsto\xi_e+q_e,\\ \xi_e\mapsto 0\end{smallmatrix}\right\},\,\forall e\in\Gamma^{[1]}}\,,$$

such that the Corolla differential acts on the parametric integrand with non-standard second Symanzik polynomial (cf. Definition 4.5 and Theorem 4.8) and after the differentiation the standard second Symanzik polynomial is obtained by setting the $\{\xi_e^\mu\}_{e\in\Gamma^{[1]}}$ in accordance with the external momenta (cf. Remark 4.6). Note also that due to the Leibnitz rule we get also contributions from differentiating the inverse external propagators P_Γ (cf. 7. from Definition 4.2) but since the differential operators for the external edges are of order 1, these contributions vanish again whilst setting the $\{\xi_e^\mu\}_{e\in\Gamma^{[1]}}$ in accordance with the external momenta (again, cf. Remark 4.6).

2. We choose the 4-vectors ξ_e^μ assigned to each edge of the graph independently, i.e. we do not assume momentum conservation until the Corolla differential acted on the scalar integrand (cf. Definition 3.7). Therefore, they can be seen as "temporary dummy variables". After applying the Corolla differential, we assume momentum conservation and they acquire the meaning of 4-momentum vectors.

3. $\mathcal{D}^i(\Gamma)$ creates, whilst acting on the corresponding parametric scalar QFT integrand $I(\Gamma)$, all possible massless pure Yang-Mills theory Feynman graphs with $|\mathfrak{C}_\Gamma|$ loops and i ghost loops. The alternating sum in the Corolla polynomial $\mathcal{C}(\Gamma)$ and hence also in the Corolla differential $\mathcal{D}(\Gamma)$ takes care of the minus sign for each closed ghost loop.

Theorem 5.6. *Then, the renormalized amplitude $\tilde{\mathcal{I}}_F^R(\Gamma)$ of all gauge theory graphs of pure Yang-Mills theory which can be achieved from Γ via graph and cycle homology [4] is then obtained via the Corolla differential $\mathcal{D}_{\mathrm{QCD}}$ (cf. Definition 5.4) and the renormalized parametric integrand with non-standard second Symanzik polynomial $I^R(\Gamma)$ (cf. Theorem 4.8 and again [1, 2, 3, 4, 6, 7, 9] for the huge topic of renormalization)*

$$\tilde{I}_F^R(\Gamma) = \mathcal{D}_{\mathrm{QCD}}(\Gamma)I^R(\Gamma) \tag{31}$$

by

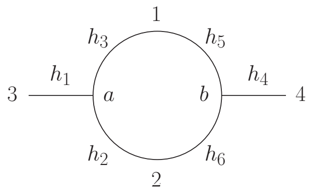
$$\tilde{\mathcal{I}}_F^R(\Gamma) = \frac{1}{\operatorname{sym}(\Gamma)}\sum_{k=0}^{\infty}\sum_{\{e_1,\dots,e_k\}\subset\Gamma^{[1]}_{\mathrm{int}}}\int \mathrm{d}\underline{A}_{\Gamma\setminus\{e_1,\dots,e_k\}}\operatorname*{Reg}_{A_1,\dots,\widehat{A_{e_1}},\dots,\widehat{A_{e_k}},\dots=0}\operatorname*{Res}_{A_{e_1},\dots,A_{e_k}=0}\tilde{I}_F^R(\Gamma)\,. \tag{32}$$

Proof. We refer to [4, Page 31] for a proof. ■

Example 5.7. We continue with Example 5.3:

Again, we have

$$\Gamma := \qquad (33)$$



and

$$\begin{aligned} \mathcal{C}(\Gamma) =& \mathcal{C}^0(\Gamma) - \mathcal{C}^1(\Gamma) \\ =& \left(\eta^{\mu_2\mu_1}\left(a_{h_{1+}} - a_{h_{1-}}\right) + \eta^{\mu_1\mu_3}\left(a_{h_{2+}} - a_{h_{2-}}\right) + \eta^{\mu_3\mu_2}\left(a_{h_{3+}} - a_{h_{3-}}\right)\right) \\ & \times \left(\eta^{\mu_1\mu_2}\left(a_{h_{4+}} - a_{h_{4-}}\right) + \eta^{\mu_2\mu_4}\left(a_{h_{5+}} - a_{h_{5-}}\right) + \eta^{\mu_4\mu_1}\left(a_{h_{6+}} - a_{h_{6-}}\right)\right) \\ & - a_{h_{1+}} a_{h_{4+}} - a_{h_{1-}} a_{h_{4-}} \,. \end{aligned} \qquad (34)$$

Choosing an anti-clockwise orientation for our embedding Riemannean manifold, external momenta p^{μ_3} and p^{μ_4} incoming and also an anti-clockwise orientation for the loop momenta, we receive:

$$\mathscr{A}_{h_{1+}} := \frac{1}{2A_2}\frac{\partial}{\partial \xi_{2\mu_3}}\,, \qquad \mathscr{A}_{h_{1-}} := \frac{1}{2A_1}\frac{\partial}{\partial \xi_{1\mu_3}}\,, \qquad (35a)$$

$$\mathscr{A}_{h_{2+}} := -\frac{1}{2A_1}\frac{\partial}{\partial \xi_{1\mu_2}}\,, \qquad \mathscr{A}_{h_{2-}} := \frac{1}{2A_3}\frac{\partial}{\partial \xi_{3\mu_2}}\,, \qquad (35b)$$

$$\mathscr{A}_{h_{3+}} := -\frac{1}{2A_3}\frac{\partial}{\partial \xi_{3\mu_1}}\,, \qquad \mathscr{A}_{h_{3-}} := -\frac{1}{2A_2}\frac{\partial}{\partial \xi_{2\mu_1}}\,, \qquad (35c)$$

$$\mathscr{A}_{h_{4+}} := \frac{1}{2A_1}\frac{\partial}{\partial \xi_{1\mu_4}}\,, \qquad \mathscr{A}_{h_{4-}} := \frac{1}{2A_2}\frac{\partial}{\partial \xi_{2\mu_4}}\,, \qquad (35d)$$

$$\mathscr{A}_{h_{5+}} := -\frac{1}{2A_2}\frac{\partial}{\partial \xi_{2\mu_1}}\,, \qquad \mathscr{A}_{h_{5-}} := \frac{1}{2A_4}\frac{\partial}{\partial \xi_{4\mu_1}}\,, \qquad (35e)$$

$$\mathscr{A}_{h_{6+}} := -\frac{1}{2A_4}\frac{\partial}{\partial \xi_{4\mu_2}}\,, \qquad \mathscr{A}_{h_{6-}} := -\frac{1}{2A_1}\frac{\partial}{\partial \xi_{1\mu_2}}\,, \qquad (35f)$$

and

$$\begin{aligned} \mathcal{D}(\Gamma) =& \left(\eta^{\mu_2\mu_1}\left(\mathscr{A}_{h_{1+}} - \mathscr{A}_{h_{1-}}\right) + \eta^{\mu_1\mu_3}\left(\mathscr{A}_{h_{2+}} - \mathscr{A}_{h_{2-}}\right) + \eta^{\mu_3\mu_2}\left(\mathscr{A}_{h_{3+}} - \mathscr{A}_{h_{3-}}\right)\right) \\ & \times \left(\eta^{\mu_1\mu_2}\left(\mathscr{A}_{h_{4+}} - \mathscr{A}_{h_{4-}}\right) + \eta^{\mu_2\mu_4}\left(\mathscr{A}_{h_{5+}} - \mathscr{A}_{h_{5-}}\right) + \eta^{\mu_4\mu_1}\left(\mathscr{A}_{h_{6+}} - \mathscr{A}_{h_{6-}}\right)\right) \\ & - \mathscr{A}_{h_{1+}}\mathscr{A}_{h_{4+}} - \mathscr{A}_{h_{1-}}\mathscr{A}_{h_{4-}} \,. \end{aligned} \qquad (36)$$

Acting with this differential operator on the scalar QFT parametric integrand with non-standard second Symanzik polynomial $I(\Gamma)$ (cf. Example 4.9), we receive [6, Example 5.15]

$$\begin{aligned} \tilde{I}_F(\Gamma) =& \mathcal{D}_{\mathrm{QCD}}(\Gamma) I(\Gamma) \\ =& g_s^2 f^{a_3a_2a_1} f^{a_4a_2a_1} \Bigg(\bigg(q^{\mu_3} q^{\mu_4} \left(2A_1^2 + 2A_2^2 + 12A_1A_2\right) \\ & - q^2 \eta^{\mu_3\mu_4} \left(5A_1^2 + 5A_2^2 + 8A_1A_2\right) \bigg) \frac{1}{\psi_\Gamma^2} 8 + \eta^{\mu_3\mu_4} \frac{1}{\psi_\Gamma} \Bigg) I(\Gamma) \,. \end{aligned} \qquad (37)$$

Then, we obtain (after renormalizing the parametric integrand $I(\Gamma)$) the renormalized Feynman amplitude with renormalization point μ [6, Example 5.15]

$$\tilde{\mathcal{I}}_F^R(\Gamma) = \frac{10}{3} g_s^2 f^{a_3 a_2 a_1} f^{a_4 a_2 a_1} \left(-q^{\mu_3} q^{\mu_4} + q^2 \eta^{\mu_3 \mu_4} \right) \ln \frac{q^2}{\mu^2} \tag{38}$$

which is transversal, as desired.

Remark 5.8. The full m-loop scattering amplitude can be obtained by applying the Corolla polynomial componentwise to the combinatorial Green's function at loop order m [4].

5.2 Inclusion of the gauge bosons of the electroweak sector

The inclusion of the gauge bosons of the electroweak sector is the next step to adapt the Corolla polynomial to the Standard Model.

Outline. First, recall that we have $W^{\pm}$-particle conservation which implies that every vertex of the electroweak sector of the Standard Model has to consist of a positive even number of $W^{\pm}$-particles, in particular every 3-valent vertex must consist of two $W^{\pm}$-particles. Therefore we define two nested sums over a 3-regular scalar QFT Feynman graph Γ in the following way: The first sum creates all possible ways to attach $W^{\pm}$-labels to Γ such that every 3-valent vertex consists of exactly two $W^{\pm}$-labeled edges. Then, the second sum creates all possible ways to attach Z- or A-labels to the unlabeled lines of the $W^{\pm}$-labeled graphs. The full details are explained in the proof after Theorem 5.12.

Therefore, we define:

Definition 5.9. Let Γ be a 3-regular graph. Then:

1. We define the equivalence class of sets $\mathscr{P}_W(\Gamma)$ to consist of all possible n-spanning forests P_W (cf. Definition 3.5) of Γ (modulo equivalence, see below) with the additional requirements that each of P_W's components is an unoriented path and furthermore fulfills one of the following two conditions:

   - The endpoints of the path are connected to external vertices (i.e. vertices with external half-edges attatched to it). In this case, enlarge this path by adding these external half-edges to it.

   - The endpoints of the path are connected to adjacent vertices (i.e. vertices which are connected via one or two edges). In this case, create one or two cycles by adding one of these edges to this path.

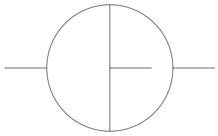
   Then, each set $P_W \in \mathscr{P}_W$ consists of pairwise disjoint paths and cycles. Finally, we establish an equivalence relation $P_W \sim P'_W$, if P_W is componentwise isomorphic to P'_W. Then $\mathscr{P}_W$ is defined to consist of all such equivalence classes $[P_W]$ of Γ, to which we refer from now on simply as P_W.

2. Let $\wp(\Gamma) := \wp\left(\Gamma^{[1]}\right)$ be the power set of all external half-edges and internal edges of Γ. In particular, we define $\mathscr{P}_Z\left(P_W\right) := \wp\left(\Gamma \setminus P_W\right)$, i.e. the power set of all external half-edges and internal edges of Γ which are not in the set P_W. The edges in each set P_Z in the set of sets $\mathscr{P}_Z\left(P_W\right)$ are labeled by a Z-label and the edges in $\Gamma \setminus \left(P_W \cup P_Z\right)$ are labeled by an A-label.

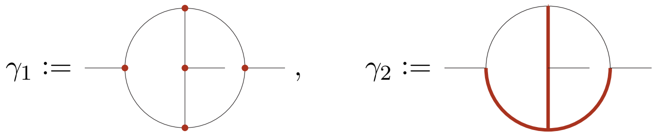
3. Let $\left|P_Z\right|$ and $\left|\Gamma \setminus P_Z\right|$ denote the number of vertices in P_Z and $\Gamma \setminus P_Z$, respectively.

4. Let $\text{iso}\,(\Gamma_{\text{labeled}})$ be the number of labeled graphs (via the sets P_W and P_Z) in the set $\{\mathscr{P}_Z(P_W)|P_W \in \mathscr{P}_W(\Gamma)\}$ isomorphic to Γ_{labeled}.

Example 5.10. To illustrate this, consider the graph:

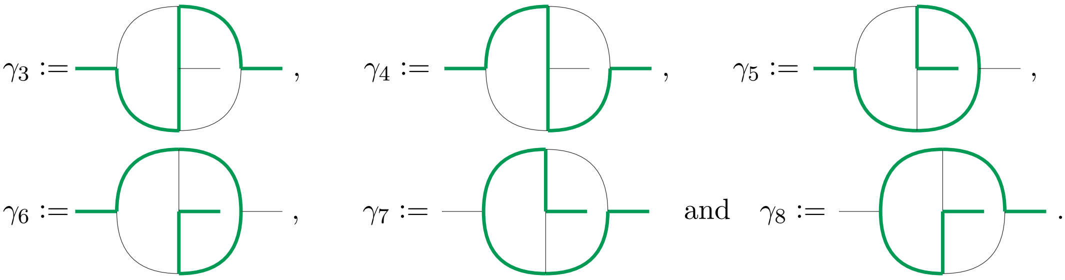
$\Gamma :=$  (39)

Since we have chosen a more complicated graph in order to show some more involved cases, we do not give all spanning n-forests but just some examples (where the red parts of Γ denote the spanning forest):



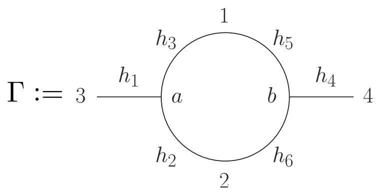
(40)

Here both forests, even though being spanning forests of Γ, are not allowed in the set $\mathscr{P}_W(\Gamma)$ since in both cases the parts of the spanning forests are not paths. The only allowed edge sets in $\mathscr{P}_W(\Gamma)$ (where the green lines represent the edge sets $P_W \in \mathscr{P}_W(\Gamma)$) are



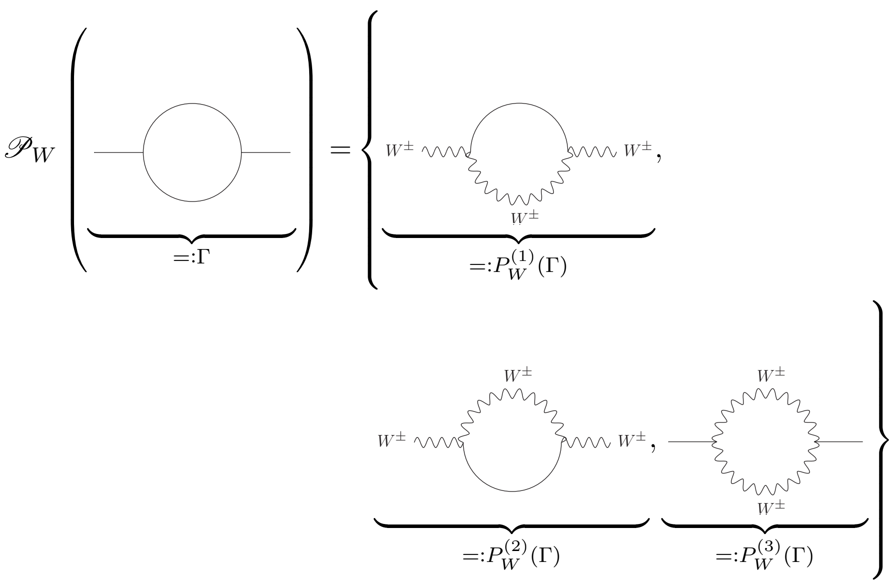
(41)

Example 5.11. In particular, for the one-loop self-energy graph



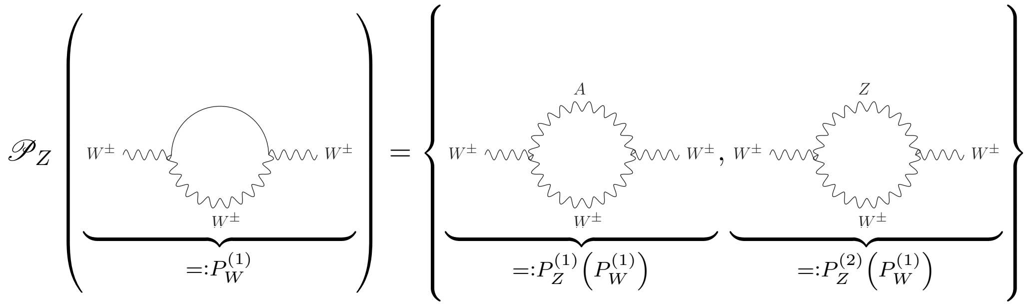
(42)

we have:

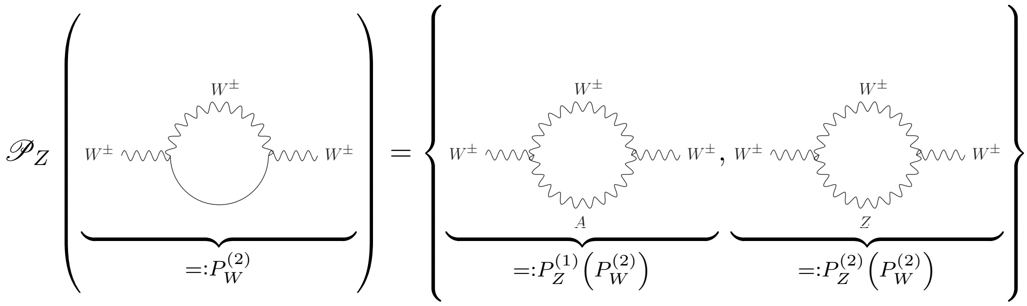


(43)

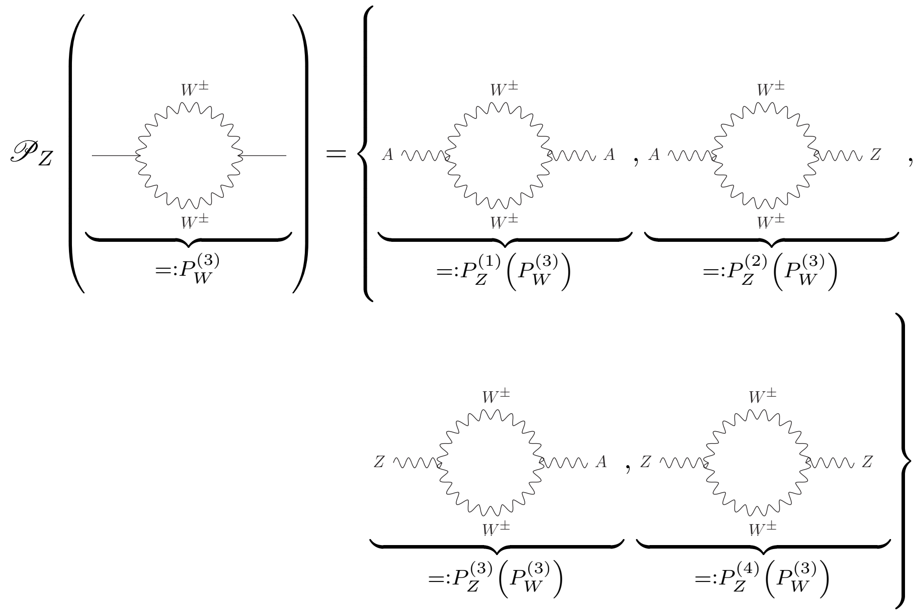
and



(44a)

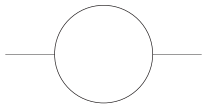
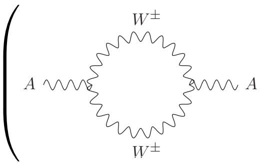


(44b)

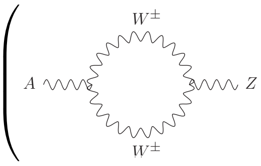


(44c)

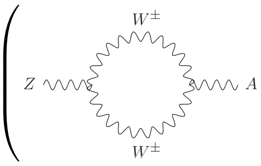
Moreover, the above defined factors read

sym () = sym (

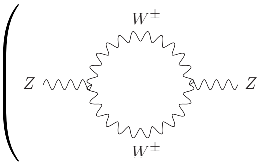
)

= sym (

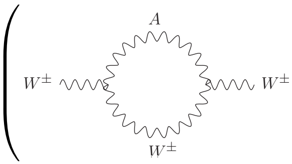
)

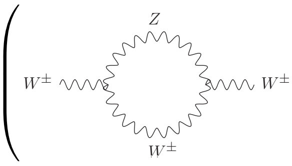
= sym (

) (45a)

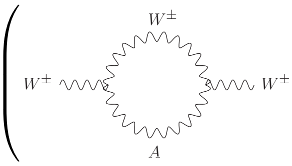
= sym (

) = 2 ,

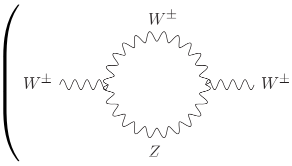
sym (

) = sym (

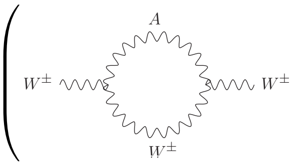
)

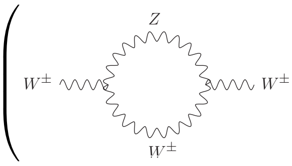
= sym (

) (45b)

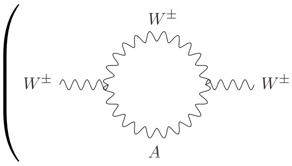
= sym (

) = 1 ,

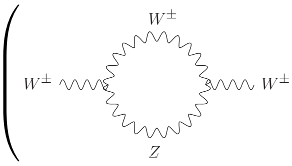
iso (

) = iso (

)

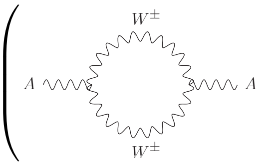
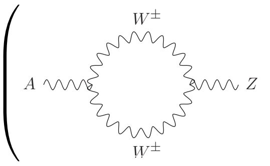
= iso (

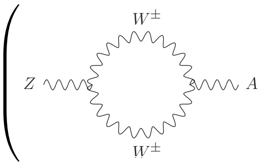
) (45c)

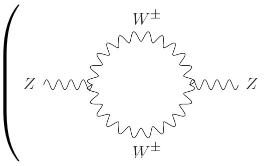
= iso (

) = 2

and

iso  = iso 

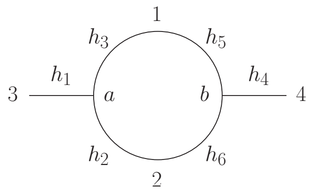
= iso    (45d)

= iso  $= 1\,,$

such that

$$\frac{\operatorname{sym}(\Gamma)}{\operatorname{sym}(\Gamma_{\text{labeled}})\operatorname{iso}(\Gamma_{\text{labeled}})} = 1\,, \qquad \forall P_Z \in \mathscr{P}_Z(P_W)\,, \quad \forall P_W \in \mathscr{P}_W(\Gamma)\,, \tag{45e}$$

where we remind the reader that we consider the one-loop self-energy graph

$\Gamma :=$  .   (46)

Then, we have:

Theorem 5.12. *Let Γ be a 3-regular scalar QFT Feynman graph. Then the gauge bosons and their corresponding ghosts of the electroweak sector of the standard model can be included into the definition of the Corolla polynomial (Definition 5.1) by multiplying the parametric integrand $I(\Gamma)$ of the corresponding scalar QFT amplitude by $J(\Gamma; m_W, m_Z)$, defined as*

$$\begin{aligned} J(\Gamma; m_W, m_Z) := \Bigg(& g_s^{\Gamma^{[0]}} \operatorname{color}(\Gamma) \\ & + \sum_{P_W \in \mathscr{P}_W(\Gamma)} \sum_{P_Z \in \mathscr{P}_Z(P_W)} \frac{\operatorname{sym}(\Gamma)}{\operatorname{sym}(\Gamma_{\text{labeled}})\operatorname{iso}(\Gamma_{\text{labeled}})} \\ & \times (g\cos\theta_W)^{|P_Z|}(e)^{|\Gamma \setminus P_Z|} e^{-\left(\sum_{e\in P_W} A_e m_W^2 + \sum_{f \in P_Z} A_f m_Z^2\right)} \Bigg)\,, \end{aligned} \tag{47}$$

such that the amplitude $\tilde{I}_F(\Gamma)$ including gluons, ghosts, electroweak gauge bosons and their corresponding ghosts can be written as[14]

$$\tilde{I}_F(\Gamma) = \mathcal{D}(\Gamma) I(\Gamma) J(\Gamma; m_W, m_Z)\,. \tag{48}$$

[14]Note that here the Corolla differential $\mathcal{D}(\Gamma)$ is used instead of $\mathcal{D}_{\text{QCD}}(\Gamma)$, since the additional constants are moved into the definition of $J(\Gamma; m_W, m_Z)$.

Proof. Recall that the Feynman rules do not allow gluons to couple to the gauge bosons of the electroweak sector. Therefore we can consider these two parts of the Standard Model separately. Additionally note that the Feynman rules for the gauge bosons of the electroweak sector have precisely the same tensor structure than the Feynman rules for the gluons, the only difference lies in the coupling constants g_s and the structure constants for the gluons or their corresponding ghosts and the coupling constants $g \cos\theta_W$ and e for the gauge bosons or their corresponding ghosts of the electroweak sector. These are taken in account by the factors $g_s^{\Gamma^{[0]}}$ color(Γ) and $(g\cos\theta_W)^{|P_Z|}(e)^{|\Gamma\setminus P_Z|}$, respectively. Moreover, notice that due to the conservation of $W^\pm$-particles each 3- or 4-valent vertex has to consist of an even number of $W^\pm$-particles. In particular, since the 4-valent vertices are created by the Corolla polynomial in the same fashion than for the gluons we only need to consider the 3-valent Feynman rules, i.e. we have to sum over all possibilities to label edges in Γ in such a way, that each 3-valent vertex consists of exactly two half-edges with label $W^\pm$ and one unlabeled half-edge with the additional requirement that edges don't change their labeling. This is precisely fulfilled by the set of sets $\mathscr{P}_W(\Gamma)$ for each set $P_W \in \mathscr{P}_W(P_Z)$. Furthermore, notice that the Feynman rules of the electroweak sector of the Standard model allow each unlabeled edge of Γ to be turned into either a Z- or an A-particle. This is precisely fulfilled by the set of sets $\mathscr{P}_Z(P_W)$ for each set $P_W \in \mathscr{P}_W$. Moreover, the corresponding ghost edges have the same masses than their corresponding gauge bosons, meaning that each component $\mathcal{D}^i(\Gamma)I(\Gamma)$ gets the same factor $J(\Gamma; m_W, m_Z)$ and thus the whole amplitude created by the Corolla differential can be multiplied by the factor given in Equation (47), such that Equation (48) holds. Since no derivatives act on the corresponding mass terms in the parametric representation of a scalar QFT, $I(\Gamma)$ can be simply multiplied by $J(\Gamma; m_W, m_Z)$. Finally, the possibly different symmetry factors of Γ_{labeled} compared to Γ are compensated by the factor $\operatorname{sym}(\Gamma) / \operatorname{sym}(\Gamma_{\text{labeled}})$ and the redundant number of isomorphic graphs is divided out by $\operatorname{iso}(\Gamma_{\text{labeled}})$. ∎

Remark 5.13. Again, as in [4], the factors $\pm i$ for the vertex and propagator Feynman rules are not explicitly given and should be taken into account for concrete calculations.

Lemma 5.14. *Let the assumptions of Theorem 5.12 hold. Then we have*

$$\frac{\operatorname{sym}(\Gamma)}{\operatorname{sym}(\Gamma_{\text{labeled}}) \operatorname{iso}(\Gamma_{\text{labeled}})} = 1, \qquad \forall \Gamma . \tag{49}$$

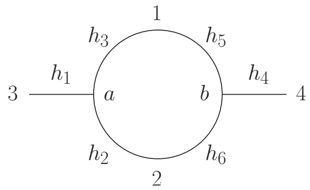
Proof. Note that the symmetry factors of the graphs in the set $\mathscr{P}_W$ are similar to QED graphs, since every vertex possesses exactly two orientable lines of the same type and one of another type. Since we have chosen to work in this thesis with unoriented $W^\pm$-edges, the symmetry factors are not all equal to 1, as in QED for unoriented fermion lines [1]. Moreover, note that the labeling with Z-labels and A-labels does not change the symmetry-factors, i.e. the elements $P_W(\Gamma)$ and $P_Z(P_W)$ have the same symmetry factors since the symmetry factor can be defined as the number of ways half-edges of adjacent vertices can be interchanged without changing the graph. But since two lines are of the same particle type, the remaining particle type is fixed and hence it does not matter if it is of Z- or A-type, the only contributions comes from the unoriented two lines of the same particle type. Furthermore, we remark that as in QED the sum of all possible orientations of unoriented edges (which is given by the factor $\operatorname{iso}(\Gamma_{\text{labeled}})$) times the symmetry factor $\operatorname{sym}(\Gamma_{\text{labeled}})$ is equal to 1 [1]. Finally, in [6, Lemma 5.1] it is proven that the symmetry factors work out correct while passing from 3-valent to 4-valent vertices by shrinking internal edges. ∎

Corollary 5.15. *In particular,* $J\left(\Gamma; m_W, m_Z\right)$ *simplifies to*

$$J\left(\Gamma; m_W, m_Z\right) := \Bigg(\operatorname{color}(\Gamma) + \sum_{P_W \in \mathscr{P}_W(\Gamma)} \sum_{P_Z \in \mathscr{P}_Z(P_W)} \left(g \cos\theta_W\right)^{|P_Z|} (e)^{|\Gamma \setminus P_Z|} \times e^{-\left(\sum_{e \in P_W} A_e m_W^2 + \sum_{f \in P_Z} A_f m_Z^2\right)} \Bigg) . \tag{50}$$

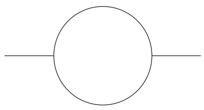
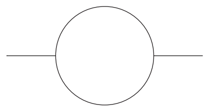
Proof. Theorem 5.12 and Lemma 5.14. ■

Example 5.16. Consider

$\Gamma :=$ 

. (51)

Then, we have (cf. Example 5.11):

$J\Bigg($  $; m_W, m_Z\Bigg) = \Bigg(\operatorname{color}\Bigg($  $\Bigg)$

$$+e^{-\left((A_3+A_2+A_4)m_W^2\right)} e^2$$
$$+e^{-\left((A_3+A_2+A_4)m_W^2 + A_1 m_Z^2\right)} \left(g\cos\theta_W\right)^2$$
$$+e^{-\left((A_3+A_1+A_4)m_W^2\right)} e^2$$
$$+e^{-\left((A_3+A_1+A_4)m_W^2 + A_2 m_Z^2\right)} \left(g\cos\theta_W\right)^2 \tag{52}$$
$$+e^{-\left((A_1+A_2)m_W^2\right)} e^2 + e^{-\left((A_1+A_2)m_W^2 + A_4 m_Z^2\right)} eg\cos\theta_W$$
$$+e^{-\left((A_1+A_2)m_W^2 + A_3 m_Z^2\right)} eg\cos\theta_W$$
$$+e^{-\left((A_1+A_2)m_W^2 + (A_3+A_4)m_Z^2\right)} \left(g\cos\theta_W\right)^2 \Bigg)$$

5.3 Inclusion of the scalar bosons of the electroweak sector

For the inclusion of the scalar bosons from the electroweak sector of the Standard Model we have to add four new particles, namely the Higgs boson h and the Goldstone bosons $\varphi^\pm$ and φ_Z.

Outline. Again, the idea will be to define two nested sums over a 3-regular scalar QFT Feynman graph in the following way: The first sum creates all possible ways to attach scalar particle labels to edges of Γ that are allowed to become a scalar edge (i.e. edges that will not become a ghost edge) and the second sum will create all possible Feynman graphs with 4-valent scalar vertices by shrinking suitable scalar labeled edges. Furthermore we define a third sum which creates all particle labelings which are allowed by the Standard Model Feynman rules. The full details are explained in the proof after Theorem 5.22.

Therefore, we need the following definitions:

Definition 5.17. Let Γ be a 3-regular scalar QFT Feynman graph. Then:

1. Let $\wp(\Gamma)$ be the power set of $\Gamma^{[1]}$, i.e. the set of all possible sets $P_{\mathrm{H/G}}$ of edges of Γ. In particular, we will be interested in the set $\wp\left(\Gamma \setminus \left(\bigcup_{k=1}^{i} C_i\right)\right)$, the power set of all edges of Γ that will not be turned into ghost edges and are free to possibly become a Higgs or Goldstone edge.

2. Let $\wp_{(4)}\left(P_{\mathrm{H/G}}\right)$ be the power set of all internal edges of Γ in the set $P_{\mathrm{H/G}}$ having 2 or 4 neighbors, that do not share a vertex with an edge which will be turned into a ghost edge (also edges that will be turned into fermions edges are not allowed either, if included) and also adjacent edges are not allowed to be in the same set $P_{(4)} \in \wp_{(4)}\left(P_{\mathrm{H/G}}\right)$, i.e. edges that are free to create a 4-valent vertex (either 2-scalar-2-gauge bosons or 4-scalar bosons vertices).

3. Let $e \in P_{(4)}\left(P_{\mathrm{H/G}}\right)$. Then we define the set of adjacent half-edges to e as $H_{(4)}(e) := \{ h_1, h_2, h_3, h_4 \}$. In particular, we are interested in the set $H_{(2)}(e) \subset H_{(4)}(e)$ defined as

$$H_{(2)}(e) := \begin{cases} \{ h_1, h_2 \} & \text{if } h_3, h_4 \notin P_{\mathrm{H/G}} \\ \emptyset & \text{if } h_1, h_2, h_3, h_4 \in P_{\mathrm{H/G}} \end{cases}$$

   for some arbitrary numbering $1, \dots, 4$, since only in the case of a 2-scalar-2-gauge bosons vertex we have to add a metric tensor with indices depending on the gauge boson edges (where the product below in the modified Corolla polynomial in Definition 5.20 over the empty set is defined to be 0). For the 4-scalar bosons vertex we just receive a coupling constant, see part 5. of this definition.

4. Let $\mathscr{L}\left(\Gamma, P_{\mathrm{H/G}}, P_{(4)}\right)$ denote the set of all possible allowed particle type labelings of the graph Γ, with scalar particle edges $P_{\mathrm{H/G}}$ and 4-valent scalar vertices $P_{(4)}$, compatable with the Standard Model Feynman rules (cf. Appendix A and [5]), such that[15]

$$L(e) \in \left\{W^{\pm}, Z, A, h, \varphi^{\pm}, \varphi_Z\right\}, \qquad e \in \Gamma^{[1]}.$$

   In the previous Subsection 5.2 this was created by the sets of sets $\mathscr{P}_W$ and $\mathscr{P}_Z$ for the special cases $P_{\mathrm{H/G}} = P_{(4)} = \emptyset$, given in Definition 5.9. Note that the labeling itself is not depending on $P_{(4)}\left(P_{\mathrm{H/G}}\right)$ since shrinking any edge $e \in P_{(4)}$ creates a valid 4-valent vertex (cf. Remark 5.23). We still give it here as an argument such that our functions $\operatorname{sym}(L)$ and $\operatorname{iso}(L)$ are well-defined.

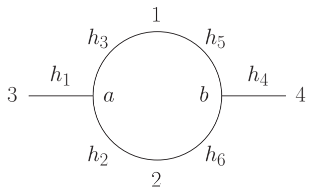
5. Let $\operatorname{coupling}\left(P_{\mathrm{H/G}}, P_{(4)}, L\right)$ be the product over all coupling constants of $\Gamma^{[0]}$ with labeling L (cf. Appendix A and [5]), including all 3-valent vertices and scalar 4-valent vertices (the 4-gauge bosons vertex coupling constants are created from the corresponding 3-valent ones by the Corolla polynomial in the usual fashion). Note that here $P_{\mathrm{H/G}}$ is itself explicitly not

[15] We denote here the gauge bosons, as well as their corresponding ghosts, by $\{ W^{\pm}, Z, A \}$ since in the following we are only interested in their componentwise coincident masses.

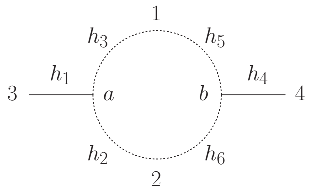
necessary as an argument for coupling $\left(P_{\mathrm{H/G}}, P_{(4)}, L\right)$ since the scalar edges are already fixed through the scalar particle labels in $L\left(\Gamma, P_{\mathrm{H/G}}, P_{(4)}\right)$, but implicitly for $P_{(4)}\left(P_{\mathrm{H/G}}\right)$ which is the reason why we still list it.

We start with examples for 1., 2. and 3. from Definition 5.17:

Example 5.18. Again, consider the one-loop graphs

$\Gamma :=$  (53a)

and

$\Gamma' :=$  . (53b)

Then, we have:

$$\begin{aligned}\wp(\Gamma) = \Big\{ & \underbrace{\emptyset}_{P^{(1)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1\}}_{P^{(2)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{2\}}_{P^{(3)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{3\}}_{P^{(4)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{4\}}_{P^{(5)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1,2\}}_{P^{(6)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1,3\}}_{P^{(7)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1,4\}}_{P^{(8)}_{\mathrm{H/G}}(\Gamma)}, \\ & \underbrace{\{2,3\}}_{P^{(9)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{2,4\}}_{P^{(10)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{3,4\}}_{P^{(11)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1,2,3\}}_{P^{(12)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1,2,4\}}_{P^{(13)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1,3,4\}}_{P^{(14)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{2,3,4\}}_{P^{(15)}_{\mathrm{H/G}}(\Gamma)}, \underbrace{\{1,2,3,4\}}_{P^{(16)}_{\mathrm{H/G}}(\Gamma)} \Big\}\end{aligned} \tag{54a}$$

$$\wp\left(\Gamma'\right) = \wp\left(\Gamma \setminus C_1\right) = \left\{ \underbrace{\emptyset}_{P^{(1)\prime}_{\mathrm{H/G}}(\Gamma')}, \underbrace{\{3\}}_{P^{(2)\prime}_{\mathrm{H/G}}(\Gamma')}, \underbrace{\{4\}}_{P^{(3)\prime}_{\mathrm{H/G}}(\Gamma')}, \underbrace{\{3,4\}}_{P^{(4)\prime}_{\mathrm{H/G}}(\Gamma')} \right\} \tag{54b}$$

and

$$\wp_{(4)}\left(P^{(l)}_{\mathrm{H/G}}\right) = \left\{ \underbrace{\emptyset}_{P^{(1)}_{(4)}\left(P^{(l)}_{\mathrm{H/G}}\right)} \right\}, \qquad 1 \leq l \leq 13 \tag{55a}$$

$$\wp_{(4)}\left(P^{(14)}_{\mathrm{H/G}}\right) = \left\{ \underbrace{\emptyset}_{P^{(1)}_{(4)}\left(P^{(14)}_{\mathrm{H/G}}\right)}, \underbrace{\{1\}}_{P^{(2)}_{(4)}\left(P^{(14)}_{\mathrm{H/G}}\right)} \right\} \tag{55b}$$

$$\wp_{(4)}\left(P^{(15)}_{\mathrm{H/G}}\right) = \left\{ \underbrace{\emptyset}_{P^{(1)}_{(4)}\left(P^{(15)}_{\mathrm{H/G}}\right)}, \underbrace{\{2\}}_{P^{(2)}_{(4)}\left(P^{(15)}_{\mathrm{H/G}}\right)} \right\} \tag{55c}$$

$$\wp_{(4)}\left(P^{(16)}_{\mathrm{H/G}}\right) = \left\{ \underbrace{\emptyset}_{P^{(1)}_{(4)}\left(P^{(16)}_{\mathrm{H/G}}\right)} , \underbrace{\{1\}}_{P^{(2)}_{(4)}\left(P^{(16)}_{\mathrm{H/G}}\right)} , \underbrace{\{2\}}_{P^{(3)}_{(4)}\left(P^{(16)}_{\mathrm{H/G}}\right)} \right\} \tag{55d}$$

$$\wp_{(4)}\left(P^{(l)\prime}_{\mathrm{H/G}}\right) = \left\{ \underbrace{\emptyset}_{P^{(1)}_{(4)}\left(P^{(l)\prime}_{\mathrm{H/G}}\right)} \right\} , \qquad 1 \leq l \leq 4 \tag{55e}$$

And for our sets $H_{(2)}(e)$, $e \in P_{(4)}\left(P_{\mathrm{H/G}}\right)$ we have (we just list the non-empty sets):

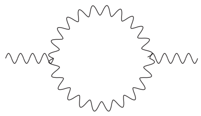
$$1 \in P^{(1)}_{(4)}\left(P^{(6)}_{\mathrm{H/G}}\right): \qquad H_{(2)}(1) = \{ (a,3), (b,4) \} \tag{56a}$$

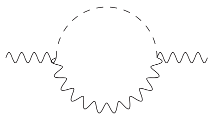
$$2 \in P^{(1)}_{(4)}\left(P^{(6)}_{\mathrm{H/G}}\right): \qquad H_{(2)}(2) = \{ (a,3), (b,4) \} \tag{56b}$$

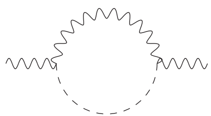
$$1 \in P^{(1)}_{(4)}\left(P^{(14)}_{\mathrm{H/G}}\right): \qquad H_{(2)}(1) = \{ (a,2), (b,2) \} \tag{56c}$$

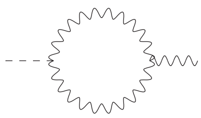
$$2 \in P^{(1)}_{(4)}\left(P^{(15)}_{\mathrm{H/G}}\right): \qquad H_{(2)}(2) = \{ (a,1), (b,1) \} \tag{56d}$$

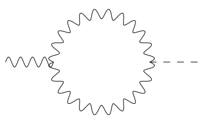
These create after applying the modified Corolla differential (cf. Definition 5.20 and Theorem 5.22) to the scalar parametric integrand the contributions of the following graphs (which is indicated by the arrow "$\rightsquigarrow$"):

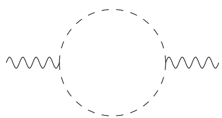
$P^{(1)}_{(4)}\left(P^{(1)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57a)

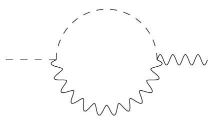
$P^{(1)}_{(4)}\left(P^{(2)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57b)

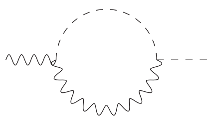
$P^{(1)}_{(4)}\left(P^{(3)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57c)

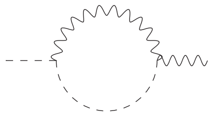
$P^{(1)}_{(4)}\left(P^{(4)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57d)

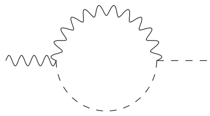
$P^{(1)}_{(4)}\left(P^{(5)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57e)

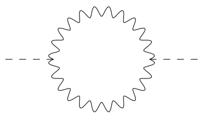
$P^{(1)}_{(4)}\left(P^{(6)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57f)

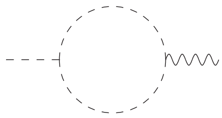
$P^{(1)}_{(4)}\left(P^{(7)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57g)

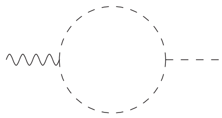
$P^{(1)}_{(4)}\left(P^{(8)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57h)

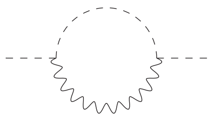
$P^{(1)}_{(4)}\left(P^{(9)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57i)

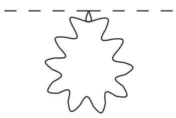
$P^{(1)}_{(4)}\left(P^{(10)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57j)

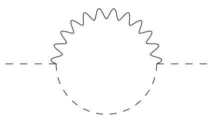
$P^{(1)}_{(4)}\left(P^{(11)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57k)

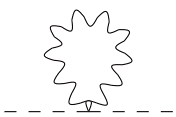
$P^{(1)}_{(4)}\left(P^{(12)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57l)

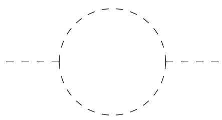
$P^{(1)}_{(4)}\left(P^{(13)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57m)

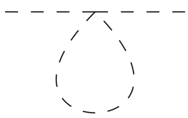
$P^{(1)}_{(4)}\left(P^{(14)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57n)

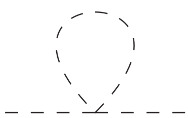
$P^{(2)}_{(4)}\left(P^{(14)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57o)

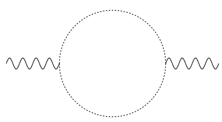
$P^{(1)}_{(4)}\left(P^{(15)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57p)

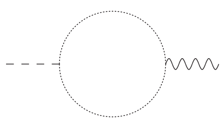
$P^{(2)}_{(4)}\left(P^{(15)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57q)

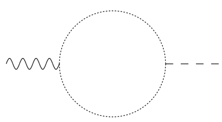
$P^{(1)}_{(4)}\left(P^{(16)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57r)

$P^{(2)}_{(4)}\left(P^{(16)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57s)

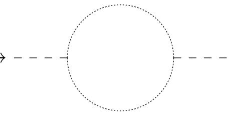
$P^{(3)}_{(4)}\left(P^{(16)}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57t)

$P^{(1)}_{(4)}\left(P^{(1)\prime}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57u)

$P^{(1)}_{(4)}\left(P^{(2)\prime}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57v)

$P^{(1)}_{(4)}\left(P^{(3)\prime}_{\mathrm{H/G}}\right) \rightsquigarrow$  (57w)

$$P^{(1)}_{(4)}\left(P^{(4)\prime}_{\mathrm{H/G}}\right) \rightsquigarrow$$

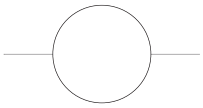


(57x)

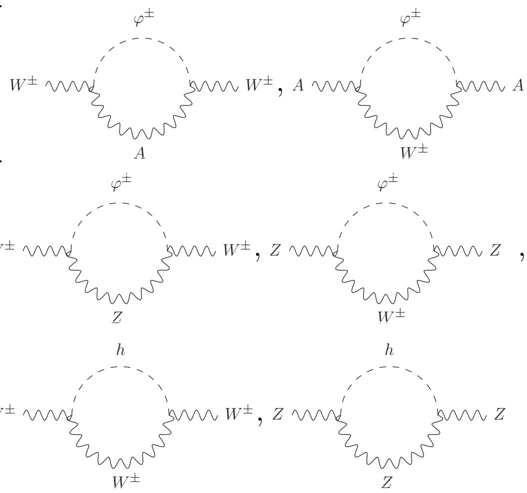
Furthermore, we have the following examples for 4. and 5. of Definition 5.17:

Example 5.19. We have

$$\mathscr{L}\Bigg($$

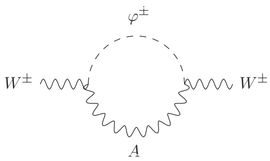


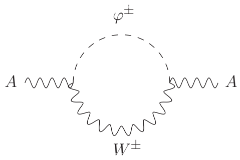
$$, \{1\}, \emptyset \Bigg) = \Bigg\{$$

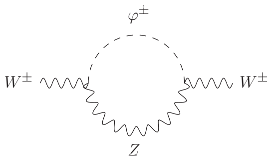


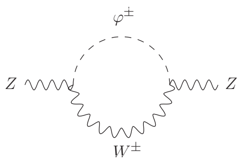
$$\Bigg\}$$ (58)

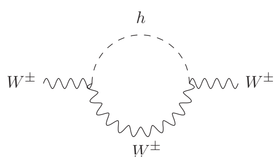
and

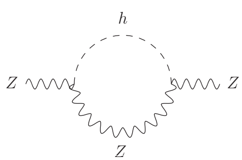
$$\operatorname{coupling}\Bigg(\{1\}, \emptyset,$$  $$\Bigg) = e^2 m_W^2\,,$$ (59a)

$$\operatorname{coupling}\Bigg(\{1\}, \emptyset,$$  $$\Bigg) = e^2 m_W^2\,,$$ (59b)

$$\operatorname{coupling}\Bigg(\{1\}, \emptyset,$$  $$\Bigg) = g^2 m_Z^2 \sin^4\theta_W\,,$$ (59c)

$$\operatorname{coupling}\Bigg(\{1\}, \emptyset,$$  $$\Bigg) = g^2 m_Z^2 \sin^4\theta_W\,,$$ (59d)

$$\operatorname{coupling}\Bigg(\{1\}, \emptyset,$$  $$\Bigg) = g^2 m_W^2\,,$$ (59e)

$$\operatorname{coupling}\Bigg(\{1\}, \emptyset,$$  $$\Bigg) = \frac{g^2}{\cos^2\theta_W} m_Z^2\,.$$ (59f)

Then we can define the Corolla polynomial with the inclusion of the gauge bosons of the electroweak sector and scalar particles as follows:

Definition 5.20 (Corolla polynomial for the electroweak sector of the Standard model (omitting fermions))**.** Let Γ be a 3-regular scalar QFT Feynman graph. Then we define the various summands of the Corolla polynomial for the electroweak sector of the Standard model (omitting fermions) by

$$\begin{aligned}
\mathcal{C}^0_{\mathrm{EW}}(\Gamma) := & \sum_{P_{\mathrm{H/G}} \in \wp(\Gamma)} \sum_{P_{(4)} \in \wp_{(4)}\left(P_{\mathrm{H/G}}\right)} \sum_{L \in \mathscr{L}\left(\Gamma, P_{\mathrm{H/G}}, P_{(4)}\right)} \\
& \Bigg[\left(\frac{\operatorname{sym}(\Gamma)}{\operatorname{sym}(L) \operatorname{iso}(L)} \right) \operatorname{coupling}\left(P_{\mathrm{H/G}}, P_{(4)}, L\right) e^{-\left(\sum_{e \in \Gamma^{[1]} \setminus P_{(4)}} A_e m^2_{L(e)}\right)} \\
& \left(\prod_{v \notin P_{\mathrm{H/G}}} \mathscr{V}_v \right) \left(\prod_{\substack{h \in P_{\mathrm{H/G}} \setminus P_{(4)}, \\ h_+, h_- \notin P_{\mathrm{H/G}} \setminus P_{(4)}}} \eta^{\mu_{e(h_+)} \mu_{e(h_-)}} \right) \\
& \left(\prod_{\substack{h_+, h_- \in P_{\mathrm{H/G}} \setminus P_{(4)}, \\ h \notin P_{\mathrm{H/G}} \setminus P_{(4)}}} \left(a_{h_+} - a_{h_-}\right) \right) \left(\prod_{\substack{e \in P_{(4)}, \\ h_1, h_2 \in H_{(2)}(e)}} \eta^{\mu_{e(h_1)} \mu_{e(h_2)}} \right) \Bigg] ,
\end{aligned} \tag{60a}$$

$$\begin{aligned}
\mathcal{C}^i_{\mathrm{EW}}(\Gamma) := & \sum_{\substack{C_1, C_2, \dots, C_i \in \mathscr{C}_\Gamma, \\ C_j \text{ pairwise disjoint}}} \sum_{P_{\mathrm{H/G}} \in \wp\left(\Gamma \setminus \bigcup_{k=1}^i C_k\right)} \sum_{P_{(4)} \in \wp_{(4)}\left(P_{\mathrm{H/G}}\right)} \sum_{L \in \mathscr{L}\left(\Gamma, P_{\mathrm{H/G}}, P_{(4)}\right)} \\
& \Bigg[\left(\frac{\operatorname{sym}(\Gamma)}{\operatorname{sym}(L) \operatorname{iso}(L)} \right) \operatorname{coupling}\left(P_{\mathrm{H/G}}, P_{(4)}, L\right) e^{-\left(\sum_{e \in \Gamma^{[1]} \setminus P_{(4)}} A_e m^2_{L(e)}\right)} \\
& \left(\prod_{j=1}^i \mathscr{G}_{C_j} \right) \left(\prod_{v \notin \bigcup_{k=1}^i C_k \cup P_{\mathrm{H/G}}} \mathscr{V}_v \right) \left(\prod_{\substack{h \in P_{\mathrm{H/G}} \setminus P_{(4)}, \\ h_+, h_- \notin P_{\mathrm{H/G}} \setminus P_{(4)}}} \eta^{\mu_{e(h_+)} \mu_{e(h_-)}} \right) \\
& \left(\prod_{\substack{h_+, h_- \in P_{\mathrm{H/G}} \setminus P_{(4)}, \\ h \notin P_{\mathrm{H/G}} \setminus P_{(4)}}} \left(a_{h_+} - a_{h_-}\right) \right) \left(\prod_{\substack{e \in P_{(4)}, \\ h_1, h_2 \in H_{(2)}(e)}} \eta^{\mu_{e(h_1)} \mu_{e(h_2)}} \right) \Bigg]
\end{aligned} \tag{60b}$$

and the Corolla polynomial by

$$\mathcal{C}_{\mathrm{EW}}(\Gamma) := \sum_{i=0}^{\infty} (-1)^i \mathcal{C}^i_{\mathrm{EW}}(\Gamma) \,. \tag{60c}$$

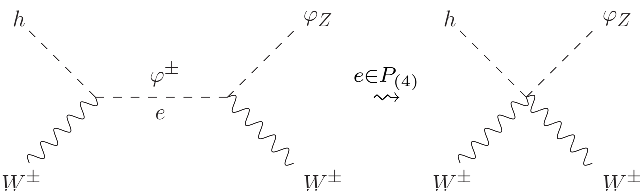
Remark 5.21. Once more, as in [4], the factors $\pm i$ for the vertex and propagator Feynman rules are not explicitly given and should be taken into account for concrete calculations.

Theorem 5.22. *Let Γ be a 3-regular scalar QFT Feynman graph. Then the full parametric integrand of quantum chromodynamics and the electroweak sector of the Standard model (excluding fermions) $\tilde{I}_F(\Gamma)$ can be obtained via acting with the sum of the two Corolla differentials for quantum chromodynamics and the electroweak sector of the Standard model $\big(\mathcal{D}_{\mathrm{QCD}}(\Gamma) + \mathcal{D}_{\mathrm{EW}}(\Gamma)\big)$ on the corresponding parametric integrand for the scalar QFT $I(\Gamma)$*[16]

$$\tilde{I}_F(\Gamma) = \big(\mathcal{D}_{\mathrm{QCD}}(\Gamma) + \mathcal{D}_{\mathrm{EW}}(\Gamma)\big)\, I(\Gamma)\,. \tag{61}$$

Proof. First note that the scalar particles h, $\varphi^{\pm}$ and φ_Z only couple to gauge bosons of the electroweak sector of the Standard Model so that we are allowed to treat the quantum chromodynamics sector and the electroweak sector separately. The contribution for the quantum chromodynamics sector via the action from $\mathcal{D}_{\mathrm{QCD}}(\Gamma)$ on $I(\Gamma)$ was already proven in Theorem 5.6. Then, notice that for a given graph Γ every edge that is not going to be turned into a ghost edge is allowed to be become a scalar edge (possibly with a restriction on the edge labeling). This is precisely created by the power set of all edges of the graph without the ghost edges $\wp\left(\Gamma \setminus \bigcup_{k=1}^{i} C_k\right)$ for a graph with i ghost loops. Moreover, the creation of 4-valent scalar vertices (2-scalar-2-gauge bosons or 4-scalar bosons) can be constructed from shrinking an internal scalar labeled edge of two 3-valent vertices with the additional restrictions of having 2 or 4 scalar edge neighbors (i.e. edges in the set $P_{\mathrm{H/G}}$) that do not share a vertex with an edge which will be turned into a ghost edge (again we remark, that also edges which will be turned into fermions edges are not allowed either, if included) and also adjacent edges are not allowed to shrink simultaneously, since all 4-valent gauge boson vertices are already created by the Corolla polynomial in the usual fashion for some $P_{\mathrm{H/G}} \in \mathscr{P}_{\mathrm{H/G}}(\Gamma)$ with two glued together 3-valent gauge boson vertices and we do only have 4-valent scalar vertices with 2 or 4 scalar edges involved and also no 4-valent vertices with ghost edges (or fermion edges) are allowed and additionally we also do not want valences greater than 4. Therefore $P_{(4)} \in \wp_{(4)}\left(P_{\mathrm{H/G}}\right)$ consists of all edges that are allowed to shrink in order to produce valid 4-valent scalar Feynman rules. Furthermore, the missing Feynman rules for the 3-valent 1-scalar-2-gauge bosons and 2-scalar-1-gauge bosons vertices are created by the additional products of the modified Corolla polynomial (cf. Definition 5.20 compared to Definition 5.1). Moreover, since the original Corolla polynomial acts only on the part of Γ which will be turned into pure gauge bosons vertices, the 4-valent 4-gauge bosons vertices are created in the same way than with the original Corolla polynomial and can be received as the corresponding residues in the shrinked edge Schwinger parameters. Additionally, the right symmetry factors are obtained by the multiplication with the fraction $\mathrm{sym}(\Gamma)/\,\mathrm{sym}(L)$ and possible redundancies are divided out by the factor $\mathrm{iso}(L)$. Finally, the corresponding particle masses are included via the exponentional term and all corresponding coupling constants for the labeling $L \in \mathscr{L}\left(\Gamma, P_{\mathrm{H/G}}, P_{(4)}\right)$ are given in coupling $\left(P_{\mathrm{H/G}}, P_{(4)}, L\right)$. ■

Remark 5.23. With the modified Corolla polynomial given in Definition 5.20 we create all 4-valent scalar vertices (2-scalar-2-gauge bosons and 4-scalar bosons vertices), given in Appendix A and [5] once or twice, where in the last case the redundancy is divided out by the factor $\mathrm{iso}(L)$. But additionally we create one further one which is not explicitly given in [5] but does not violate obvious particle conservation laws:



(62)

[16] Recall $\mathcal{D}_{\mathrm{QCD}}(\Gamma) := g_s^{\Gamma^{[0]}} \mathrm{color}(\Gamma)\mathcal{D}(\Gamma)$ from Definition 5.1 and Definition 5.4.

If we want to omit this graph, we can simply alter the definition of the labeling $\mathscr{L}\left(\Gamma, P_{\mathrm{H/G}}, P_{(4)}\right)$ such that we take $P_{(4)}$ as an explicit argument for our labeling function so that this vertex will not show up (cf. 4. from Definition 5.17).

6 Conclusion

The aim of this thesis was to generalize the Corolla polynomial to the bosons of the electroweak sector of the Standard model. Therefore all the relevant graph theoretic notions (cf. Section 3), the parametric representation of scalar quantum field theories (cf. Section 4) and the Corolla polynomial and differential for pure Yang-Mills theory (cf. Subsection 5.1) were studied. The inclusion of the bosons of the electroweak sector of the Standard model was successfully done by first working out the combinatorics of labeling a 3-regular scalar QFT Feynman graph with labels of the gauge bosons of the electroweak sector of the Standard Model (cf. Subsection 5.2) and then by working out the additional tensor structures arising from the inclusion of the Feynman rules for the scalar particles of the electroweak sector of the Standard Model (cf. Subsection 5.3). It showed up that at least for the gauge bosons of the electroweak sector of the Standard model the symmetry factors work out correct (cf. Corollary 5.15). Furthermore a relatively compact notation could be found for the inclusion of the scalar particles of the Standard model, compared to the standard Feynman rules (cf. Definition 5.20, Theorem 5.22, Appendix A and [5]).

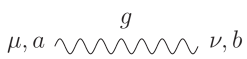
As projects for future work, several topics related to the Corolla polynomial are of particular interest: First of all, although it is in principle clear [4], the combinatorics for the inclusion of fermions to the Corolla polynomial for the electroweak sector of the Standard model should be worked out. Secondly, as we believe that this approach is particularly useful for computer calculations, since derivations can be done with much less afford than integrations, it would be useful to bring this approach on a computer. And thirdly, since it is believed that the Corolla polynomial can also be generalized to the case of quantum gravity [4], it would be interesting to see what we could learn from this approach and it's underlying combinatorics.

A Feynman rules

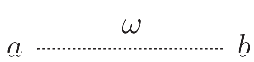
Here we list all Feynman rules $\Phi(\Gamma)$ for $\Gamma \in \mathcal{R}_V \cup \mathcal{R}_E$ which are relevant in this thesis, taken from [5] with all appearing signs chosen positive, i.e. $\eta_G = \eta_s = \eta_e = \eta = \eta_Z = 1$, and using the Feynman gauge, i.e. $\xi_G = \xi_A = \xi_W = \xi_Z = 1$:

A.1 QCD Feynman rules

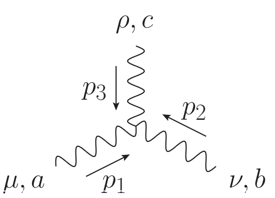
Gluon propagator:

Φ () $= -i\delta_{ab}\dfrac{\eta_{\mu\nu}}{k^2 + i\epsilon}$ (63)

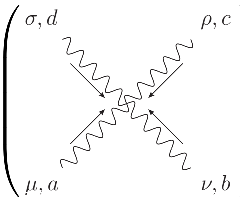
Ghost propagator:

Φ () $= \delta_{ab}\dfrac{i}{k^2 + i\epsilon}$ (64)

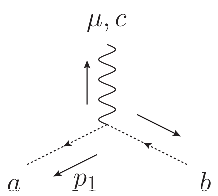
Triple Gauge Interactions:

Φ () $= -g_s f^{abc}\left[\eta^{\mu\nu}(p_1 - p_2)^\rho + \eta^{\nu\rho}(p_2 - p_3)^\mu + \eta^{\rho\mu}(p_3 - p_1)^\nu\right]$ (65)

Quartic Gauge Interactions:

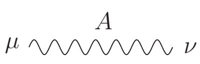
Φ () $= -ig_s^2\Big[f^{eab}f^{ecd}(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})$ $+f^{eac}f^{edb}(\eta^{\mu\sigma}\eta^{\rho\nu} - \eta^{\mu\nu}\eta^{\rho\sigma})$ $+f^{ead}f^{ebc}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma})\Big]$ (66)

Ghost Interactions:

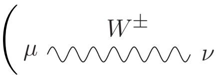
Φ () $= -g_s f^{abc} p_1^\mu$ (67)

A.2 Feynman Rules for the Electroweak Theory

γ-Propagator:

Φ () $= -i\dfrac{\eta_{\mu\nu}}{k^2 + i\epsilon}$ (68)

$W^\pm$-Propagator:

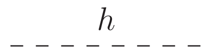
Φ () $= -i\dfrac{\eta_{\mu\nu}}{k^2 - m_W^2 + i\epsilon}$ (69)

Z-Propagator:

$$\Phi\left(\right) = -i\frac{\eta_{\mu\nu}}{k^2 - m_Z^2 + i\epsilon} \tag{70}$$

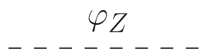
h-Higgs Propagator:

$$\Phi\left(\right) = \frac{i}{p^2 - m_h^2 + i\epsilon} \tag{71}$$



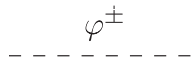
φ_Z-Goldstone Propagator:

$$\Phi\left(\right) = \frac{i}{p^2 - m_Z^2 + i\epsilon} \tag{72}$$



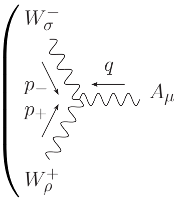
$\varphi^{\pm}$-Goldstone Propagator:

$$\Phi\left(\right) = \frac{i}{p^2 - m_W^2 + i\epsilon} \tag{73}$$

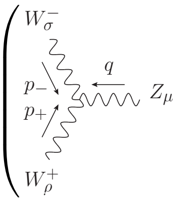


Triple Gauge Interactions:

$$\Phi\left(\right) = -\,ie\left[\eta^{\sigma\rho}(p_- - p_+)^\mu + \eta^{\rho\mu}(p_+ - q)^\sigma + \eta^{\mu\sigma}(q - p_-)^\rho\right] \tag{74a}$$

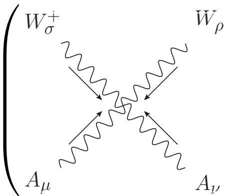


$$\Phi\left(\right) = -\,ig\cos\theta_W\left[\eta^{\sigma\rho}(p_- - p_+)^\mu + \eta^{\rho\mu}(p_+ - q)^\sigma + \eta^{\mu\sigma}(q - p_-)^\rho\right] \tag{74b}$$

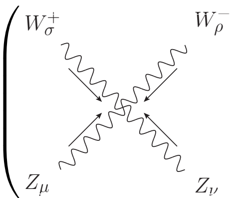


Quartic Gauge Interactions:

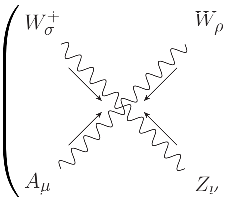
$$\Phi\left(\right) = -\,e^2\left[2\eta^{\sigma\rho}\eta^{\mu\nu} - \eta^{\sigma\mu}\eta^{\rho\nu} - \eta^{\sigma\nu}\eta^{\rho\mu}\right] \tag{75a}$$

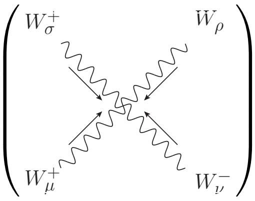


$$\Phi\left(\right) = -\,ig^2\cos^2\theta_W\left[2\eta^{\sigma\rho}\eta^{\mu\nu} - \eta^{\sigma\mu}\eta^{\rho\nu} - \eta^{\sigma\nu}\eta^{\rho\mu}\right] \tag{75b}$$



$$\Phi\left(\right) = -\,ieg\cos\theta_W\left[2\eta^{\sigma\rho}\eta^{\mu\nu} - \eta^{\sigma\mu}\eta^{\rho\nu} - \eta^{\sigma\nu}\eta^{\rho\mu}\right] \tag{75c}$$

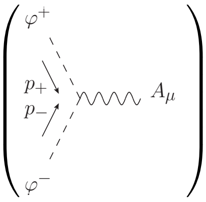


Φ 

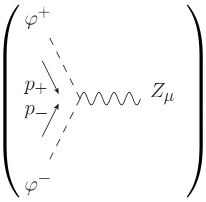
$$= ig^2\left[2\eta^{\sigma\mu}\eta^{\rho\nu} - \eta^{\sigma\rho}\eta^{\mu\nu} - \eta^{\sigma\nu}\eta^{\rho\mu}\right] \tag{75d}$$

(75e)

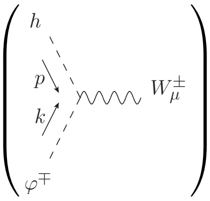
Triple Higgs-Gauge and Goldstone-Gauge Interactions:

Φ 

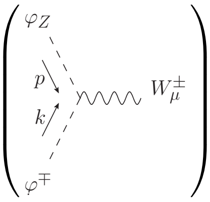
$$= -ie(p_+ - p_-)^\mu \tag{76a}$$

Φ 

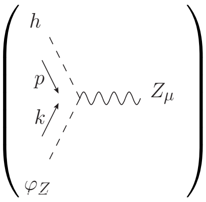
$$= -ig\frac{\cos 2\theta_W}{2\cos\theta_W}(p_+ - p_-)^\mu \tag{76b}$$

Φ 

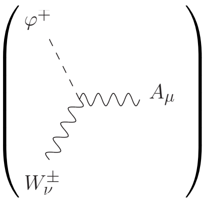
$$= \pm\frac{i}{2}g(k-p)^\mu \tag{76c}$$

Φ 

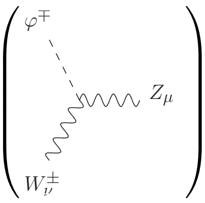
$$= -\frac{g}{2}(k-p)^\mu \tag{76d}$$

Φ 

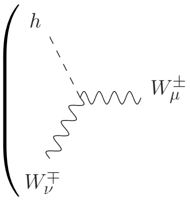
$$= -\frac{g}{2\cos\theta_W}(k-p)^\mu \tag{76e}$$

Φ 

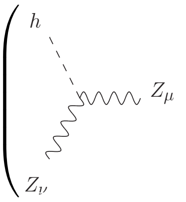
$$= iem_W\eta^{\mu\nu} \tag{76f}$$

Φ 

$$= -igm_Z\sin^2\theta_W\eta^{\mu\nu} \tag{76g}$$

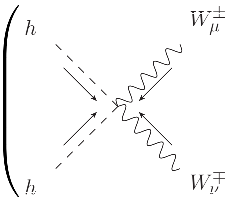
$$\Phi\left(\right.$$ 

$$\left.\right) = igm_W\eta^{\mu\nu} \tag{76h}$$

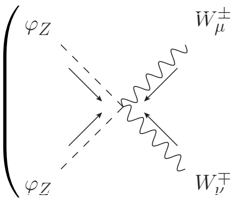
$$\Phi\left(\right.$$ 

$$\left.\right) = i\frac{g}{\cos\theta_W}m_Z\eta^{\mu\nu} \tag{76i}$$

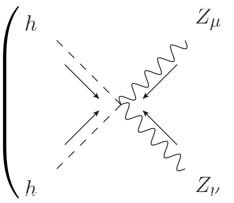
Quartic Higgs-Gauge and Goldstone-Gauge Interactions:

$$\Phi\left(\right.$$ 

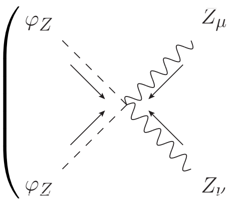
$$\left.\right) = \frac{i}{2}g^2\eta^{\mu\nu} \tag{77a}$$

$$\Phi\left(\right.$$ 

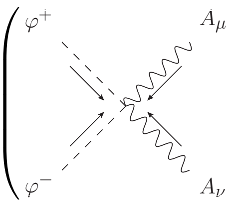
$$\left.\right) = \frac{i}{2}g^2\eta^{\mu\nu} \tag{77b}$$

$$\Phi\left(\right.$$ 

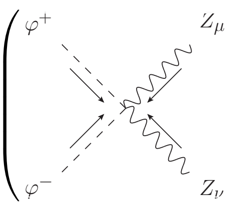
$$\left.\right) = \frac{i}{2}\frac{g^2}{\cos^2\theta_W}\eta^{\mu\nu} \tag{77c}$$

$$\Phi\left(\right.$$ 

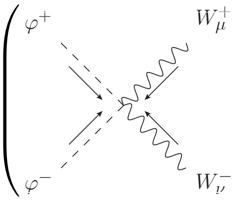
$$\left.\right) = \frac{i}{2}\frac{g^2}{\cos^2\theta_W}\eta^{\mu\nu} \tag{77d}$$

$$\Phi\left(\right.$$ 

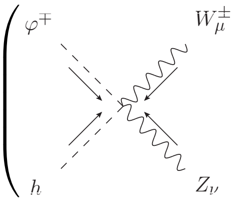
$$\left.\right) = 2ie^2\eta^{\mu\nu} \tag{77e}$$

$$\Phi\left(\right.$$ 

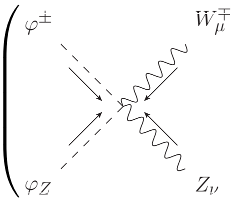
$$\left.\right) = \frac{i}{2}\left(\frac{g\cos 2\theta_W}{\cos\theta_W}\right)^2\eta^{\mu\nu} \tag{77f}$$

Φ 

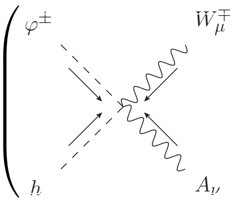
$$= \frac{i}{2} g^2 \eta^{\mu\nu} \tag{77g}$$

Φ 

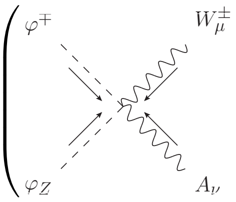
$$= -\, i g^2 \frac{\sin^2\theta_W}{2\cos\theta_W} \eta^{\mu\nu} \tag{77h}$$

Φ 

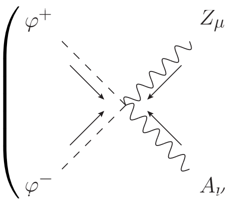
$$= \mp\, g^2 \frac{\sin^2\theta_W}{2\cos\theta_W} \eta^{\mu\nu} \tag{77i}$$

Φ 

$$= \frac{i}{2} e g \eta^{\mu\nu} \tag{77j}$$

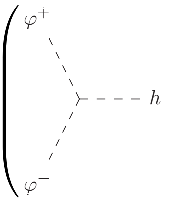
Φ 

$$= \mp\, \frac{1}{2} e g \eta^{\mu\nu} \tag{77k}$$

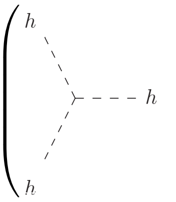
Φ 

$$= i e g \frac{\cos 2\theta_W}{\cos\theta_W} \eta^{\mu\nu} \tag{77l}$$

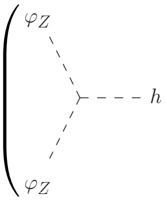
Triple Higgs and Goldstone Interactions:

Φ 

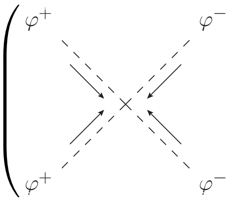
$$= -\, \frac{i}{2} g \frac{m_h^2}{m_W} \tag{78a}$$

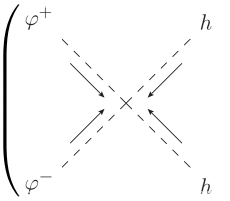
Φ 

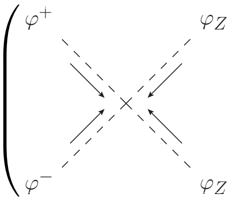
$$= -\, \frac{3}{2} i g \frac{m_h^2}{m_W} \tag{78b}$$

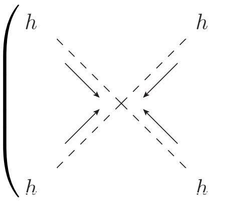
Φ () $= -\frac{i}{2} g \frac{m_h^2}{m_W}$ (78c)

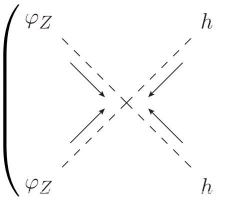
Quartic Higgs and Goldstone Interactions:

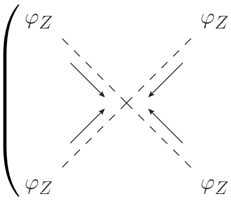
Φ () $= -\frac{i}{2} g^2 \frac{m_h^2}{m_W^2}$ (79a)

Φ () $= -\frac{i}{4} g^2 \frac{m_h^2}{m_W^2}$ (79b)

Φ () $= -\frac{i}{4} g^2 \frac{m_h^2}{m_W^2}$ (79c)

Φ () $= \frac{3}{4} i g^2 \frac{m_h^2}{m_W^2}$ (79d)

Φ () $= -\frac{i}{4} g^2 \frac{m_h^2}{m_W^2}$ (79e)

Φ () $= -\frac{3}{4} i g^2 \frac{m_h^2}{m_W^2}$ (79f)

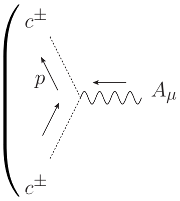
Ghost Propagators:

Φ () $= \frac{i}{k^2 + i\epsilon}$ (80a)

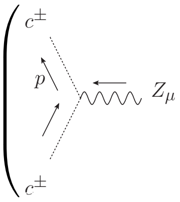
Φ () $= \frac{i}{k^2 - m_W^2 + i\epsilon}$ (80b)

Φ () $= \frac{i}{k^2 - m_Z^2 + i\epsilon}$ (80c)

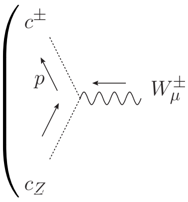
Ghost Gauge Interactions:

Φ 

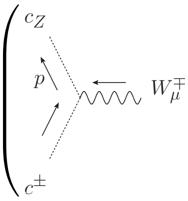
$$= \mp\, iep^{\mu} \tag{81a}$$

Φ 

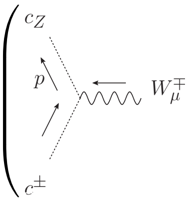
$$= \mp\, ig\cos\theta_W p^{\mu} \tag{81b}$$

Φ 

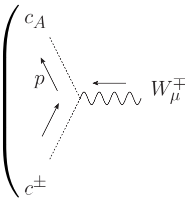
$$= \pm\, ig\cos\theta_W p^{\mu} \tag{81c}$$

Φ 

$$= \pm\, iep^{\mu} \tag{81d}$$

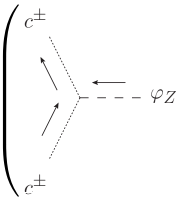
Φ 

$$= \pm\, ig\cos\theta_W p^{\mu} \tag{81e}$$

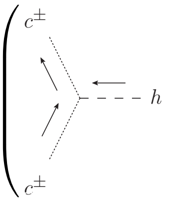
Φ 

$$= \pm\, iep^{\mu} \tag{81f}$$

Ghost Higgs and Ghost Goldstone Interactions:

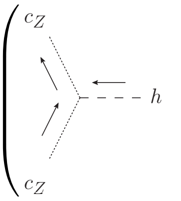
Φ 

$$= \pm\, \frac{g}{2} m_W \tag{82a}$$

Φ 

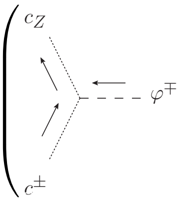
$$= -\, \frac{i}{2} g m_W \tag{82b}$$

Φ



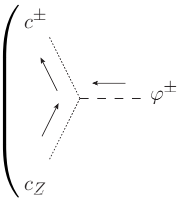
$$= -\,\eta_G \frac{ig}{2\cos\theta_W} m_Z \tag{82c}$$

Φ



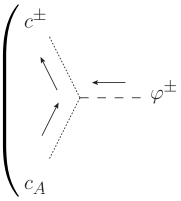
$$= \frac{i}{2} g m_Z \tag{82d}$$

Φ



$$= -\,ig \frac{\cos 2\theta_W}{2\cos\theta_W} m_W \tag{82e}$$

Φ



$$= -\,iem_W \tag{82f}$$

B Statement of authorship

I hereby declare that I noticed the 2010 version of the "Studien- und Prüfungsordnung für den Masterstudiengang Physik". The present master thesis and the results therein are worked out by me and only me, if not indicated otherwise. I used nothing but the indicated references in Appendix C. Furthermore, this master thesis is the first which is submitted by me personally in the subject "Physik" and has not been submitted to any other scientific institution.

Berlin, September 30, 2015:

David Prinz

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