INTRODUCTION TO QUANTUM FIELD THEORY (KREIMER, WINTER 17/18)

EXERCISE 1 (OCT 25 2017, TO BE HANDED IN NOV 15 2017)

- 1.(20 points)
  Using Fourier transforms $f(\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \hat{f}(\vec{k}) \leftrightarrow \hat{f}(\vec{k}) = \int d^3\vec{x} e^{-i\vec{k}\cdot\vec{x}} f(\vec{x})$, derive expressions for annihilators and generators $\alpha(k), \alpha^\dagger(k)$ in terms of ϕ, $\partial_{x_0}\phi$. Here,
  $$\phi(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega(\vec{k})} \left(\alpha^\dagger(k) e^{ix\cdot k} + \alpha(k) e^{-ix\cdot k} \right),$$
  and $\omega((\vec{k})) = \sqrt{\vec{k}\cdot\vec{k} + \mu^2}$.
- 2. (20 points)
  Let $\Box_x = \partial^2_{x_0} - \partial^2_{x_1} - \partial^2_{x_2} - \partial^2_{x_3}$. Show
  $$(\Box_x + \mu^2) T(\phi(x)\phi(y)) = -i\delta^{(4)}(x - y),$$
  and
  $$\langle 0|T\phi(x)\phi(y)|0\rangle = \lim_{\epsilon\to 0^+} \int \frac{d^4k}{(2\pi)^4} \frac{-ie^{i(x-y)\cdot k}}{k^2 - \mu^2 + i\epsilon}.$$