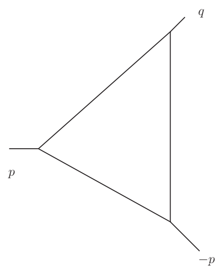
INTRODUCTION TO QUANTUM FIELD THEORY (KREIMER, WINTER 17/18)

EXERCISE 4 (DEC 13 2017, TO BE HANDED IN JAN 17 2018)

- 1.(40 points)
Consider the integral representation of the Γ-function
$$\Gamma(z) = \int_0^\infty x^{-z} e^x dx$$
and the formula for Gauian Euclidean integration
$$\int_{\mathbb{R}^D} e^{-\frac{A}{2}k^2} d^D k = \left(\frac{2\pi}{A}\right)^{D/2}, \, A > 0,$$
to compute:
$$I(D;\alpha,\beta;q^2) := \int_{\mathbb{R}^D} \frac{1}{(k^2)^\alpha ((k+q)^2)^\beta} \frac{d_E^D k}{\mu^{-2\epsilon}}, \ \alpha,\beta \in \mathbb{C}.$$
Consider the analytic continuation to complex D and determine the regions for complex D, α, β where the integral can be defined.
- 2. (40 points)
Compute the one-loop vertex graph t



in a massless field theory with a real scalar field ϕ which interacts cubically: $L_{int} = Z_g g\phi^3/3!$. Consider the one- loop approximation to the 1PI vertex function, given by the triangle graph t. What is its degree of divergence in $D = 6$ dimensions of spacetime?
Set one of the external momenta, say p, to zero. The other ones are then $q, -q$. Set $D = 6 - 2z$, with a limit $z \to 0$ to be understood at the end.
Show: the unrenormalized Feynman rules Φ give $\Phi(t) = I(6; 2, 1; q^2)$.
Let $Z_g = 1 + g^2 c_{1,g} + \mathcal{O}(g^4)$ be the Z-factor in the interaction Lagrangean above.
Determine $c_{1,g} = c_{1,g}(D, \mu^2)$ and the renormalized Feynman rules
$$\Phi_R(t) := lim_{z\to 0}(\Phi(t) + c_{1,g})$$
such that we have $\Phi_R(t)_{|q^2=\mu^2} = 0$.
Hint: $(q^2)^{-z} = 1 - z\ln(q^2) + \mathcal{O}(z^2)$.