INTRODUCTION TO QUANTUM FIELD THEORY (KREIMER, WINTER 17/18)

EXERCISE 5 (JAN 17 2018, TO BE HANDED IN JAN 31 2018)

- 1.(30 points)
  Give all 1PI graphs Γ in scalar ϕ^3 theory with two and three external legs and up to three closed loops.
- 2.(50 points)
  Assign a weight 0 to a four-valent vertex, a weight -1 to a three-valent vertex and a weight 2 to an internal edge. Consider
  $$\omega_D(\Gamma) = D|\Gamma| - 2E_\Gamma - V_\Gamma,$$
  with E_Γ the number of edges of Γ, V_Γ the number of three-valent vertices, and $|\Gamma|$ the number of independent loops, for a scalar theory with 3- and 4-valent vertices. Show that at $D = 4$ it is zero for all graphs with four external edges, $+1$ for all graphs with three external edges, and is $+2$ for all graphs with two external edges.

Consider a Clifford algebra represented by γ-matrices with $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$. Set $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. Note:$\{\gamma_5, \gamma_\mu\} = 0$, $\gamma_5^2 = 1$.

- 3.(20 points)
  Show: $tr(\gamma_{\mu_1} \dots \gamma_{\mu_{2+1}}) = 0$, $tr(\gamma_{\gamma_5}\gamma_{\mu_1} \dots \gamma_{\mu_{2+1}}) = 0$.
- 4.(30 points)
  What is $tr(\gamma_{\mu_1} \dots \gamma_{\mu_2})$ in terms of metric tensors?