Differential calculus on conformal manifolds, conformal holonomy groups and applications to spinors

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The holonomy group of a semi-Riemannian manifold (M, g) is the Lie group of all parallel displacements along loops closed in a fixed point $x \in M$ with respect to the Levi-Civita connection of the metric g. As we know, special holonomy groups correspond to special kinds of geometries. Moreover, the holonomy groups allow us for example to detect all kinds of geometries that admit parallel spinors.

In the lecture I will explain the corresponding notions for conformal manifolds and thereby review some standard methods from conformal differential geometry, which one often meets in geometric as well as in physics papers. Let (M, c) be a conformal manifold, that means a manifold equipped with a conformal class of metrics $c := [g] = \{e^{2\sigma} g \mid \sigma \in C^\infty(M)\}$. Contrary to the metric case, in the conformal case there is no distinguished connection ∇ on the tangent bundle of (M, c) which can be used for an invariant differential calculus on the conformal manifold. Instead, in conformal geometry we use the so-called Cartan connections, which differ from the principle bundle connections we know so far, to define the differential calculus. It turns out, that there is a distinguished Cartan connection (the so-called normal conformal Cartan connection) on the first prolongation of the bundle of conformal frames, which in conformal geometry plays the same role as the Levi-Civita connection is playing in the metric cases. We use this distinguished Cartan connection and its induced covariant derivative (tractor connection) to define the holonomy group of a conformal manifold. As application we show that twistor spinors (conformal Killing spinors) on (M, g) correspond to fixed elements of the conformal holonomy group of $(M, [g])$ acting on spinors. Therefore, the conformal holonomy groups allow us to detect all kinds of geometries that admit conformal Killing spinors.