See also Problem 2 and 3 from Homework 9

Problem 1
Compute the cohomology rings $H^\ast(\mathbb{R}P^n; \mathbb{Z})$ and $H^\ast(\mathbb{R}P^\infty; \mathbb{Z})$.

Problem 2
(1) Show that $H^\ast(\coprod_{\alpha} X_{\alpha}; R) \cong \bigoplus_{\alpha} H^\ast(X_{\alpha}; R)$ as rings.
(2) Show that $\tilde{H}^\ast(\bigvee_{\alpha} X_{\alpha}; R) \cong \bigoplus_{\alpha} \tilde{H}^\ast(X_{\alpha}; R)$ as rings if the base points $x_{\alpha} \in X_{\alpha}$ used for the construction of the wedge admit an open neighborhood which contracts onto $x_{\alpha}$.
(3) Show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$. Compute their homology and cohomology groups with various coefficients.