Marko Roczen

# 1-Semiquasihomogeneous Singularities of Hypersurfaces in Characteristic 2

Abstract:
In arbitrary characteristic different from 2, the singularities with semiquasihomogeneous equations characterized by the condition to have Saito-invariant 1 are the "classical" quasihomogeneous ones, known over the field of complex numbers as simple elliptic singularities (Saito, [10]). Here we find them in characteristic 2 as well: In odd dimensions and for weights $\tilde{E}_6$ and $\tilde{E}_8$ non-quasihomogeneous equations appear.

## 0. The problem

$k$ denotes an algebraically closed field. Let $X$ be a finite set of indeterminates $x$ equipped with positive weights $w(x) \in \mathbb{Q}$ and $f \in k[[X]]$ be a formal power series consisting of monomials of weight $\geq 1$ such that $f_1$ (:= sum of terms of total degree 1) defines an isolated singularity (i.e. the partial derivatives generate an ideal which is primary for the maximal ideal in $k[[X]]/(f)$). Then we associate to $f$ the "Saito-invariant" $s := |X| - 2\sum_{x\in X} w(x)$. We say "$f$ is $s$-semiquasihomogeneous" (or short: "s-sqh") with respect to the given weights. For $f = f_1$, $f$ is said to be "1-quasihomogeneous". The case of $s < 1$ gives the rational double points (the simple singularities or, equivalently, the absolutely isolated Cohen Macaulay double points, cf. [3], [6], [4]). Here the "boundary case" of $s = 1$ is considered, which corresponds in the complex-analytic case to the simple elliptic singularities ([10]). Note however, that for *char* $k = 2$ not all of those singularities arise from dimension 2, so here they better will be referred to only as 1-semiquasihomogeneous. As for the simple singularities, the case of characteristic 2 is most complicated in the sense of stable equivalence for different dimensions. From the point of representations (considering the Auslander-Reiten quiver of maximal Cohen-Macaulay modules over the local ring of the singularity), the usual Knörrer-periodicity has to be replaced by Solberg's periodicity (taking dimensions *mod* 2), and the results of Kahn (cf. [5]) may apply at least to some of the singularities found here.

## 1. The quasihomogeneous case

Write $X = \{X_0, \ldots, X_n\}$ and $w(X_i) = w_i$. We always assume $w_i \leq \frac{1}{2}$; this is no loss of generality (cf. e.g. [6]). Let $k[X]_1$ denote the polynomials which are sum of monomials of weight 1 (set of "quasihomogeneous polynomials" with respect to the given weights). Then we have the following

**Cancellation property**: Let $f, g \in k[X]_1$ define isolated singularities, and let $q_1,\ q_2 \in k[Y]$ be nondegenerate quadratic forms in a finite set $Y$ of new variables of weight $\frac{1}{2}$. Suppose $f + q_1$ can be transformed into $g + q_2$ by an automorphism

$\Phi$ of $k[X,Y]$ preserving the grading. Then there exists an automorphism $\Psi$ of $k[X]$ which preserves the grading and such that $f = g \circ \Psi$.

This is a consequence of the following (cf. [6])

**Proposition (Saito, Knop):** Choose $f \in k[[X]]$ defining an isolated singularity.

(i) If $Y \subseteq X$, then one of the following is satisfied:

(a) There exists $X^\alpha \in supp(f)$ such that $X^\alpha \in k[Y]$, or

(b) There exists an injective map $\varphi : Y \hookrightarrow X - Y$ and a map $\psi : Y \to \mathbb{N}^Y$ such that $Y^{\psi(y)} \cdot y \cdot \varphi(y) \in supp(f)$ for every $y \in Y$.

(ii) Assume $f$ is quasihomogeneous of degree 1. Then up to an automorphism of $k[X]$ which preserves the grading, $f = f_1 + \sum_{x \in A} x\phi(x)$, where $A = \{x \in X,\ w(x) > \frac{1}{2}\}$ and $\phi : A \hookrightarrow X - A$ is an injection, $f_1 \in k[X - (A \cup \phi(A))]$. Now, choose all $w_i \le \frac{1}{2}$ and denote $Q := \{x \in X |\ w(x) = \frac{1}{2}\}$, $R := X - Q = \{x \in X |\ w(x) < \frac{1}{2}\}$.

Up to a graded automorphism[1], $f$ is of the following form:

(a) $f = f_1 + q,\ f_1 \in k[R]$, and $q \in k[Q]$ a nondegenerate quadratic form.

(b) $char\ k = 2$, and there exists $x_0 \in Q$ such that $f = f_1 + f_2 \cdot x_0 + x_0^2 + q$, where $q \in k[Q - \{x_0\}]$ ($q$ nondegenerate quadratic form), $f_i \in k[R]$ for $i = 1, 2$.

We deduce a

**Proof of the cancellation property:**
Let $f + q_1 = (g + q_2) \circ \Phi$. In case of part (ii) (a) of the preceding proposition, we may assume $X = R$, i.e. $f, g \in (X_0, \ldots, X_n)^3$, $w(X_i) < \frac{1}{2}$, thus $\Phi(X_i) \in k[X]$, and after a linear change of coordinates in $Y$, $\Phi(Y_i) = Y_i$.

Now let $char\ k = 2$ and suppose $f$ has the form (ii) of (b), $f + q_1 = f_1 + f_2 X_0 + X_0^2 + q$, where $f_i \in k[X_1, \ldots, X_n]$ and $q \in k[Y]$ is a nondegenerate quadratic form. We may assume $g + q_2 = g_1 + g_2 X_0 + X_0^2 + q$, and also $g_i, f_i \in k[Y]$, $|Y| = m$ even and $q = Y_1 Y_2 + \ldots + Y_{m-1} Y_m$ (classification of quadratic forms in characteristic 2). Then, if $f = g \circ \Phi$, $\Phi$ graded. We obtain $\Phi(R) \subseteq k[R]$, $R = \{X_1, \ldots, X_n\}$, $\Phi$ induces a linear transformation in the variables $\{X_0\} \cup Y\ mod(X_1, \ldots, X_n)^2$, fixing $X_0^2 + q(Y)\ mod(X_1, \ldots, X_n)^2$. Thus we may assume $\Phi(X_0) = X_0 + \phi_0,\ \Phi(Y_i) = Y_i + \phi_i,\ \phi_i \in k[X]$ of weight $\frac{1}{2}$. But $q$ is nondegenerate, thus $\phi_1 = \ldots = \phi_m = 0$.

**Definition:** Choose $f \in k[[X]]$ and $g \in k[[X']]$.

(i) $f, g$ are said to be right equivalent if $X = X'$ and there exists an automorphism $\Phi$ of $k[[X]]$ such that $f = g \circ \Phi$. In this case, we write $f \overset{r}{\sim} g$ (without loss of generality, $\Phi$ can be choosen homogeneous of degree 0 if $f \in k[X]_1$, $g \in k[X]_1$ for a fixed weight $w$).

[1] tacitly assumed to be of degree 0

(ii) Assume there exist nondegenerate quadratic forms $q \in k[Z]$, $q' \in k[Z']$ respectively in finite sets $Z$, resp. $Z'$ of new variables such that $f + q \overset{r}{\sim} g + q'$. Then $f, g$ are said to be stable-equivalent[2]. We write $f \overset{s}{\sim} g$. The polynomials $f + q$, $g + q'$ respectively will be referred to as "quadratic suspensions" of $f, g$ respectively.

Thus, the above cancellation property says: If $f$, $g$ (as above) have the same number of variables and $f \overset{s}{\sim} g$, then $f \overset{r}{\sim} g$

If $f \overset{s}{\sim} g$, then $s(f) = s(g)$, and always $0 \leq s(f) < |X|$. The classes of $f$ having $s(f) < 1$ are precisely the quasihomogeneous forms of the simple singularites ADE (cf. [6], [4]); their behavior under the canonical local resolution is studied in [7].

For the 1-qh polynomials we have the following

**Theorem:** Let $f \in k[X]$ be a polynomial defining an isolated singularity such that $f$ is quasihomogeneous for some weight $w$ with $s = 1$.

Then $w$ is (up to permutation) one of the weights

$$\tilde{E}_6 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \ldots, \frac{1}{2}),\ \tilde{E}_7 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \ldots, \frac{1}{2}),\ \tilde{E}_8 = (\frac{1}{3}, \frac{1}{6}, \frac{1}{2}, \ldots, \frac{1}{2})$$

and $f$ is stable-equivalent with one of the following polynomials ($t \in k$ denotes a parameter):

**Case A:** $char(k) \neq 2$

$\tilde{E}_6$: $f = X_1(X_1 - X_0)(X_1 - tX_0) - X_0X_2^2, \quad t \neq 0, 1$

$\tilde{E}_7$: $f = X_0X_1(X_1 - X_0)(X_1 - tX_0), \quad t \neq 0, 1$

$\tilde{E}_8$: $f = X_0(X_0 - X_1^2)(X_0 - tX_1^2), \quad t \neq 0, 1$

**Case B:** $char(k) = 2$

1. $n$ odd

| | | |
|---|---|---|
| $\tilde{E}_6(0)$: | $X_0^3 + X_1^2X_2 + X_1X_2^2 + X_3^2$ | |
| $\tilde{E}_6(t)$: | $X_0^3 + tX_2^3 + X_1^2X_2 + X_0X_1X_2 + X_3^2,$ | $t \neq 0$ |
| $\tilde{E}_7(t)$: | $X_0X_1(X_1 + X_0)(X_1 + tX_0), \quad t \neq 0,\ 1$ | |
| $\tilde{E}_8(t)$: | $X_0(X_0 + X_1^2)(X_0 + tX_1^2), \quad t \neq 0,\ 1$ | |

[2]Note, the condition implies that the total number of variables has to be the correct one.

2. $n$ even

$$\begin{array}{lll} \tilde{E}_6(0): & X_0^3 + X_1^2X_2 + X_1X_2^2 & \\ \tilde{E}_6(t): & X_0^3 + tX_2^3 + X_1^2X_2 + X_0X_1X_2, & t \neq 0 \end{array}$$

$$\begin{array}{lll} \tilde{E}_{7,1}(t): & X_0^2 + X_0X_1^2 + X_1X_2^2(tX_1 + X_2) & \\ \tilde{E}_{7,2}(t): & X_0^2 + X_0X_1X_2 + X_1X_2(tX_1 + X_2)^2, & t \neq 0 \end{array}$$

$$\begin{array}{lll} \tilde{E}_8(t): & X_0^2 + X_0X_1X_2 + X_1(X_1 + X_2^2)(X_1 + tX_2^2), & t \neq 0 \end{array}$$

**Proof:** To start with, we need the following

**Lemma:** With the previous notations, assume $s = |X| - 2\sum_{x\in X} w(x) = 1$, i.e.

$$\sum_{x\in R} w(x) \;=\; \frac{1}{2}(|R| - 1)$$

and such that there exists a polynomial $f \in k[X]_1$ with an isolated singularity. Then

(i) $|R| \neq 0, 1$

(ii) $S := \{x \in X |\ \frac{1}{3} < w(x) < \frac{1}{2}\} = \emptyset$

(iii) $|R| \leq 3$ with equality at most if $w_0 = w_1 = w_2 = \frac{1}{3}$ (up to permutation of indices of the $X_i$).

(iv) If $|R| = 2$, then $w_0 = w_1 = \frac{1}{4}$, or $w_0 = \frac{1}{3},\ w_1 = \frac{1}{6}$ (again, indices may permute).

**Proof of the Lemma:** (i) is an obvious consequence of $s = 1$.
To show (ii), (iii), apply (i) in the above proposition: Choose maps $\varphi,\ \psi$ with the property (b) and obtain:

$$\begin{aligned} \frac{1}{2}(|R| - 1) &= \sum_{x\in R} w(x) = \\ &= \sum_{x\in S} w(x) + \sum_{x\in S} w(\varphi(x)) + \sum_{x\in R-(S\cup\varphi(S))} w(x) \\ &= \sum_{x\in S} w(x) + \sum_{x\in S}(1 - w(x) - w(S^{\psi(x)})) + \sum_{x\in R-(S\cup\varphi(S))} w(x) \\ &= \sum_{x\in S}(1 - w(S^{\psi(x)})) + \sum_{x\in R-(S\cup\varphi(S))} w(x) \leq \frac{2}{3}|S| + \frac{1}{3}|R - S\cup\varphi(S)| \end{aligned}$$

(note that $S^{\psi(x)} \cdot x \cdot \varphi(x) \in supp(f)$ for all $x$, i.e. $w(S^{\psi(x)}) + w(x) + w(\varphi(x)) = 1$; also, $\varphi(S) \subseteq R$). Thus

$$\frac{1}{2}(|R|-1) \leq \frac{1}{3}(2|S| + |R - (S \cup \varphi(S))|) = \frac{1}{3}|R|.$$

To prove (iv), we may assume $w_0 + w_1 = \frac{1}{2}$, $w_i = \frac{1}{2}$ for $i > 1$, i.e. for $w_0 = w_1$ we are done. Assume $w_0 > w_1$, then $\frac{1}{4} < w_0 < \frac{1}{2}$. If $X_0^3 \notin supp(f)$, then no power of $X_0$ is in $supp(f)$, and (i) (a) in the Proposition implies (using $Y = X_0$) that one of the monomials $X_0^{\alpha+1}X_i$ ($\alpha \in \mathbb{N}$, $i \in \{1, \ldots, n\}$) is in $supp(f)$. This implies $w_0 = \frac{1}{2\alpha}$ or $w_0 = \frac{1}{2(\alpha+1)}$ (contradiction, since $\alpha \in \mathbb{N}$). Thus $X_0^3 \in supp(f)$, i.e. $w_0 = \frac{1}{3}$, $w_1 = \frac{1}{6}$.

Now, a detailed case by case analysis gives the

**Proof of the Theorem:**

Choose e.g. the case of $\tilde{E}_6$ in even dimension, i.e. here without loss of generality in dimension 2. Then in coordinates $(x0 : x1 : x2)$, the corresponding equation $f = 0$ defines a smooth curve $C$ of degree 3 in the projective plane. We obtain the above normal form after a linear change of coordinates. In $char\ k = 2$ we have two cases: $\tilde{E}_6(0)$ if the elliptic curve is supersingular, $\tilde{E}_6(t)$, with $t \neq 0$ otherwise.

For the weights $\tilde{E}_7$, $\tilde{E}_8$, a geometric analysis of the relevant forms is necessary, giving different equations in even and odd dimensions for $char\ k = 2$.
We apply the proposition to obtain the list of equations; choose e.g. $f$ of weight $\tilde{E}_7$, $char\ k = 2$:

We may assume $X = \{X_0, \ldots, X_n\}$ with

(a) $n = 1$, $w_0 = w_1 = \frac{1}{4}$, $f = f(X_0, X_1)$ homogeneous of degree 4 and defining an isolated singularity, i.e. $f$ with 4 different zeroes on $\mathbb{P}^1$.

(b) $n = 2$, $w_0 = \frac{1}{2}$, $w_1 = w_2 = \frac{1}{4}$, $f = x_0^2 + gx_0 + h$, $g \in k[X_1, X_2]$ homogeneous of degree 2, $h \in k[X_1, X_2]$ homogeneous of degree 4.

If (b1) $g = 0$, then coordinates can be choosen such that $X_1X_2^3 \notin supp\ h$, thus $V(X_1, X_0^2 + g(X_1, X_2)) \subseteq sing(f)$, i.e. the singular locus has positive dimension. Now assume (b2) $g = X_1^2$, then $f = X_0^2 + X_0X_1^2 + h(X_1, X_2)$. Write $h(X_1, X_2) = \sum_{\nu=0}^{4} h_\nu X_1^\nu X_2^{4-\nu}$. Then $f$ defines an isolated singularity iff $h_1 \neq 0$; we may assume $h_1 = 1$. A coordinate transformation $X_0 := X_0 + aX_1^2 + bX_1X_2 + cX_2^2$ brings $h$ into the form $h = X_1X_2^2(tX_1 + X_2)$.

The case (b3) $g = X_1X_2$ is done in a similar way.

**Remark:** Note that also in $char\ k = 2$, the equations for $\tilde{E}_6$ can be written in a form such that $\tilde{E}_6(0)$ and $\tilde{E}_6(t)$, $t \neq 0$ are in the same 1-parameter family: Take $n = 2$ and let $C(s)$ be the curve defined in the projective plane by

$$X_0^3 + X_1^3 + X_2^3 + sX_0X_1X_2 = 0$$

where $s \in k$. For $s^3 \neq 1$ this is an elliptic curve with absolute invariant $j = \frac{s^{12}}{(s^3+1)^3}$, and $\tilde{E}_6(0)$ is the cone over an elliptic curve with invariant 0, thus isomorphic to the cone over $C(0)$. For fixed $t \neq 0$, the equation $ts^{12} + s^9 + s^6 + s^3 + 1 = 0$ has 12 different solutions $s$. We obtain several $C(s)$ with invariant $j = \frac{1}{t}$.
Thus any 2-dimensional quasihomogeneous singularity of type $\tilde{E}_6$ is obtained as cone over some $C(s)$.

**Corollary:** Let $f \in k[X]$ be quasihomogeneous of some weight $w = w(f)$ and assume $s = s(f) \leq 1$. Then $w$ is uniquely determined up to permutation in the class of quasihomogeneous functions which are stable equivalent $f$. Especially, the number $s$ is well defined on the equivalence class.

**Remark:** In the case considered here, $w$ (up to permutation) and therefore $s(f)$ depends only on the complete local ring of the singularity. It is not known to the author, if this is generally so for $s(f) > 1$ (but it is always true for $k = \mathbb{C}$ by [10]).

## 2. Normal forms of semiquasihomogeneous functions

Now let $f = f_1 + f_{>1}$ be a formal power series which contains no monomials of weight $< 1$ with respect to the given weight $w$. Put $f_1 :=$ sum of terms of weight 1 in $f$ and assume $f_1$ defines an isolated singularity.

$f$ is said to be contact equivalent with a power series $g$, if the $k$-algebras $k[[X]]/(f)$ and $k[[X]]/(g)$ of formal power series are isomorphic.

The following result reduces the part $f_{>1}$ into a normal form without changing $f_1$ and the contact equivalence class of $f$. $T(f_1)$ denotes the "Tjurina-algebra",

$$T(f_1) := k[[X]]/(f_1, \frac{\partial f_1}{\partial X_0}, \ldots, \frac{\partial f_1}{\partial X_n}).$$

We have $dim_k(T(f_1) < \infty$.

**Theorem:** Let $(\overline{e_1}, \ldots, \overline{e_s})$ denote any maximal linear independent set of classes in $T(f_1)$ of monomials $e_i$ having weight $> 1$ ("superdiagonal monomials"). Then $f = f_1 + f_{>1}$ is contact equivalent with $f_1 + c_1e_1 + \ldots + c_se_s, \quad c_i \in k$.

**Proof:** Let $w = (\frac{m_0}{d}, \ldots, \frac{m_n}{d})$ with positive integers $m_i$, $d$. Denote $o_m(h)$ the total order of the initial term of a power series $h \in k[[X_0, \ldots, X_n]]$ with respect to $(m_0, \ldots, m_n)$.

If the classes of superdiagonal monomials $\{e_1, \ldots, e_s\}$ form a basis of the subspace generated by all superdiagonal monomials in the Tjurina algebra $T(f_1)$, then the similar assertion is true for any fixed order $d'$, i.e. let $\{e_{i_1}, \ldots, e_{i_q}\}$ be the subset of monomials such that $o_m(e_{i_j}) = d'$, then this is a basis for the subspace in $T(f_1)$ generated by the classes of all monomials having $o_m = d'$ ($f_1$ is homogeneous).
Obviously, an inductive convergence argument gives the result, if we show the following

**Lemma:** Let (after some permutation) $e_1, \ldots, e_r$ be the monomials of order $o_m(e_i) = d' > d$ in $\{e_1, \ldots, e_s\}$. Then $f$ is contact equivalent with a series

$$f_1 + f'_{>1} + \sum_{i=1}^{r} c_i e_i + h,$$

where $f'_{>1}$ is the sum of terms of order $o_m < d'$ in $f_{>1}$, $c_i \in k$ und $h \in k[[X_0, \ldots, X_n]]$ has order $o_m(h) > d'$.
(Note that the case is included, where $\{e_1, \ldots, e_r\}$ is the empty set.)

**Proof of the Lemma:** Choose $c_i \in k$ such that

$$g - \sum_{i=1}^{r} c_i e_i = q \cdot f_1 + \sum_{i=0}^{n} v_i \frac{\partial f_1}{\partial X_i}$$

for some $q,\ v_i \in k[[X_0, \ldots, X_n]]$ and $g$ the sum of monomials of order $d'$ in $f_{>1}$.

Without loss of generality, $q$ and $v_i$ are quasihomogeneous for $(m_0, \ldots, m_n)$ of order

$$o_m(q) = d' - d =: \delta > 0$$
$$o_m(v_i) = d' - (d - m_i) = \delta + m_i > m_i,$$

respectively. We obtain

$$(*) \quad f_1 + f'_{>1} + \sum_{i=0}^{r} c_i e_i = (1-q)(f_1 + f_{>1}) + q f_{>1} - \sum_{i=0}^{n} v_i \frac{\partial f_1}{\partial X_i} + p,$$

where in the right hand term $o_m(q f_{>1}) > d'$, $v_i \frac{\partial f_1}{\partial X_i}$ is quasihomogeneous with $o_m(v_i \frac{\partial f_1}{\partial X_i}) = d'$, and $o_m(p) > d'$.
Assume without loss of generality $m_0 \geq m_1 \geq \ldots \geq m_n$. Let $X_i := X'_i - v_i(X')$, then $o_m(v_i) > m_i = o_m(X_i)$ implies: The linear part of this coordinate transformation has a lower triangular matrix $(a_{ij})$ with $a_{ii} = 1$, and $a_{ij} \neq 0$ for $i > j$ is possible only if $m_i > m_j$. The above substitution sends

$$(**) \quad f_1(X) \mapsto \quad f_1(X') - \sum_{i=0}^{r} v_i(X') \frac{\partial f_1(X')}{\partial X'_i} + \textit{terms in } X' \textit{ of order } o_m > d'$$

(if we take the same weights for the $X'$).
By $(*)$, we have

$$(***) \quad (1-q(X))f(X) \equiv f_1 + \sum_{i=0}^{n} v_i(X)\frac{\partial f_1}{\partial X_i} + (f'_{>1}(X) + \sum_{i=0}^{r} c_i e_i(X))$$

*mod* terms of order $o_m > d'$. If we apply $(**)$ and remember $o_m(v_i) > d' - d$, the substitution above transforms the right hand side of $(***)$ into

$$f_1(X') + (f_{>1}(X') + \sum_{i=0}^{r} c_i e_i(X')) + h(X'),$$

where $o_m(h) > d'$. This completes the proof.

Note that we do not need any assumption on *char* $k$. If *char* $k = 0$, by Euler's formula we have $f_1 \in (\frac{\partial f_1}{\partial X_0}, \dots, \frac{\partial f_1}{\partial X_n})$, i.e. the Tjurina-algebra $T(f_1)$ coincides with the Milnor-algebra $M(f_1) = k[[X]]/(\frac{\partial f_1}{\partial X_0}, \dots, \frac{\partial f_1}{\partial X_n})$, and in this case the result coincides with ([1], 12.6).

### 3. Results in the 1-semiquasihomogeneous case

Using a computer[3], from the theorem in section 2. we obtain easily:

| semiquasihomogeneous singularities with $s = 1$ in characteristic 2 | | | |
|---|---|---|---|
| type | Tjurina-number | maximal set of linearly independent superdiagonal monomials | total number |
| case 1: dimension $\equiv 1$ *mod* 2 | | | |
| $\tilde{E}_6(0)$ | 16 | $X_0X_1X_3,\ X_1X_2X_3,\ X_0X_2X_3,\ X_0X_1X_2X_3$ | 4 |
| $\tilde{E}_6(t)$ | 16 | $X_1^2X_3,\ X_2^2X_3,\ X_1X_2X_3,\ X_2^3X_3$ | 4 |
| $\tilde{E}_7$ | 9 | $\emptyset$ | 0 |
| $\tilde{E}_8$ | 12 | $X_0X_1^5$ | 1 |
| case 2: dimension $\equiv 0$ *mod* 2 | | | |
| $\tilde{E}_6(0)$ | 8 | $\emptyset$ | 0 |
| $\tilde{E}_6(t)$ | 8 | $\emptyset$ | 0 |
| $\tilde{E}_{7,1}(t)$ | 10 | $\emptyset$ | 0 |
| $\tilde{E}_{7,2}(t)$ | 10 | $\emptyset$ | 0 |
| $\tilde{E}_8(t)$ | 10 | $\emptyset$ | 0 |

[3] calculations done with REDUCE 3.5

Thus e.g. for $n$ odd, the 1-sqh singularities with first term $\tilde{E}_8$ (as in the theorem of section 1) are given by adding a constant multiple the monomial $X_0X_1^5$. If the coefficient is not zero, an easy coordinate transformation leads to the only non quasihomogeneous 1-sqh singularity of that weight; it is given by the equation $X_0(X_0+X_1^2)(X_0+tX_1^2)+X_0X_1^5=0$ ($t \notin \{0,1\}$) with Tjurina number 11.

# References


[1] Arnold, V.I., Gusejn-Zade, S.M, Varchenko, A.N., Singularities of differentiable maps, Vol. I, Birkhäuser, Monographs in Mathematics, Vol. 82, Boston - Basel - Stuttgart (1985)

[2] Arnold, V.I., Normal forms for functions near degenerate critical points, the Weyl groups $A_k$, $D_k$ and $E_k$ and Lagrangian singularities, Func. Anal. Appl. 6 (1972) 4, 3-25

[3] Artin, M., Coverings of the rational double points in characteristic $p$, Complex Analysis and Algebraic Geometry, eds. Baily, W.L. Jr. and Shioda, T., Cambridge 1977, 11-22

[4] Greuel, G.M., Kröning, H., Simple singularities in positive characteristic, Math. Z. 203 (1990), 339-354

[5] Kahn, C. P.. Reflexive modules on minimally elliptic singularities, Math. Ann. 285 (1989), 141-160

[6] Knop, F., Ein neuer Zusammenhang zwischen einfachen Gruppen und einfachen Singularitäten, Invent. math. 90 (1987), 579-604

[7] Roczen, M., Recognition of simple singularities in positive characteristic, Math. Z. 210 (1992), 641-654

[8] Roczen, M., Quasihomogeneous singularities of hypersurfaces with Saito-invariant 1 in positive characteristic, Preprint 1995

[9] Saito, K., Quasihomogene isolierte Singularitäten von Hyperflächen, Inventiones Math. 14 (1971), 123 - 142

[10] Saito, K.,Einfach elliptische Singularitäten, Inventiones Math. 23 (1974), 289 - 325



Author's adress:

Institut für Mathematik
Humboldt-Universität zu Berlin

Ziegelstr. 13 A
D - 10099 Berlin, Germany
e-mail: roczen@mathematik.hu-berlin.de