Alfred Gray`s Curves and Surfaces

adapted to *Mathematica* v. 9.0 by

Rolf Sulanke

Finished July 16, 2013

Introduction

In the ninetieth, till to his sudden and unexpected death in Bilbao 1998, Alfred Gray developed intensively applications of *Mathematica* to Euclidean differential geometry of curves and surfaces. As a result of his work he published in 1994 his trend-setting book [G1] opening completely new approaches to differential geometry in teaching and research. Already in the eightieth and later Alfred Gray visited our Institute of Mathematics at Humboldt University several times. He gave reports about his work and introduced us already into the first versions of *Mathematica.* He cooperated with Hubert Gollek who translated his book into German, see [G2]. Now the third edition [G3] of the book appeared accompanied by a series of *Mathematica* notebooks which correspond to the sections of the book. They can be seen and downloaded from the website [G4] of the publishing house.

In connection with his book Alfred Gray wrote about 20 *Mathematica* Packages and some notebooks which he left us generously for use in teaching and research (May 1998). This *Mathematica* Code is almost identical with the packages contained in the zip-file Gray.zip which can be downloaded from the Wolfram Library Archive [G5]. I have taken the packages [G5] as the starting point for adapting Gray`s code to the actual *Mathematica* version 9.0. Also on the website of Mohammad Ghomi [MG] one can find applications of these packages in teaching differential geometry. I cite the first paragraphs of the file readme.txt contained in [G5] (here printed in italics}:

README (Version 2.0)

*The subdirectory CandS contains all the files of the form *.m.*
They can be used with Mathematica. For more information see the book
"Modern Differential Geometry of Curves and Surfaces via Mathematica,
Second Edition" (CRC Press, 1998). A gallery of surfaces is available:
http://bianchi.umd.edu, from which updates are available. Updates
are also available via anonymous ftp from bianchi.umd.edu. You can
contact me at gray@bianchi.umd.edu.

Alfred Gray, April, 1998.

Instructions for use:

1. Make a subdirectory of your home directory called CandS
*and copy all of files of the form *.m into it.*

*2. To access the *.m files using version 2.2 of Mathematica,*
enter the following command into a Mathematica session:

AppendTo[$Path,"~/CandS"]

This command can also be added to your file init.m;
then the files will be automatically accessible.
*The *.m files will be accessible using version 3.x of Mathematica,*
provided they are copied into the subdirectory ExtraPackages.

3. To use the main miniprograms, enter one of the following commands in a Mathematica session:

Needs["CAS`"]
or
<<CAS.m

This will load the following files:

CSPROGS.m (miniprograms for manipulating curves and surfaces)
CURVES.m (definitions of parametrizations and implicit equations of curves)
SURFS.m (definitions of parametrizations and implicit equations of surfaces)
PLTPROGS.m (useful plotting programs commands that supplement those of Mathematica)

Unfortunately, the part printed in magenta, is not valid anymore. The four packages mentioned in the last paragraph are loaded into this notebook by the initialization; the start package CAS.m is not necessary. I adapted the packages CSPROGS.m and PLTPROGS.m to *Mathematica* v. 9.01. Many of the miniprograms of these packages are tested in the corresponding subsections below. The packages CURVES.m and SURFACES.m are catalogs of parameter presentations of plane and space curves, resp. surfaces in the Euclidean 3-space. I used some of them, but I didn't feel the necessity to test them all; I didn't change any. Also I didn't change the other packages of the file Gray.zip.

I joined the adapted and the unchanged files of Gray.zip with the current notebook into the new zip-file gray1.zip. This file and other *Mathematica* notebooks and packages can be downloaded from my homepage. Working with this stuff can students help to understand Euclidean differential geometry and to acquire experience in applying *Mathematica*. I will be grateful for comments, criticism, and corrections. Write to
sulanke@mathematik.hu-berlin.de.

A completely different method of programming and applying *Mathematica* to differential geometry is available from DigiArea, see the software package atlas².

Initialization

Loading Gray`s four basic packages

1. Download the package gray1.zip from my homepage and extract all files into your working directory. Download.
2, Open the notebook cands-1.nb and insert the Path of your working directory in the shape corresponding to your operating system as argument of SetDirectory into the next cell. Here is an example valid in the OS Windows:

In[1]:=
```
SetDirectory["E:\\mymath\\diffgeo\\Gray_Alfred\\CandS"];
```

3. Save the notebook under a new name (to preserve the original notebook for further applications)
4. Activate Evaluation/Evaluate Initialization Cells from the menu.

In[2]:=
```
Get["CSPROGS.m"]
```

```
The Packages  CSPROGS.m, CURVES.m,  SURFS.m,  PLTPROGS.m are loaded!
```

Now the notebook is ready for your own work. You can use all the miniproograms listed in the next chapter. To start a session you have to do step 4 only; steps 1 - 3 are necessary only once before the first use of the notebook.

Remarks.1. If you want to test the original packages of Alfred Gray yourself, or if you work with *Mathematica* version 3, start step 1 with downloading Gray.zip from [G5].
2. To avoid error messages I deleted in Gray`s original packages the call of obsolete *Mathematica* packages of *Mathematica* v.2.2; see the Appendix.

The usages

CSPROGS

The package contains 186 of Alfred Gray`s miniprograms. I couldn't test all of them. The user should try the wanted program and adapt it, if necessary, to the actual *Mathematica* version.

```
Names["CSPROGS`*"];
```

```
Length[%]
```

```
186
```

```
Attributes[fff]
```

```
{Protected}
```

The subsections corresponding to a miniprogram start with the usage, followed by a simple example. For more information see or copy the definition of the program from the package..

In[3]:=
```
? CSPROGS`*
```

▼ CSPROGS`

analytictominimal	ggg	plotintrinsicea
analytictominimalpolar	gggg	plotintrinsicsurfrev
approxcurve	gmap	plotsurfrevfromgc
arclength	grad	plottl
arclengthea	harmonictoanalytic	plottp
arclengtheaprime	hcursq	polarize
arclengthprime	helical	polarsurf
asymeq1	hessian	prindev
asymeq2	infarea	reflectgraphics
binormal	inscurve	reflection
binormalsurf	intrinsic	rf
bjorling	intrinsicea	rollcurve
bracket	inversion	rotation2d
brioschicurvature	inversioncurve	rotation3d
canalsurf	inversionsurf	rt
christoffel	involute	rtoc
circle3point	involutends	ruled
clairautu	J	seashell
clairautv	jacobian2	sff
combescure	k1	singmonge
commutator	k2	singmongepolar
cone	kappa	solvegeoeqs
contrapedal	kappa2	stereoinj
cross	kappa23d	stereoinjcomplex
csprogs	kappa2c	stereoproj
ctor	kappa2ea	stretch
curveonsurf	kappa2implicit	studycurvature
cylinder	kappa2polar	surfacejacobian
darbouxvector	kappan	surfacejacobianmatrix
der	kmu	surfacenormal
descenttime	kpi	surfast
descenttimen	laplacian	surfrev
descenttimends	length	surfrevfromgc
descenttimeprime	lengthea	tandev
directionalD	lengthean	tangent
divergence	lengtheands	tangentkmu
ds2	lengthn	tangentkpi
ee	lengthnds	tangentline
eee	mcurvature	tangentplane
eeee	meancurvature	tau
evolute	meancurvaturevf	tl
evolute3d	metricmatrix	totalscalarcurvature
ff	monge	translategraphics

fff	mongepolar	translation
ffff	nateq	transsurf
filament	nestedevolutes	tubecurve
focalk1	normal	tubeplanecurve
focalk2	normalcurvature	turningangle
focalkmu	normalfactor	turningnumber
focalkpi	normalline	twist
frenetframe	normalsurf	umeqs
gaussbrioschi	osculatingcircle	unitnormal
gaussiancurvature	oval	unitspeednds
gcursq	paramtoimp	wei
gcurvature	parcurve	weicurvature
genhel	parsurf	weidata
gensurfrev	pedal	weimetric
geo	pedsurf	weingarten
geodesiccurvature	perpsurf	weinormal
geodesictorsion	planeframe	weipolar
geomincurve	plotintrinsic	weiprime
gg	plotintrinsic3d	writhe

Click the name to get the usage!

Simple Examples

In this section we consider examples for Alfred*s miniprograms running without problems also in *Mathematica* v. 9.

approxcurve

darbouxvector

evolute, evolute3d

filament

jacobian2. surfacejacobian. surfacejacobianmatrix

monge. singmonge. singmongepolar

plotintrinsicsurfrev

polarsurf

prindev

reflection. reflectgraphics. rf

CURVES

The package CURVES.m is a library of parameter representations of curves in the Euclidean plane or space combined with Plot-commands composed by Alfred Gray. I tested many of the functions; all tests gave reasonable results.

```
Names["CURVES`*"];
```

```
Length[%]
```

```
214
```

```
Attributes[agnesi]
```

```
{Protected}
```

In[10]:= `? CURVES`*`

▼ CURVES`

agnesi	deltoidimplicit	nephroidimplicit
agnesiimplicit	deltoidinvolute	nephroidpolar
airyspiral	deltoidunitspeed	nephroidunitspeed
alysoid	devilimplicit	newtonphillipsimplicit
alysoidkappa2	diamond	ngon
alysoidnds	duererimplicit	nielsenspiral
alysoidprime	eight	oneparametersubgroup
archimedesspiral	eightknot	pacman
archimedesspiralpolar	elasticainflect	parabola
ast3d	elasticanoninflect	parabolaimplicit
astroid	ellipse	parabolicspiral
astroidimplicit	ellipsebis	perseusimplicit

astroidimplicit	ellipsebis	perseusimplicit
baseballseam	ellipseimplicit	piriform
bellcurve	ellipseinc	piriformimplicit
bernstein	epicycloid	predatorpreyimplicit
besselcurve	epicycloidunitspeed	pseudocatenary
besseln	epispiral	pseudocatenarykappa2
beziercurve	epispiralpolar	pseudocatenarynds
bicorn	epitrochoid	pseudocatenaryprime
bicylinder	epitrochoid3d	pseudospheregeodesic
biquadratic	fermatspiral	pseudosphericalloxodrome
bow	fermatspiralpolar	rectangle
bowpolar	folium	regularpolygon
bowtie	foliumimplicit	reuleauxpolygon
bulletnose	foliumpolar	rose
bulletnoseimplicit	genhelix	rosepolar
cardioid	genparabola	sc
cardioidimplicit	hankelspiral	scarab
cardioidpolar	helix	scimplicit
cardioidunitspeed	helixunitspeed	scunitspeed
cartesianimplicit	hippopede	seiffertspiral
cartesianoval	hippopedeimplicit	semicubic
cassini	horopter	semicubicimplicit
cassiniimplicit	hyperbola	serpentine
cassinipolar	hyperbolabis	serpentineimplicit
catenary	hyperbolaimplicit	sinhspiral
catenaryunitspeed	hyperbolicspiral	sinn
cayleysextic	hyperbolicspiralpolar	sinoval
circle	hypocycloid	sphericalcardioid
circleimplicit	hypocycloidinvolute	sphericalellipse
circleinvolute	hypocycloidunitspeed	sphericalhelix
circleinvoluteunitspeed	hypotrochoid	sphericalloxodrome
circleunitspeed	kampyle	sphericalloxodromeunitspeed
cissoid	kampylepolar	sphericalnephroid
cissoidimplicit	kappacurve	sphericalspiral
clelia	kappacurveimplicit	spring
clothoid	keplerimplicit	strophoid
clothoidnds	keplerorbit	strophoidimplicit
clothoidprime	keplerorbitpolar	talbot
cnccoprofile	kochsnowflake	tanhspiral
cnchyprofile	lagrangecurve	teardrop
cochleoid	lehr	teeth
cochleoidpolar	lehrkappa2	tennisballseam
conchoid	lehrnds	tooth
conchoidimplicit	lemniscate	torusknot

conicalhelix	lemniscatebis	tractrix
coshspiral	lemniscateimplicit	tractrixminus
cosn	lemniscatepolar	tractrixplus
cothspiral	lemniscateunitspeed	tractrixunitspeed
cpcprofile	limacon	triangle
crosscurve	limaconimplicit	trident
crosscurveimplicit	limaconpolar	tridentimplicit
cubicalellipse	line	trisectrix
cubicparabola	line3d	tschirnhausen
cubicparabolaimplicit	lissajous	twicubic
curves	lissajous3d	twistedn
cycloid	lituus	veryflat
cycloidunitspeed	lituuspolar	viviani
delaunay	logistic	watt
delaunaykappa2	logspiral	wattimplicit
delaunaynds	logspiralpolar	wigglyellipse
deltoid	logspiralunitspeed	zcprofile
deltoidbis	nephroid	

Click the name to get the usage!

SURFS

The package SURFS.m is a library of parameter representations of surfaces combined with Plot-commands composed by Alfred Gray. I tested many of the functions; all tests gave reasonable, sometimes complex results.

```
Names["SURFS`*"];
```

```
Length[%]
```

```
199
```

```
Attributes[apple]
```

```
{Protected}
```

In[11]:=
```
? SURFS`*
```

▼ SURFS`

agnesirev	equihom1implicit	paraboliccoor
apple	equihom2implicit	paraboliccyclide
astell	equihom3implicit	paraboloid
astroidmincurve	equihom4implicit	paraboloidpolar
bohdom	equihom5implicit	pear
bour	equihom6implicit	pearbis
bourmincurve	exptwist	perturbedms
bourpolar	exptwistasym	perturbedmspolar
boy	flattorus	pillow

cardioidmincurve	funnel	plane
catalan	funnelasym	planepolar
catalandef	gencube	planetocat
catalanmincurve	genhypparab	plucker
catenoid	genoctahedron	pluckerpolar
catenoidasym	geopolar	poincare
catenoidiso	goursatimplicit	poindisk
catenoidmincurve	handkerchief	pseudocrosscap
cayleysexticmincurve	heart	pseudosphere
circleinvolutemin	helicoid	pseudospherebis
circleinvolutemincurve	helicoidprin	pseudospheretcheb
circularcone	heltocat	rconoid
circularcylinder	henneberg	richmond
circularmembrane	hennebergmincurve	richmondmincurve
circularmembranemovie	hy2sheet	richmondpolar
circularmembraneplot	hy2sheetimplicit	roman
cissoidmincurve	hyperbolamincurve	rosemin
clothoidmin	hyperbolicparaboloid	rosemincurve
clothoidmincurve	hyperbolicparaboloidimplicit	scherk
cnccosurfrev	hyperboloid	scherk5
cnccotcheb	hyperboloidimplicit	scherk5implicit
cnchyprofilemincurve	hypocycloidinvolutemin	scherkimplicit
cnchysurfrev	hypocycloidinvolutemincurve	scherktowermincurveprime
cnchytcheb	hypocycloidmincurve	schwarzschildplane
cossurface	invertedcircularcone	scmincurve
cossurfaceimplicit	invertedcircularcylinder	shoe
costa	invertedtorus	shoeasym
costamincurve	jorgemeeksmincurve	sievert
cpcprofilemincurve	jorgemeeksmincurveprime	sinovaloid
cpcsurfrev	jorgemeekspolar	sinsurface
crosscap	kazoolaimplicit	sinsurfaceimplicit
crosscapimplicit	kidney	snail
deltoidinvolutemin	klein4	sphere
deltoidinvolutemincurve	kleinbottle	sphereimplicit
deltoidmin	kleinbottlebis	squaremembrane
deltoidmincurve	kleinbottlewri	squaremembranemovie
dini	kuen	squaremembraneplot
dinihelicoid	kuentcheb	stereographicellipsoid
dinitcheb	kummerimplicit	stereographicsphere
earth	lemniscatemin	stereographicspherepolar
eighteight	lemniscatemincurve	surfs
eightsurface	liouville	swallowtail
ellhypcyclide	logmincurve	tetrahedral

ellipsemin	logminpolar	thomsen
ellipsemincurve	logspiralmincurve	torus
ellipsoid	menn	torusasym
ellipsoidimplicit	mercatorellipsoid	torusbis
ellipticalhyperboloidminus	mercatorsphere	torusimplicit
ellipticalhyperboloidplus	moebcirc	tractrixmincurve
ellipticcoor	moebiusstrip	twiflat
ellipticparaboloid	monkeysaddle	twisphere
ellipticparaboloidimplicit	monkeysaddlepolar	twocuspsimplicit
enneper	nephroidmincurve	veronese
ennepercatenoidmincurve	nielsenspiralmincurve	wallis
ennepercatenoidpolar	nodoid	whitneyumbrella
ennepermincurve	nodoidnds	wrinkles
enneperpolar	parabolamin	
epicycloidmincurve	parabolamincurve	

Click the name to get the usage!

PLTPROGS

The packagePLTPROGS.m contains miniprograms useful for graphical presentations of curves and surfaces. It starts with the Definition

```
pltprogs := Print[Select[Names["PLTPROGS`*"], # != "pltprogs" &]]
```

```
pltprogs
```

```
{barycenter, borderedpolygon, centerlines, dip, elrot, filledcurve,
 graphics2dtosurfrev, lathe, linetolines, orth4d, paramcurveplot1,
 paramcurveplot2, plotpatch2d, polygonholes, stereo4d, wireframe}
```

The result is the list of names of the Context PLTPROGS`, with exception of "pltprogs". All these programs are defined by Alfred Gray. I commented out Mathematica programs constructed for working with graphics in the programs Acrospin or Geomview being obsolete now. . Other definitions are commented out since they don' t work in Mathematica v. 9.

```
Names["PLTPROGS`*"];
```

```
Length[%]
```

```
17
```

```
Attributes[stereo4d]
```

```
{Protected}
```

We obtain an interactive list of the remaining usages in this Context by

In[12]:= `? PLTPROGS`*`

▼ PLTPROGS`

barycenter	filledcurve	paramcurveplot1	stereo4d
borderedpolygon	graphics2dtosurfrev	paramcurveplot2	wireframe
centerlines	lathe	plotpatch2d	
dip	linetolines	pltprogs	
elrot	orth4d	polygonholes	

Click the name to get the usage!

In the following subsections we consider Alfred's definitions in detail. Each subsection starts with the definition copied as a not evaluable cell and finishes with an example.

barycenter

borderedpolygon

centerline

checkerboard

elrot

filledcurve

graphics2dtosurfrev

lathe

linetolines

nomesh

orth4d

paramcurveplot

plotpatch2d

polygonholes

wireframe

stereo4d

Appendix

References

Gray1

[G1] Alfred Gray. Modern Differential Geometry of Curves and Surfaces.
CRC Press. 1994

Gray2

[G2] Alfred Gray. Differentialgeometrie. Klassische Theorie in moderner Darstellung. (Übersetzung aus dem Amerikanischen H. Gollek).
Spektrum Akademischer Verlag, Heidelberg.Berlin.Oxford. 1994.

Gray3

[G3] Alfred Gray, Simon Salamon, Elsa Abbena. Modern Differential Geometry of Curves and Surfaces with *Mathematica*.
Third ed. CRC Press. 2006.

Gray4

[G4] http://alpha01.dm.unito.it/personalpages/abbena/gray/

Gray5

[G5] Gray.zip. http://library.wolfram.com/infocenter/Books/3759/

MG

[MG] Mohammad Ghomi. http://people.math.gatech.edu/~ghomi/Classes/Math426/index.html

Obso

Obsolete Contexts

Epilog