

Lie Algebras

by

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Summary

In this notebook we define basic operations for Lie algebras and calculate the Killing forms of the following Lie algebras:

the general linear Lie algebras $\mathfrak{gl}(n, K)$,

the special linear Lie algebras $\mathfrak{sl}(n, K)$,

the orthogonal Lie algebras $\mathfrak{o}(n)$,

the pseudo-orthogonal Lie algebras $\mathfrak{o}(n, 1)$,

the symplectic Lie algebras $\mathfrak{sp}(2n, K)$,

the unitary Lie algebras $\mathfrak{u}(n)$,

the special unitary Lie algebras $\mathfrak{su}(n)$.

Here K denotes the fields of the real or the complex numbers. Other real forms or complex Lie algebras can be treated similarly; we leave the details to the interested user. Furthermore we construct an orthonormalization procedure appropriate for arbitrary symmetric bilinear forms defined on spaces of matrices, in particular for Killing forms. By the help of this orthonormalization we calculate the index of the considered Killing forms (in the real case), and compare it with the index listed in [3]. It is well known that the Killing forms play an important role in the classification of semisimple Lie algebras and the geometric applications of semisimple Lie groups, see e.g. the book S. HELGASON [1]. Applications for finding geodesics in the Möbius geometry of spaces of m -spheres are contained in [2]. In principle the tools developed in this notebook give the possibility to perform any calculation in the series of Lie algebras listed above.

Keywords

1 Introduction, Initialization

2 The general and the special linear Lie algebras

3 The orthogonal Lie algebras $\mathfrak{o}(n)$ **4 The pseudo-orthogonal Lie algebras $\mathfrak{o}(n,k)$** **5 The symplectic Lie algebras $\mathfrak{sp}(2m, K)$** **6 The unitary and the special unitary Lie algebras $\mathfrak{u}(n)$, $\mathfrak{su}(n)$** **References**

- [1] S. HELGASON: Differential Geometry, Lie Groups, and Symmetric Spaces, ACADEMIC PRESS, 1978.
- [2] R. SULANKE: Spheres in <http://mathsource.com/Content/Enhancements/Geometry/0210-092>.
- [3] J. TITS: Tabellen zu den einfachen Lie Gruppen und ihren Darstellungen, L.N. in Math. Nr. 40, Springer-Verlag 1967
- [4] PEKKA JANHUNEN: Declare.m, in <http://www.mathsource.com/Content22/Enhancements/Algebraic/0202-149>.

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For comments, hints, and corrections the author will be very thankful.