Loxodromes

by

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Summary

The aim of this notebook is to show that the loxodromes besides of the circles are the curves of constant conformal curvature in the Möbius plane. We calculate the conformal natural parameter of a plane curve and the moving Frenet Frame of the loxodromes. The relation between Euclidean and Möbius invariants are deduced. On the way we define and apply basic concepts of Euclidean differential geometry of plane curves. The notebook originated as a first attempt to calculate conformal invariants of curves with Mathematica. Some Modules and functions defined here ad hoc are contained in a may be changed form in later created Mathematica packages.

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Keywords

Plane Curves, spherical curves, arclength, curvature, conformal curvature, natural conformal parameter, spirals, loxodromes

Initialization

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1. List of Symbols and their Usages

1.1. Symbols in the Package vectorcalc.m

1.2. Symbols in the Package euvec.m

1.3. Symbols in the Package neuvec.m

1.4. Symbols in the Global Context

2. The Loxodromes

2.1. Logarithmic Spirals as Orbits of 1-parameter Groups of Conformal Transformations

2.2. Loxodromes as Images of the Spirals under Inverse Stereographic Projection

3. Euclidean Theory of Plane Curves

In this section we construct Modules for the geometry of plane Euclidean curves. For a detailed presentation of Euclidean differential geometry with *Mathematica* see the book [2] of Alfred Gray.

3.1 Curves in the Euclidean Plane

3.2. Control Test

4. Frenet Formulas in the Möbius Plane S^2

The differential geometry of curves in the n-dimensional Möbius space is treated in [1]. The notebook [3] containes a complete set of *Mathematica* tools for the differential geometry of curves in the Möbius plane. The presentation in the current notebook loxodromes.nb is earlier than and independent of the notebook [3].

4.1 Conformal Embedding of the Euclidean Space

4.2. Loxodromes as Conformal Embeddings of the Logarithmic Spirals

4.3. Möbius Geometry of Curves in S^2 Applied to the Loxodromes