Reliable Methods of Mathematical Modeling 8
– Book of Abstracts –

Matheon Workshop
Humboldt-Universität zu Berlin, Unter den Linden 6, 10117 Berlin, Germany

Monday, July 31st – Thursday, August 3rd, 2017

Fleurianne Bertrand

A posteriori error estimation for planar linear elasticity by stress reconstruction

The nonconforming triangular piecewise quadratic finite element space by Fortin and Soulie can be used for the displacement approximation and it's combination with discontinuous piecewise linear pressure elements is known to constitute a stable combination for incompressible linear elasticity computations. In this talk, we present a stress reconstruction procedure and resulting guaranteed a posteriori error estimator for linear elasticity, all results holding uniformly in the incompressible limit. The stress has to be reconstructed in the next-to-lowest order Raviart-Thomas spaces such that its anti-symmetric part vanishes in average on each element and the auxiliary conforming approximation is constructed under the constraint that its divergence coincides with the one for the nonconforming approximation. Local efficiency and effectiveness are illustrated by adaptive computations involving different Lamé parameters. This is joint work with Marcel Moldenhauer and Gerhard Starke.

Dietrich Braess

The two-energies principle and an a posterior error bound for the biharmonic equation

The application of the two-energies principle to pdes of fourth order is not a simple extension of the theory for pdes of second order. When discontinuous Galerkin (DG) methods with continuous elements are considered, no unknown generic constants occur in the a posteriori error bound, since the constant in the term with the data oscillation was determined by Carstensen. It may be more important for the treatment of some problems in engineering that the DG methods may be considered as being located between the primal variational formulation and the saddle-point methods. Conservation of mass, equilibration of stresses, and similar properties can be obtained by computations without indefinit matrices as they are encountered in the implemenation of mixed methods.

Philipp Bringmann

Rate optimal adaptive least-squares finite element scheme for the Stokes equations

The universality of the least-squares finite element method (LS-FEM) and its built-in a posteriori error control by the computable residual of the least-squares functional has enjoyed some ongoing attention over the years. The proof of optimal convergence rates of an adaptive LS-FEM relative to the notion of nonlinear approximation classes faces two major difficulties. First, although the least-squares functional is a reliable and efficient error estimator, it does not involve any mesh-size factor which reduces under refinement. This seemingly prevents its reduction property which is a crucial part in all known quasi-optimality proofs as in Carstensen, Feischl, Page, and Praetorius (Comp. Math. Appl. 67(6): 1195–1253, 2014). It is therefore necessary to base the adaptive mesh-design on some novel explicit residual-based a posteriori error estimator with exact solve as it is suggested by Carstensen and Park (SIAM J. Numer. Anal. 53: 43–62, 2015) for the Poisson model problem with homogeneous Dirichlet boundary data. Second, since the first-order divergence LS-FEM measures the flux errors in $H(\mathrm{div})$, the data resolution error measures the L^2-norm of the right-hand side f minus the piecewise polynomial approximation $\Pi_k f$ without a meshsize factor. This enforces a separate marking strategy with an overall abstract theory by Carstensen and Rabus (arXiv:1606.02165 [math.NA], 2016). This talk presents the extension of these techniques to the Stokes equations in 2 dimensions with inhomogeneous Dirichlet boundary data which has been recently established by Bringmann and Carstensen (Numer. Math. 135(2): 459–492, 2017). Numerical experiments confirm the theoretically proven optimal convergence rates. Further generalizations to 3-dimensional linear elasticity with inhomogeneous Neumann boundary data (joint work with Carsten Carstensen and Gerhard Starke) and higher polynomial degrees for an h-adaptive LS-FEM (joint work with Carsten Carstensen) are possible.

Florent Chave

A Hybrid High-Order method for Darcy flows in fractured porous media

We develop a novel Hybrid High-Order method for the simulation of Darcy flows in fractured porous media. The discretization hinges on a mixed formulation in the bulk region and on a primal formulation inside the fracture. Salient features of the method include a seamless treatment of nonconforming discretizations of the fracture, as well as the support of arbitrary approximation orders on fairly general meshes. For the version of the method corresponding to a polynomial degree greater or equal to 0, we prove convergence rate of $k + 1$ of the discretization error measured in an energy-like norm. In the error estimate, we explicitly track the dependence of the constants on the problem data, showing that the method is fully robust with respect to the heterogeneity of the permeability coefficients, and it exhibits only a mild dependence on the square root of the local anisotropy of the bulk permeability. The numerical validation on a comprehensive set of test cases confirms the theoretical results.

Albert Cohen

Reduced model based state estimation from data measurements

One typical scenario in data assimilation is the following: one observes m linear measurements of a function u which is solution to a PDE where certain parameters are unknown. The measurement functionals are picked from a certain dictionnary D, for example whenplacing sensors at m chosen locations. The state estimation problem then consists in recovering u from these measurements. One possible approach to this problem exploits the fact that the family of solution for all potential parameter values is well approximated by linear spaces of moderate dimension n. Such spaces are typically obtained by reduced model techniques, such as reduced bases, proper orthogonal polynomial expansions in the parametric variable. The numerical method achieves a reconstruction which has the accuracy of the best approximation from the n-dimensional space to the unknown solution u, up to a multiplicative constant which takes the form of an inverse inf-sup constant between the approximation space and the spacegenerated by the Riesz representers of the linear forms giving rise to the measurements. One issue discussed in this talk is how to select the measurement functionals within D to maintain this constant of reasonable size, with m as small as possible. In particular, we present a greedy algorithm allowing for a stepwise selection process of reasonable computational cost, and we analyze its properties.

Martin Eigel

Adaptive stochastic Galerkin FE and tensor compression for random PDEs

The Stochastic Galerkin FEM (SGFEM) is a popular approach for computing the propagation of uncertainties with random PDEs. This task occurs naturally in real-world problems when the data of the model has to be assumed as random by nature or uncertain, e.g. due to unknown parameters, or material and production deviations. Random fields used in the PDEs are then typically described depending on a countable (infinite) number of random variables. This parametrization carries over to the operator and the functional stochastic solution. The discretization of the stochastic space in generalized polynomial chaos and tensorization with some FE space inevitably leads to very high-dimensional problems. These often only become numerically tractable if some model reduction, compression and adaptivity is carried out. We discuss some recent developments of a posteriori adaptive SGFEM for affine and non-affine random fields. In particular, we examine a novel adaptive approach for random PDEs with lognormal (i.e. unbounded) coefficients represented in a modern hierarchical tensor format.

Marco Favino

Efficient numerical methods in cardiac mechanics

Electromechanical coupling in the heart is a complex multi-physics problem described by a system of PDEs and ODEs, that describes phenomena spanning several temporal and spatial scales. Due to the complexity of the problem, the construction of a robust discretization schemes and efficient simulation methods is far from being a trivial task. On the one hand, simulation of electrophysiology with mono- or bi-domain system requires fine spatial meshes and small time-step sizes in order to catch the steep gradients in the action potential and to correctly simulate the stiff ODEs describing the gating variables. While the solution of the spatial problem can be optimally done with algebraic, geometric, or semi-geometric multigrid, time integration still remains the bottleneck for accurate simulations. On the other hand, solid mechanics can be simulated on coarser meshes but the main difficulty is given by the generalised saddle-point structure of the problem, for which no Lagrangian function exists. In this talk, we will discuss the derivation and the performances of high order exponential time integration methods for cellular models in electrophysiology and a novel Augmented-Lagrangian, Uzawa-like, multigrid approach for the efficient solution of the mechanical problem.

Markus Faustmann

Discrete interior regularity and applications

In this talk, we present discrete Caccioppoli-type inequalities for the single-layer and double-layer potential. These interior regularity estimates provide a control of a stronger norm on a subdomain by a weaker norm on a larger domain provided a (local) orthogonality holds. We present two applications of these results. First, we provide sharp local a-priori estimates for Symm's integral equation on polyhedral Lipschitz domains in the stronger L^2-norm. Hereby, the local error can be bounded by a local best-approximation and a global error in a very weak norm. With duality arguments, which rely on elliptic shift theorems that involve both the interior and exterior problems, we prove faster local convergence of the BEM. As a second application, we provide existence results for approximations to inverse matrices by $\mathcal{H}$-Matrices as well as the existence of approximate LU-decompositions for black-box-preconditioning. By applying the Caccioppoli-type estimates in a fully discrete setting, we prove exponential convergence in the block rank.

Julian Fischer

An analysis of variance reduction methods in numerical stochastic homogenization

The theory of stochastic homogenization of linear elliptic PDEs predicts that a medium whose e.g. heat conductivity varies randomly on a small scale behaves on large scales like a homogeneous medium with a constant effective heat conductivity. A widely used method for the approximation of the effective coefficient is the representative volume element (RVE) method: A finite sample volume of the random medium is chosen, say, a cube with side length L; by computing the homogenization corrector on this sample volume, one may obtain an approximation for the exact effective coefficient. However, the approximation of the effective coefficient by the RVE method is a random quantity, as it depends on the sample of the random medium. It turns out that the leading-order contribution to the error of this approximation is actually caused by the random fluctuations of the approximation around its expected value. To increase the accuracy of approximations, it is therefore desirable to reduce the variance. We provide a rigorous analysis of the variance reduction concepts in stochastic homogenization introduced by Le Bris, Legoll, and Minvielle, including a - rather particular - counterexample for which the variance reduction methods fail and a wide class of random coefficient fields for which they succeed.

Dietmar Gallistl

Rayleigh–Ritz approximation of the inf-sup constant for the divergence

This contribution proposes a compatible finite element discretization for the approximation of the inf-sup constant for the divergence. The new approximation replaces the H^{-1}-norm of a gradient by a discrete H^{-1}-norm which behaves monotonically under mesh-refinement. By discretizing the pressure space with piecewise polynomials, upper bounds to the inf-sup constant are obtained. The scheme enables an approximation with arbitrary polynomial degrees. It can be viewed as a Rayleigh–Ritz method and it gives monotonically decreasing approximations of the inf-sup constant under mesh refinement. In particular, the computed approximations are guaranteed upper bounds for the inf-sup constant. The novel error estimates prove convergence rates for the approximation of the inf-sup constant provided it is an isolated eigenvalue of the corresponding non-compact eigenvalue problem; otherwise, plain convergence is achieved. Numerical computations on uniform and adaptive meshes are presented.

[1] D. Gallistl. *Rayleigh–Ritz approximation of the inf-sup constant for the divergence.* Preprint 2017.

Joscha Gedicke

Residual-based a posteriori error analysis for symmetric mixed Arnold-Winther FEM

This talk introduces an explicit residual-based a posteriori error analysis for the symmetric mixed finite element method in linear elasticity after Arnold-Winther with pointwise symmetric and div-conforming stress approximation. Opposed to a previous publication, the residual-based a posteriori error estimator of this talk is reliable and efficient and truly explicit in that it solely depends on the symmetric stress and does neither need any additional information of some skew symmetric part of the gradient nor any efficient approximation thereof. Hence it is straightforward to implement an adaptive mesh-refining algorithm obligatory in practical computations. Numerical experiments verify the proven reliability and efficiency of the new a posteriori error estimator and illustrate the improved convergence rate in comparison to uniform mesh-refining. A higher convergence rate for piecewise affine data is observed in the stress error and reproduced in non-smooth situations by the adaptive mesh-refining strategy. This is joint work with Carsten Carstensen and Dietmar Gallistl.

Jan Giesselmann

Modelling error estimates and model adaptation in compressible flows

Compressible fluid flows may be described by different models of different levels of complexity. One example are the compressible Euler equations which are the limit of the Navier-Stokes-Fourier (NSF) equations for vanishing heat conduction and viscosity. Arguably, the NSF system provides a more accurate description of the flow since viscous effects, which are neglected in Euler's equation, play a dominant role in certain flow regimes, e.g. thin regions near obstacles. However, viscous effects are negligible in large parts of the computational domain where convective effects dominate. Thus, it is desirable to avoid the effort of handling viscous terms in these parts of the domain, that is, to use the NSF system only where needed and simpler models, on the rest of the computational domain. To this end we derive an a-posteriori estimator for the modelling error which is based on the relative entropy stability framework and reconstructions of the numerical solution. This is a crucial step in the construction of numerical schemes handling model adaptation in an automated manner.

Jay Gopolakrishnan

The FEAST algorithm for eigenvalues using DPG discretizations

A filtered subspace iteration called the FEAST algorithm is a popular method in numerical linear algebra for approximating targeted spectral clusters. When used to approximate a part of the spectrum of an unbounded differential operator, it is typical to discretize the operator. We examine the spectral errors caused by a Discontinuous Petrov Galerkin (DPG) discretization. New results on the convergence of the perturbed subspace iteration as well as estimates on the gap between approximate and exact eigenspaces will be presented.

Friederike Hellwig

Nonlinear Discontinuous Petrov-Galerkin Methods

The discontinuous Petrov-Galerkin method is a minimal residual method with broken test spaces and is introduced for a nonlinear model problem. Given a nonlinear function $\varphi \in C^2(0, \infty)$ with some growth conditions, it considers the stress variable $\sigma(\nabla u) = \varphi(|\nabla u|)\nabla u$ and the equilibration equation

$$f + \operatorname{div} \sigma(\nabla u) = 0 \qquad \text{a.e. in } \Omega \tag{1}$$

for some prescribed source term f in the domain Ω. The lowest-order dPG method for this problem is established and equivalently characterized as a mixed formulation, a reduced formulation, and a weighted non-linear least-squares method. Quasi-optimal a priori and reliable and efficient a posteriori estimates are obtained for the abstract nonlinear dPG framework for the approximation of a regular solution. The variational model example allows for a built-in guaranteed error control despite inexact solve. The subtle uniqueness of discrete minimizers is monitored in numerical examples. This is joint work with Carsten Carstensen, Philipp Bringmann and Peter Wriggers.

[1] C. Carstensen, P. Bringmann, F. Hellwig, P. Wriggers. *Nonlinear discontinuous Petrov- Galerkin methods*. submitted 2017.

[2] E. Zeidler. *Nonlinear functional analysis and its applications. II/B*. Springer-Verlag (1990).

[3] C. Carstensen, L. Demkowicz, J. Gopalakrishnan. *A Posteriori Error Control for DPG Methods*. SIAM Journal on Numerical Analysis 52.3 (2014), pp. 1335–1353.

[4] A. Cohen, W. Dahmen, G. Welper, L. Weggler. *Adaptivity and variational stabilization for convection-diffusion equations*. ESAIM Math. Model. Numer. Anal. 46.5 (2012), pp. 1247–1273.

Zhongyi Huang

Bloch decomposition-based stochastic Galerkin/collocation method for Schroedinger Equation with Random Inputs

In this talk, we focus on the analysis and numerical methods for the Schroedinger equation with lattice potential and random inputs. Here we recall the well-known Bloch decomposition-based split-step pseudo-spectral method where we diagonalize the periodic part of the Hamilton operator so that the effects from dispersion and periodic lattice potential are computed together. Meanwhile, for the random non-periodic external potential, we utilize the generalize polynomial chaos with Galerkin procedure to form an ode system which can be solved analytically. Furthermore, we analyse the convergence theory of the stochastic collocation method for the linear Schroedinger equation with random inputs. We provide sufficient conditions on the random potential and initial data to ensure the spectral convergence.

Boris N. Khoromskij

Reliable approximatin method for elliptic problems with rank structured quasi-periodic coefficients (Part 2)

We consider quasi-periodic elliptic problems with coefficients admitting the low rank tensor type representation. In this case, algebraic operations can be efficiently performed in tensor structured formats. Under moderate assumptions the storage and solution complexity of our approach is shown to depend only weakly (merely linear-logarithmically) on the frequency parameter $1/\epsilon$. The approach is well suited for applying the quntized-TT (QTT) tensor approximation to highly oscillating functions discretized on large d-dimensional $n \times n \ldots \times n$ tensor grids, leading to the logarithmic complexity scaling in the univariate grid size, $O(d \log^p n)$. We construct a spectrally equivalent preconditioner that allows the low rank tensor representation. Finally, the geometrically convergent PCG iteration accomplished with the adaptive rank truncation is shown to provide the accurate solution in the logarithmic time.

Raphael Kruse

Error analysis of a new randomized time-stepping method for nonlinear evolution equations with time-irregular coefficients

In this talk, we consider the numerical approximation of nonlinear and non-autonomous evolution equations of the form

$$u'(t) + A(t)u(t) = f(t), \qquad t \in (0,T),\ u(0) = u0,$$

where A and f may be discontinuous with respect to the time variable. In this non-smooth situation, it is notoriously difficult to construct numerical algorithms with a positive convergence rate. In fact, it can be shown that any deterministic algorithm depending only on point evaluations may fail to converge if, for instance, A and f only satisfy an L^2-integrability condition with respect to t. Instead, we propose to apply a randomized version of the backward Euler method to such time-irregular evolution equations. We obtain convergence rates with respect to the mean-square norm under considerably relaxed time regularity conditions. An important ingredient in the error analysis consists of a well-known variance reduction technique for Monte Carlo methods, the stratified sampling. We demonstrate the practicability of the new algorithm in the case of a fully discrete approximation of a more explicit parabolic PDE. This talk is based on joint work with Monika Eisenmann (TU Berlin), Mihály Kovaćs, and Stig Larsson (both Chalmers University of Technology).

Ulrich Langer

Functional type error control for stabilised space-time IgA approximations to parabolic problems

The paper is concerned with reliable space-time IgA schemes for parabolic initial-boundary value problems. We deduce a posteriori error estimates and investigate their applicability to space-time IgA approximations. Since the derivation is based on purely functional arguments, the estimates do not contain mesh dependent constants and are valid for any approximation from the admissible (energy) class. In particular, they imply estimates for discrete norms associated with stabilized space-time IgA approximations. Finally, we illustrate the reliability and efficiency of presented error estimates for the approximate solutions re-covered with IgA techniques on a model example.

Rui Ma

Nonconforming mixed simplicial and conforming mixed triangular prism elements for the linear elasticity problem

We propose two families of mixed finite elements for solving the classical Hellinger-Reissner mixed problem of the linear elasticity equations. First, we construct a new family of nonconforming mixed simplicial elements. Compared with those of nonconforming mixed simplicial elements in literature, the new stress shape function spaces have explicit and unique forms. Second, a family of conforming mixed triangular prism elements is constructed by product of elements on triangular meshes and elements in one dimension. The well-posedness is established for all elements with $k \geq 1$, which are of $k+1$ order convergence for both the stress and displacement. Besides, a family of reduced stress spaces is proposed by dropping the degrees of polynomial functions associated with faces.

Leszek Marcinkowski

Adaptive Domain Decomposition Method for Multiscale Problems in 3D

In our presentation we consider the second order elliptic problem with highly varying heterogeneous coefficients. We discuss variants of classical domain decomposition method, namely, overlapping Schwarz method with adaptive coarse spaces. The adaptive coarse space is constructed by adding a number of eigenfuctions of some carefully defined lower dimension interface based eigenvalue problems. We present theoretical results and some numerical tests. In case when the coarse space is large enough the convergence is independent of the variations of the contrast.

Stephanie Meier-Rohr

A Posteriori Error Estimation for Elliptic Homogenization Problems

We present an a posteriori error estimate for the approximation of an elliptic partial differential equation (PDE) with periodic coefficients via homogenization (see, e.g., [1], [2]). The arising homogenized PDE and the cell problem are numerically solved by a Galerkin finite element method. The fully discrete solution is not a Galerkin approximation of the original PDE and, hence, we consider majorants of functional type by the approach of [5]. In [6] a posteriori error majorants are derived for the error purely related to homogenization. In [7] combined a posteriori modelling-discretization errors are discussed purely related to variable coefficients. Here, we will present an error estimator for the fully discrete problem including the combined modelling-discretization majorant of the homogenized problem and the discretization majorant of the homogenized and cell problems (alternative approaches include, e.g., the a posteriori estimate of residual type for the heterogeneous multiscale discretizations (cf. [1])). Finally, we will present numerical experiments that illustrate the efficiency and sharpness of the majorants. This is joint work with Stefan Sauter and Sergey Repin.

[1] Abdulle, A. *A priori and a posteriori error analysis for numerical homogenization: a unified framework*. Ser. Contemp. Appl. Math. CAM, 16: 280–305, 2011.

[2] Cioranescu, D. and Donato, P. *An introduction to homogenization, volume 17 of Oxford Lecture Series in Mathematics and its Applications*. The Clarendon Press Oxford University Press, New York, 4: 1999.

[3] Jikov, V. V., Kozlov, S. M., and Oleinik, O. A. *Homogenization of differential operators and integral functionals*. Springer, 2012.

[4] Meier-Rohr, S. *A Posteriori Error Estimation for Elliptic Homogenization Problems*. PhD dissertation in preparation, 2017.

[5] Repin, S. *A posteriori estimates for partial differential equations*. Walter de Gruyter, Berlin, 2008.

[6] Repin, S., Samrowski, T., and Sauter, S. *Combined a posteriori modeling-discretization error estimate for elliptic problems with complicated interfaces*. ESAIM: Mathematical Modelling and Numerical Analysis, 46 (6): 1389–1405, 2012.

[7] Repin, S., Samrowski, T., and Sauter, S. *Estimates of the modeling error generated by homogenization of an elliptic boundary value problem*. Journal of Numerical Mathematics, 24(1): 1–15, 2016.

Christian Merdon

Pressure-robust mixed finite element methods for the Navier-Stokes equations

Inf-sup stable mixed finite element methods for the steady incompressible Stokes equations that relax the divergence constraint lead to a pressure-dependence in the velocity approximation that scales with the inverse of the viscosity and so gives rise to a locking phenomenon. A recently proposed variational crime is able to restore the pressure-robustness for a large class of classical finite element methods without compromising convergence rates and inf-sup stability. This talk explains the necessary modifications for the instationary (Navier–)Stokes equations, where in particular the velocity time derivative requires an improved space discretization. Semi discrete and fully discrete a priori velocity and pressure error estimates are derived and show beautiful robustness properties. Some numerical examples illustrate the findings and situations where pressure-robust methods are superior compared to their classical siblings. This is joint work with Alexander Linke and Naveed Ahmed.

Rüdiger Müller

A posteriori error analysis for coupled bulk-surface diffusion

We consider the finite element discretization of coupled diffusion in the bulk and on the surface of some domain. Due to the coupling, error estimates have to take into account the approximation errors related to the polyhedral domain approximation, in addition to discretization errors in space and time. We first study a stationary prototype problem and derive reliable and efficient a posteriori error control for adaptive FEM. Of particular interest is the case of only piecewise smooth domain boundaries and solutions of low regularity. Then, a posteriori error estimates for instationary and nonlinear problems are developed. We discuss in particular quasi-optimal $L_\infty(L^2)$ estimates and parameter robustness of the estimators.

Neela Nataraj

A posteriori error estimates for the finite element approximations of the von Karman equations

In this talk, we consider the von Karman plates that describe the bending of thin elastic plates defined on polygonal domains. Conforming and non-conforming finite element methods are employed to approximate the displacement and Airy stress functions. Reliable and efficient a posteriori error estimates are developed. This is a joint work with Prof. Carsten Carstensen and Dr. Gouranga Mallik.

Peter Oswald

Fem Divergence on distorted triangulations

The finite element method (FEM) is a workhorse for solving partial differential equations in a variational setting. FEM theory (approximation by piecewise polynomials) is well-established if the underlying domain partitions are well-shaped, e.g., locally quasi-uniform and regular. For triangular meshes in two dimensions, the celebrated maximum angle condition is sufficient for most purposes. This talk is about the necessity of this condition. Using the simplest case of lowest order conforming, nonconforming, and mixed FEM discretizations of the Poisson problem with analytic solutions over special sequences of distorted triangulations of the unit square, we show that these methods may converge arbitrarily slow or even not converge at all in the energy norm, depending on how much the maximum angle condition is violated. For conforming linear fi elements, this is due to the deterioration of the best approximation error while for the nonconforming P1 element the consistency error is the problem.

[1] P. Oswald. *Divergence of the FEM: Babuška-Aziz triangulations revisited*. Appl. Math. 60, 5 (2015), 473-484.

[2] P. Oswald. *Nonconforming P1 elements on distorted triangulations: Lower bounds for the discrete energy norm error*. submitted, INS Preprint No. 1618, Uni Bonn, 2016.

Dirk Pauly

Functional A Posteriori Error Estimates for Electro-Magneto Statics

We present a general Hilbert space theory for functional a posteriori error estimates applicable to first order systems of closed linear operators. As a main application we will discuss the first order systems of electro-magneto statics.

Philipp Petersen

Faithful projection of frames to bounded domains

Representation systems from applied harmonic analysis, such as wavelets, ridgelets, curvelets, or shearlets, have played a central role in many tasks in mathematical signal and image processing in the last couple of decades. Moreover, for instance wavelets, have been very successfully employed to discretize partial differential equations. Nonetheless, we observe a dichotomy between the theory and the applications in many of their applications since real world data, e.g., an image, is usually defined on bounded domains, whereas the systems mentioned above are mostly defined on a global domain. While for wavelet systems there are many constructions available which also yield bases or frames on bounded domains, such a construction is mostly unknown for other systems. In this talk we will demonstrate a quite general method to lift frame constructions from $\mathbb{R}^2$ to bounded domains by leveraging on existing wavelet frame constructions on such domains. One particular example is a shearlet frame which, with the help of our novel method, can be adapted to yield a frame on a bounded domain that admits a string of very beneficial properties. More specifically, this shearlet system is a frame for Sobolev spaces $H^s(\Omega)$ for suitable subsets Ω of $\mathbb{R}^2$. Additionally, it also admits very fast N-term approximations with respect to its primal and dual frame of functions that have curvilinear singularities. Finally, the system is efficiently computable by using well-established algorithms for the shearlet system on $\mathbb{R}^2$. On top of the theoretical results we shall demonstrate the potential of the newly developed frame for the adaptive solution of PDEs numerically. This is joint work with Massimo Fornasier, Philipp Grohs, Gitta Kutyniok, Jackie Ma, Mones Raslan, and Felix Voigtlaender.

Dirk Praetorius

Optimal convergence rates for adaptive FEM for compactly perturbed elliptic problems

We consider adaptive FEM for problems, where the corresponding bilinear form is symmetric and elliptic up to some compact perturbation. We prove that adaptive mesh-refinement is capable of overcoming the preasymptotic behavior and eventually leads to convergence with optimal algebraic rates. As an important consequence of our analysis, one does not have to deal with the a priori assumption that the underlying meshes are sufficiently fine. In particular, our analysis covers adaptive mesh-refinement for the Helmholtz equation from where our interest originated. The talk is based on joint work with Alex Bespalov (University of Birmingham, UK) and Alexander Haberl (TU Wien, Austria)

Mones Raslan

Adaptive solution of PDEs using boundary-adapted shearlets

One of the most important areas of applied mathematics is the numerical solution of partial differential equations (PDEs). In recent years a new approach to tackle this task emerged, using tools from harmonic analysis. For example, wavelet frames for the Sobolev space $H^s(\Omega)$ were developed as an important tool for solving elliptic PDEs defined on a bounded domain $\Omega \subset \mathbb{R}^2$. Unfortunately wavelets fail to provide good approximation rates when considering functions with curvilinear singularities. However, it was proven that shearlets exhibit optimal approximation rates for functions of this type. Unfortunately there are no straightforward constructions of pure shearlet systems on bounded domains. Therefore in this talk we present a frame for the Sobolev space $H^s(\Omega)$ which is mainly (but not fully) built out of shearlets. Since boundary conditions on the PDEs should be incorporated, the frame will also consist of wavelets adapted to the boundary of Ω. A discussion of approximation rates of the novel construction shall be underlined by numerical experiments which highlight the advantages of this new system, e.g. compared to pure wavelet systems. At the end of the talk we give an overview over the tasks that still lie ahead of us. This is joint work with Philipp Grohs, Gitta Kutyniok, Jackie Ma, and Philipp Petersen.

Sergey Repin

Reliable approximation method for elliptic problems with rank structured quasi-periodic coefficients (part 1)

This talk presents the first part of our joint work with Boris Khoromskii, in which we consider a new iteration method for solving a elliptic boundary value problems with periodic or quasi-periodic coefficients. The amount of problems covered by the method is much wider than in the classical homogenization method. Our method is based on using a "simplified" operator A_0 instead of the original operator A, which inversion is much simpler than the inversion of A. For a wide class of quasi-periodic coefficients we prove the contraction property and establish explicit estimates of the contraction factor. Moreover, we establish simple relations that suggest in a sense optimal A_0 and compute the corresponding contraction factor. Moreover, we deduce fully computable two-sided a posteriori estimates able to control the quality of approximations on any iteration. The method is well adapted to the case where the coefficients of A admit low rank representations.

Rita Riedlbeck

A posteriori error estimates for poro-mechanical problems

We present an a posteriori error estimate for poro-mechanical problems. The estimate is based on equilibrated reconstructions of the Darcy velocity and of the total stress tensor. Both reconstructions are obtained from mixed finite element solutions of local Neumann problems posed over patches of elements around mesh vertices. The Darcy velocity is reconstructed using Raviart–Thomas finite elements and the stress tensor using Arnold–Falk–Winther finite elements with a weak symmetry constraint.

Michele Ruggeri

isubsection*A convergent second-order implicit-explicit tangent plane scheme for the Landau-Lifshitz-Gilbert equation

We consider the numerical approximation of the Landau-Lifshitz-Gilbert (LLG) equation, which describes the dynamics of the magnetization in ferromagnetic materials. The numerical integration of the LLG equation poses several challenges: strong nonlinearities, a nonconvex pointwise constraint, an intrinsic energy law, and the presence of nonlocal field contributions, which prescribe the coupling with other partial differential equations (PDEs). We extend the tangent plane scheme from [1] and propose an algorithm in which all the lower-order contributions, e.g., the expensive-to-compute stray field, are treated explicitly in time by using an Adams-Bashforth approach [2]. The numerical integrator is computationally attractive in the sense that, after the first time-step, only one linear system has to be solved per time-step. Despite this modification, the resulting scheme still fulfills a formal second-order convergence in time. Under appropriate assumptions, the convergence towards a weak solution of the problem is unconditional, i.e., no CFL-type condition on the time-step size and the spatial mesh size is needed for the stability of the scheme. One particular focus is on the efficient treatment of coupled systems, e.g., the coupling of the LLG equation with the eddy current equation, for which we show that decoupling the time integration of the two PDEs is very attractive in terms of computational cost and still leads to time-marching algorithms that are (unconditionally) convergent and of second-order in time. Numerical experiments underpin our theoretical findings and demonstrate the applicability of the method for the simulation of practically relevant problem sizes. This is joint work with C.-M. Pfeiler, D. Praetorius, and B. Stiftner (TU Wien).

[1] F. Alouges, E. Kritsikis, J. Steiner, and J.-C. Toussaint. Numerische Mathematik, 128, 407-430 (2014).

[2] C.-M. Pfeiler, D. Praetorius, M. Ruggeri, and B. Stiftner. in preparation (2017).

Mira Schedensack

Low-order non-conforming finite elements for the 3D Stokes equations

In this talk, generalizations of the two dimensional nonconforming method of Kouhia and Stenberg are discussed. That method has one conforming and one nonconforming component. The generalization with one conforming and two non-conforming components does not satisfy a discrete Korn inequality, while the generalization with two conforming and one nonconforming components does not fulfill a discrete inf-sup condition. A stabilization with P2 functions or with face bubble functions in one component lead to a stable discretization.

Ian Smears

Guaranteed, locally space-time efficient, and polynomial-degree robust a posteriori error estimates for high-order discretizations of parabolic problems

We present a posteriori error estimates in parabolic energy norms for space-time discretisations based on arbitrary-order conforming FEM in space and discontinuous Galerkin methods in time. Using the heat equation as a model problem, we show a posteriori error estimates in a norm of $L^2(H^1) \cap H^1(H^{-1})$-type that is suitably extended to functions of the nonconforming discrete space. The estimators give guaranteed upper bounds on the error, and locally space-time efficient lower bounds. Furthermore, the efficiency constants are robust with respect to the discretisation parameters, including the polynomial degrees in both space and time, and also with respect to refinement and coarsening between time-steps, thereby removing the need for the transition conditions required in earlier works. This is joint work with Alexandre Ern and Martin Vohralik (ENPC and Inria).

Ernst P. Stephan

Higher order FEM for the obstacle problem of the p-Laplacian

We consider two higher order finite element discretizations of an obstacle problem with the p-Laplacian differential operator for $p \in (1, \infty)$. The first approach is a nonlinear variational inequality in the primal variable u only. The second formulation is a primal-dual mixed formulation where the dual variable represents the signed residual of the variational inequality from the first approach. These two formulations are equivalent and, under mild assumptions on the obstacle, even on the discrete level when using biorthogonal basis functions. We prove a priori error estimates as well as a general a posteriori error estimate which is valid for both formulations. We present numerical results on the improved convergence rates of adaptive schemes (mesh size adaptivity with and without polynomial degree adaptation) for the singular case $p = 1.5$ and the degenerated case $p = 3$. We also present numerical results on the mesh independency and on the polynomial degree scaling of the discrete inf-sup constant when using biorthogonal basis functions for the dual variable defined on the same mesh with the same polynomial degree distribution. This is joint work with Lothar Banz (Salzburg) and Bishnu Lamichhane (Newcastle).

Johannes Storn

Asymptotic exactness of the least-squares finite element residual

The discrete minimal least-squares functional $LS(f;U)$ is equivalent to the squared error $||u-U||^2$ in first-order div least-squares finite element methods and so leads to an embedded reliable and efficient a posteriori error control. This presentation enfolds a spectral analysis to prove that this natural error estimator is asymptotically exact in the sense that the ratio $LS(f;U)/||u-U||^2$ tends to one as the underlying mesh-size tends to zero for all kinds of conforming discretizations. A priori knowledge about the continuous and the discrete eigenspectrum allows for the computation of a guaranteed error bound with a reliability constant smaller than that from the equivalence constants. The abstract analysis applies to the Poisson model problem, the Helmholtz equation, the linear elasticity, and the time-harmonic Maxwell equations. This is joint work with Carsten Carstensen.

Steffen Weißer

The dual-weighted residual estimator realized on polygonal meshes

In recent years the use of polygonal and polyhedral meshes for the discretization of boundary value problems increased. One of the promising features is the flexibility of the element shapes in the discretization. Especially in mesh adaptivity, the use of polygonal (2D) and polyhedral (3D) discretizations is very promising. When refining elements locally, there is no need for post-processing in order to maintain the mesh admissibility and there is no need to handle hanging nodes explicitly. In this presentation, the classical residual-based error estimator is revised on polygonal meshes. Furthermore, goal-oriented local mesh adaptivity and a posteriori error estimation is considered employing the BEM-based FEM as discretization strategy. It turns out that the dual solution, involved in the dual-weighted residual estimator, can be processed in a new fashion on polygonal meshes. Instead of using patched meshes, it is possible to perform the post-processing on a single element in order to treat the dual solution. The theoretical achievements and algorithmic developments are substantiated with the help of several numerical experiments.

Shuo Zhang

Order reduced methods for fourth order problems

Many model problems in applied sciences are in the formulation of fourth order problems. Their discretizations have been drawing wide interests. In this talk, some recent progress on designing and implementing order reduced methods for fourth order problems by the speaker and his collaborators will be presented. First, a routine framework will be given which transforms the primal fourth order problem to a stable system on a series of low order spaces. The well-posedness of the generated system and the equivalence between the primal formulation and the order reduced formulation can be guaranteed provided mild hypotheses. This way, the framework works for various fourth order problems. The order reduced formulation admits discretisation with low-regularity finite element spaces, and convenience may thus be expected. Beside the ease on utilizing popular finite element packages, it can be seen that the order reduced formulation is friendly to designing fast solvers. Then, some examples are given to illustrate the implementation of the framework. A series of examples on the low-regularity discretisation of boundary value problems about several typical fourth order operators are discussed, finite element schemes are constructed for each of them, and optimal convergence rates are obtained. A second series of examples are on fourth order eigenvalue problems. With the low-regularity finite element spaces utilised, nested discretizations can be admitted. We will focus on, beyond the schemes themselves, the designing and implementation of associated multilevel method and their optimal efficiency. Examples include linear and nonlinear, and self-adjoint and non-self-adjoint problems. Some related topics such as, e.g., adaptivity can also be mentioned if time permits.