Optimisation in Electricity Pool Markets

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Summary

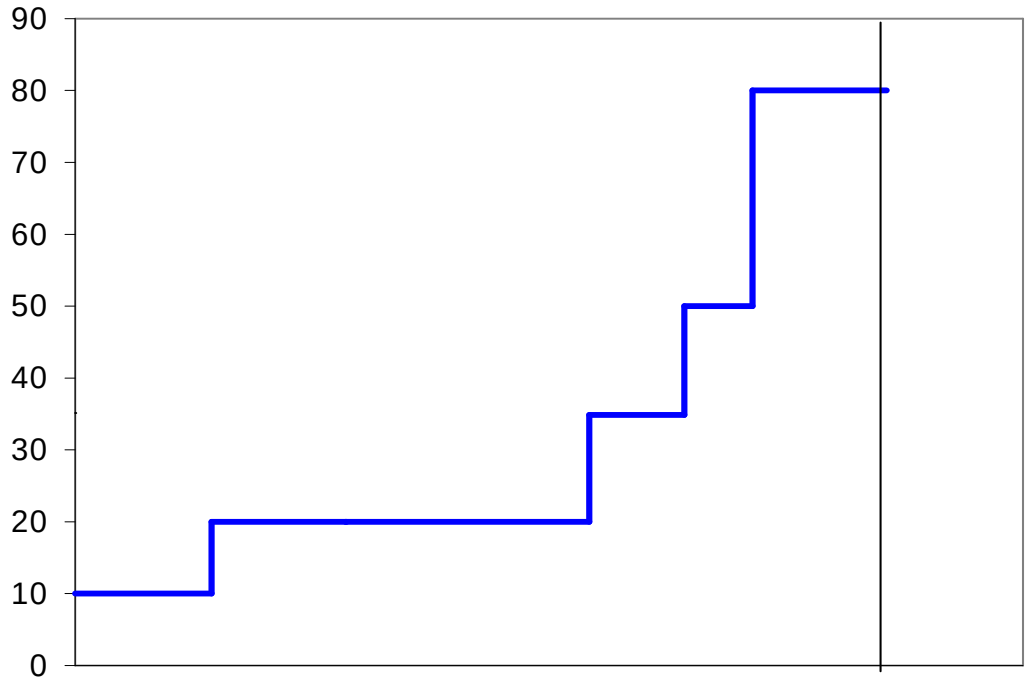
- Wholesale electricity market pools
- Optimizing electricity offers in oligopolies
- Nash-Equilibrium models
- Optimizing electricity offers in practice

Wholesale electricity pool markets

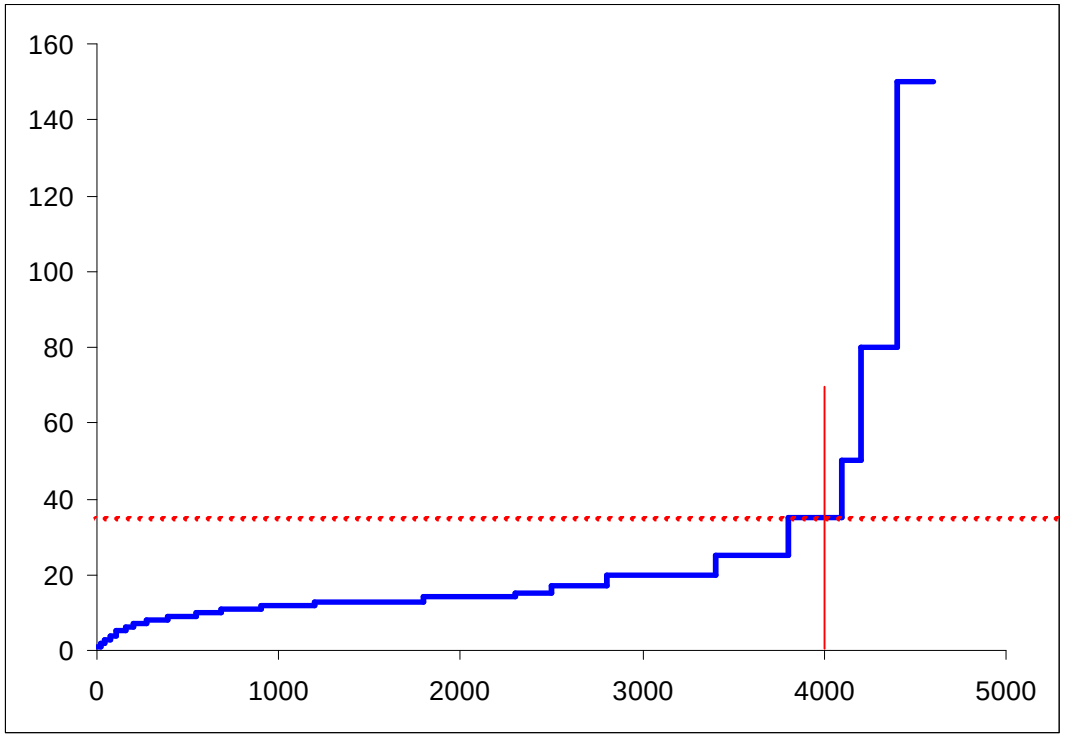
Each trading period:

- generators submit offers of power and prices that they wish to be paid per MW for dispatch;
- a merit order for dispatch is based on offer prices;
- last offer dispatched (partially) sets the system marginal price.

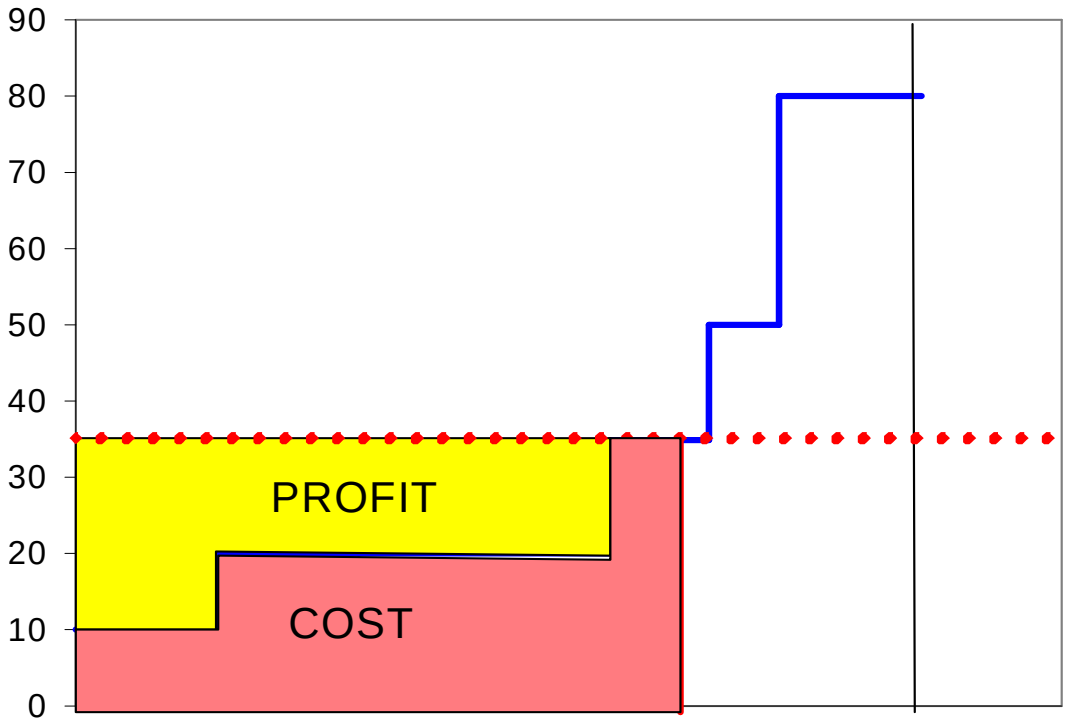
Offer stacks are piecewise constant functions



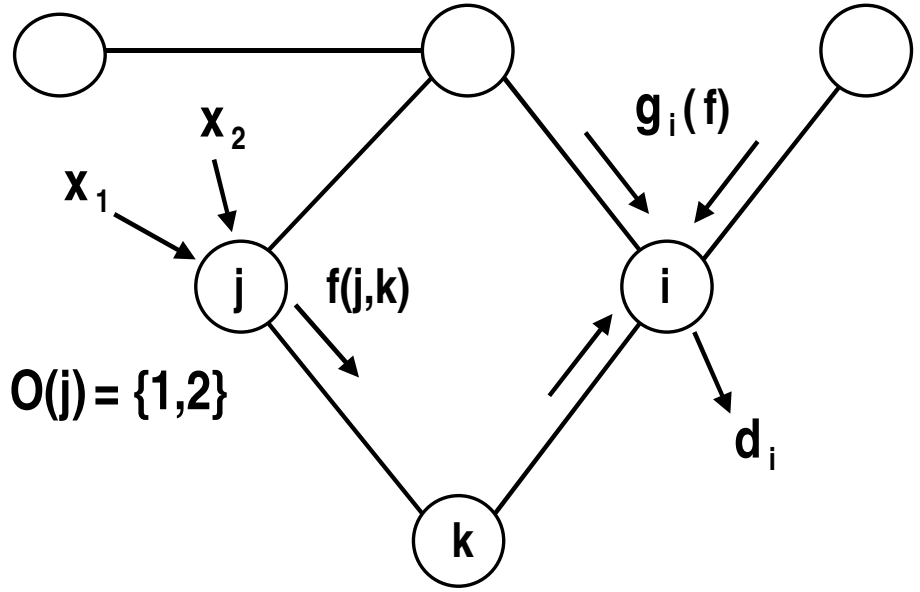
Industry offer stack



Dispatch of generator's offer stack



Pricing model in a transmission network



P: minimize $\sum_i \sum_{t \in O(i)} c_t x_t$

subject to $g_i(f) + \sum_{t \in O(i)} x_t = d_i, \quad i = 1, 2, \ldots, n$

$(x, f) \in X$

Contracts

A *contract for differences* (or hedge contract) for a quantity Q at a strike price f pays the issuer of the contract $Q(f - p)$ where p is the spot price.

If issuing a contract, the generator seeks to

$$\text{maximize } R(q, p) = \ qp - C(q) + Q(f - p).$$

Issuing a hedge contract can influence offering behaviour.

What is the optimal offer stack ?

- For simplicity focus on the single-node case where $Q = 0$.
- In perfect competition generators cannot influence the clearing price.
- In an oligopoly generators can influence the price by increasing their offer price or withholding supply.
- There are uncertainties in:

  the competitor's behaviour;

  the load;

  the cost of generating resources.

Oligopoly - Supply function models

- Generator chooses supply function to maximize its profit.
- Assume the rest-of-market offers a combined stack $T(x)$ giving their marginal offer price for quantity x.
- Inverse of $T(x)$ is the supply function $S(p) =$ amount rest-of-market offers at price p.
- Supply function equilibria with elastic demand first investigated by Klemperer and Meyer, 1989.
- Used extensively in electricity market sudy of England and Wales by Green and Newbery, 1992,...

Optimal supply function for a generator ("us")

p	clearing price
$S(p)$	supply function of rest of market
q	amount we are dispatched
$D(p)+h$	demand function plus random "shock" h
$D(p)+h-S(p)$	demand we must meet
$C(q)$	cost of generating q

For each h choose $p=p(h)$ so as to maximize

$$R=p(D(p)+h-S(p))-C(D(p)+h-S(p)).$$

Under appropriate conditions $(p(h), D(p)+h-S(p(h))$ defines a non-decreasing parameterized curve defining our optimal supply function response to the rest of the market .

Example 1: Rest of market offers S(p)=p

$$\begin{aligned} S(p) &= p \\ C(q) &= 0.1q^2 \\ D(p) &= 10 - p \end{aligned}$$

$$R(p) = p(10 - p + h - p) - 0.1(10 - p + h - p)^2$$

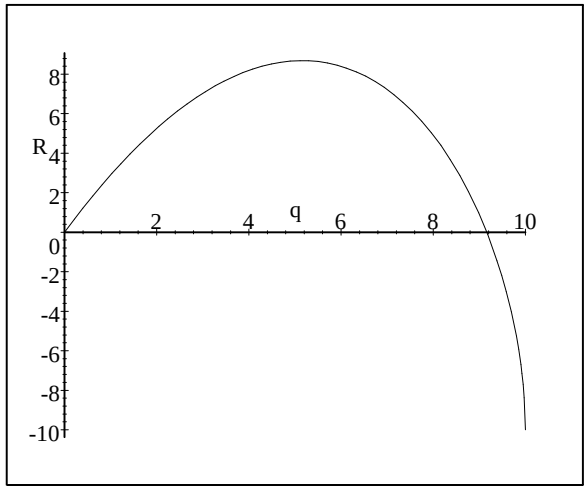
$$p = \frac{35}{12} + \frac{7}{24}h$$

$$q = \frac{25}{6} + \frac{5}{12}h$$

We offer $S^*(p) = \frac{10}{7}p$

Example 2: Inelastic demand

$$
\begin{aligned}
S(p) &= p^2 \\
D(p) &= 0 \\
T(q) &= q^{\frac{1}{2}} \\
C(q) &= 0.1q^2 \\
R(q) &= q(h-q)^{\frac{1}{2}} - 0.1q^2
\end{aligned}
$$



R as a function of q ($h = 10$)

Example 2 (continued)

$$R = q(h-q)^{\frac{1}{2}} - 0.1q^2$$

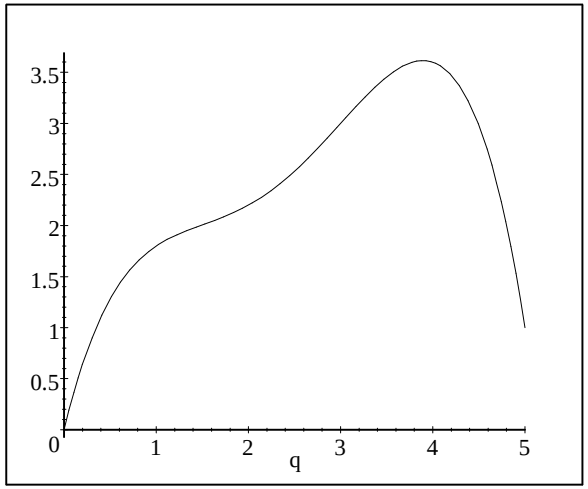
$h = 2$	$q = 1.190$	$p = (h-q)^{\frac{1}{2}} = 0.90$
$h = 4$	$q = 2.268$	$p = (h-q)^{\frac{1}{2}} = 1.31$
$h = 6$	$q = 3.279$	$p = (h-q)^{\frac{1}{2}} = 1.65$
$h = 8$	$q = 4.237$	$p = (h-q)^{\frac{1}{2}} = 1.94$
$h = 10$	$q = 5.154$	$p = (h-q)^{\frac{1}{2}} = 2.20$

We offer $S^*(p) = \ldots$

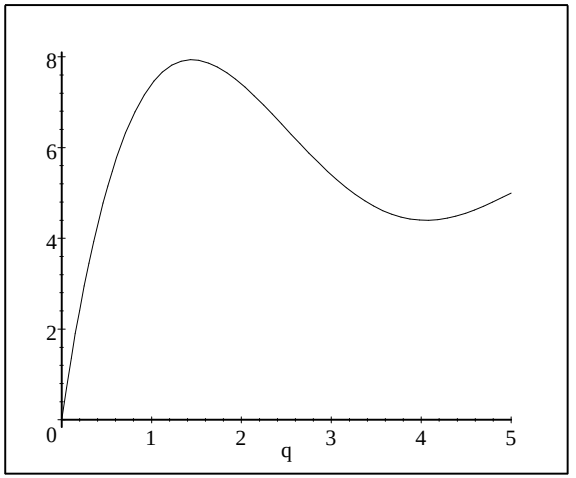
Example 3

$$
\begin{aligned}
S(p) &= (10(p-1))^{\frac{1}{3}} \\
T(x) &= 0.1x^3 + 1 \\
C(\cdot) &= 0 \\
R(q) &= (0.1(h-q)^3 + 1)q
\end{aligned}
$$

Choose q so as to maximise $R(q)$



$R(q)$ for $h = 3$



$R(q)$ for $h = 5$

Theorem [Anderson & Philpott, 2001]

Assume

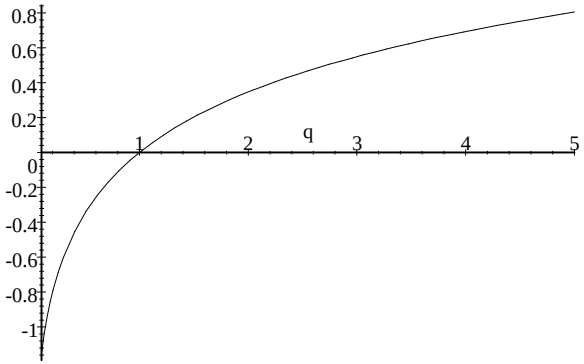
$C(q)$	is an increasing convex function;
$S(p) - D(p)$	has a log concave inverse.

Suppose

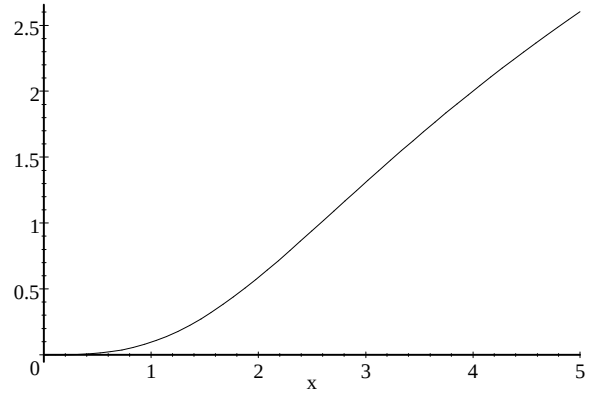
q_1 is the optimal offer for demand shock h_1 (giving clearing price p_1);
q_2 is the optimal offer for demand shock h_2 (giving clearing price p_2).

If $h_1 < h_2$ then $q_1 \leq q_2$ and $p_1 \leq p_2$.

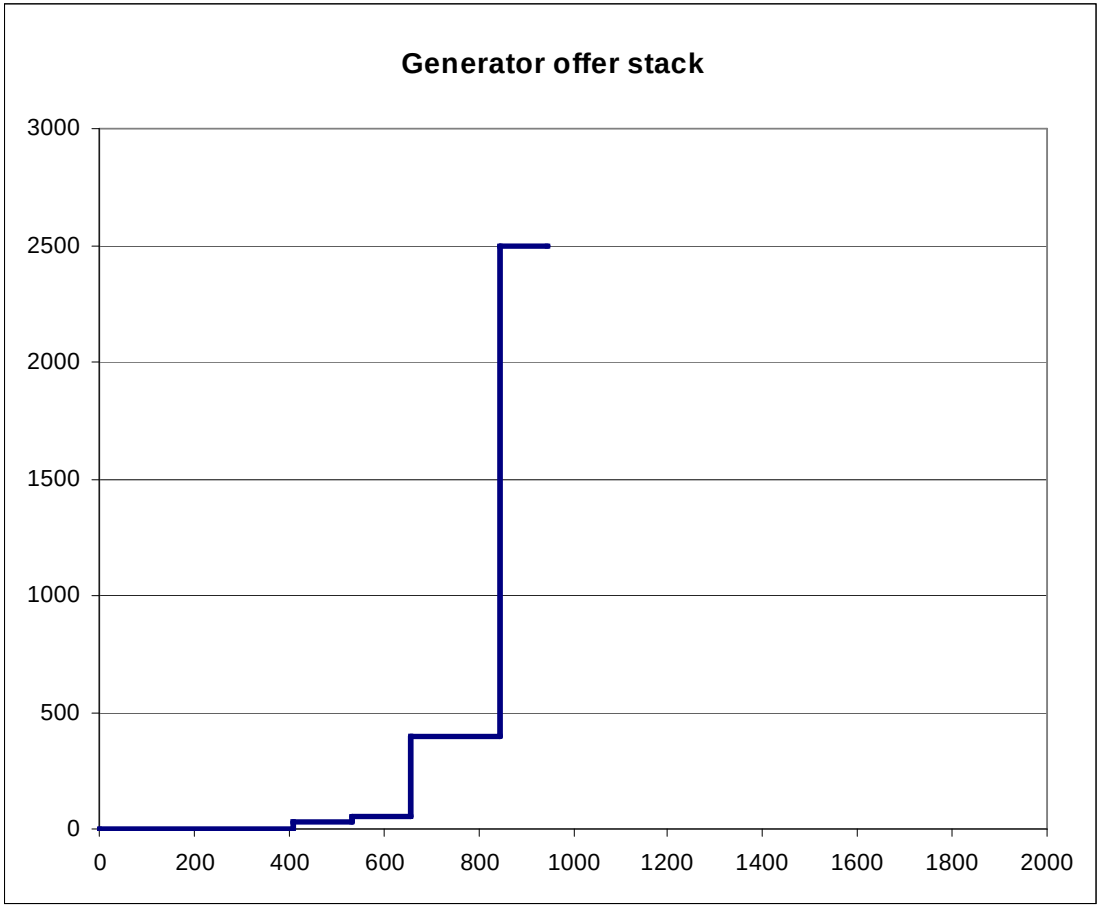
One of these curves is not concave

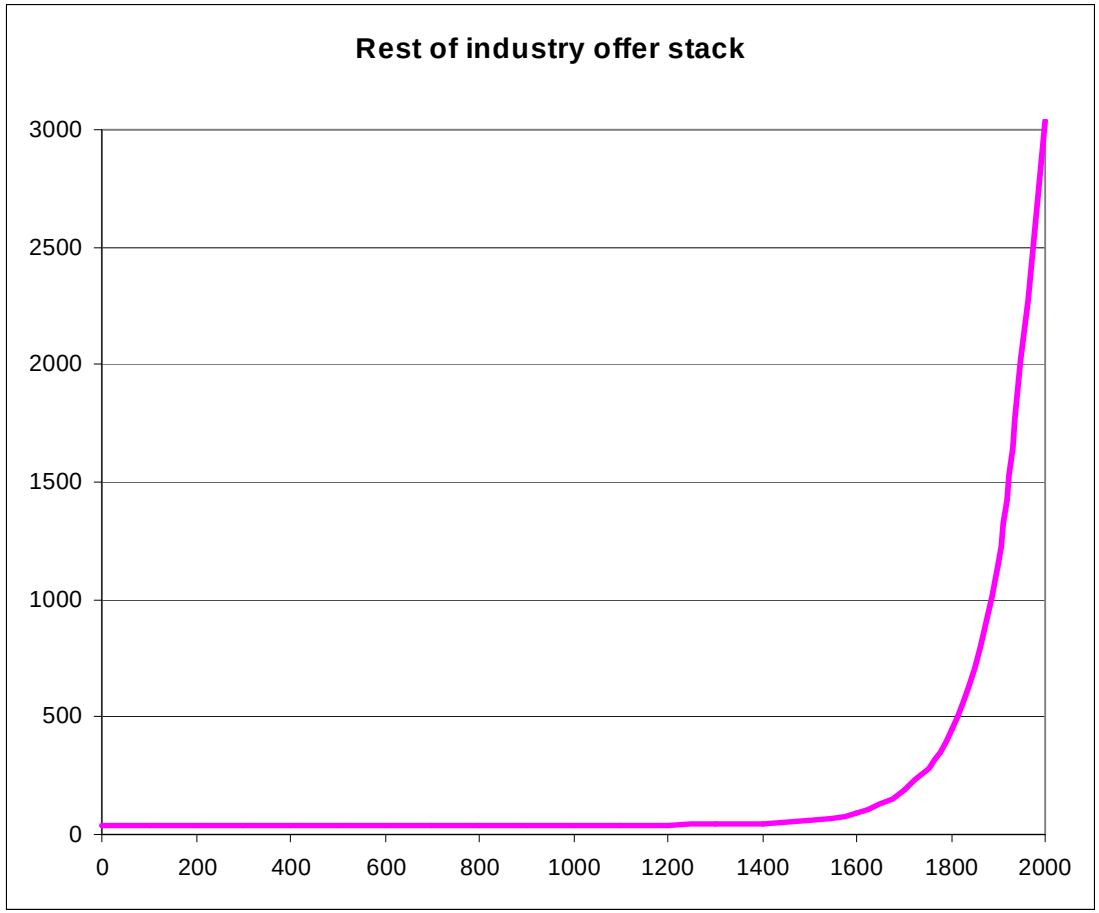


Example 2: Plot of $\log(q^{\frac{1}{2}})$



Example 3: Plot of $\log(0.1x^3 + 1)$


Generator offer stack
3000
2500
2000
1500
1000
500
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000


Rest of industry offer stack
3000
2500
2000
1500
1000
500
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000

Supply functions to maximize expected return

$q(p)$	amount we offer to the market
p	clearing price
$S(p)$	rest of market supply function
$D(p)$	demand function
$C(q)$	cost of generating
h	demand shock (uncertain)
$F(h)$	Pr(demand shock $\leq h$)
$R(q,p)$	$pq - C(q)$

If the demand shock is h, and we offer the curve $q(p)$ then the clearing price p will satisfy

$$q(p) + S(p) = D(p) + h.$$

The expected return is

$$E[R(q(p),p)] = \int R(q(p),p) dF(q(p) + S(p) - D(p)).$$

The market distribution function

[Anderson and Philpott, 2001]

Definition: $\psi(q,p) =$ the probability of not being fully dispatched if we make a single offer of q at price p.

$$\begin{aligned}\psi(q,p) &= \mathsf{Pr}(h < q + S(p) - D(p)) \\ &= F(q + S(p) - D(p)).\end{aligned}$$

Suppose we offer the curve $q(p)$. The expected return is the integral

$$E[R(q(p),p)] = \int_0^\infty R(q(p),p) d\psi(q(p),p).$$

Behaviour of Optimal Supply Curves

Let

$$Z(q,p) = \frac{\partial R}{\partial q}\frac{\partial \psi}{\partial p} - \frac{\partial R}{\partial p}\frac{\partial \psi}{\partial q}.$$

If $Z(q,p)$ is continuous then:

- An optimal curve can have $0 < q'(p) < \infty$ only where $Z(q,p) = 0$;
- The rest of the optimal curve is made up of horizontal and vertical sections.

Nash equilibria in symmetric duopoly

Suppose the rest of the market offers $S(p)$, and we offer the curve $q(p)$.
If the demand shock h has a probability density function f then

$$\psi(q,p) = F(q + S(p) - D(p)) = \int_{-\infty}^{q+S(p)-D(p)} f(h)dh.$$

$$\begin{aligned} Z(q,p) &= \frac{\partial R}{\partial q}\frac{\partial \psi}{\partial p} - \frac{\partial R}{\partial p}\frac{\partial \psi}{\partial q} \\ &= (p - C'(q(p)))(S'(p) - D'(p))f(q(p) + S(p) - D(p)) \\ &\quad -q(p)f(q(p) + S(p) - D(p)). \end{aligned}$$

$$Z(q,p) = 0 \Rightarrow (p - C'(q(p)))(S'(p) - D'(p)) = q(p).$$

For symmetric equilibrium $q(p) = S(p)$ so

$$S'(p)(p - C'(S(p))) = S(p) + D'(p)(p - C'(S(p))).$$

Supply-function equilibrium with elastic demand

[Klemperer and Meyer, 1989]

- Assume two identical generators at the same node. Suppose $C(\cdot)$ is an increasing differentiable convex function, $D(\cdot)$ is a decreasing differentiable concave function.

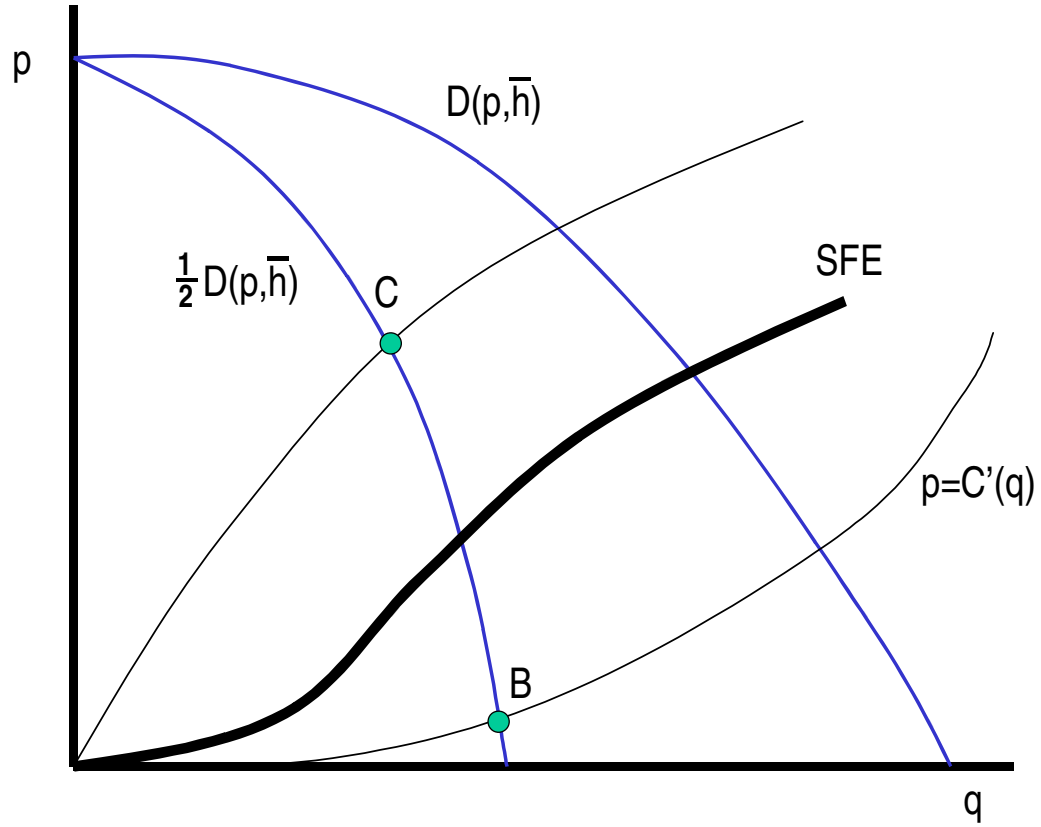
- If $h = 0$ (no uncertainty) then there are an infinite number of symmetric SFE passing through equilibrium point.

- If h has density $f(h) > 0$ everywhere on $[h_0, \infty]$ then there is a symmetric SFE which solves

$$S'(p)(p - C'(S(p)) = S(p) + D'(p)(p - C'(S(p)).$$

- Result can be extended to n firms.

Supply-function equilibrium with elastic demand



Given SFE, fix $h = \bar{h}$: clearing price is between Bertrand (B) and Cournot (C) equilibria given that generators optimise using $D(p, \bar{h})$.

Supply-function equilibrium with inelastic demand

Suppose n generators at the same node have identical convex cost functions $C(\cdot)$, and the maximum demand realization at the node is H. Let a be an arbitrary constant satisfying

$$a \geq (\frac{H}{n})^{1-n} C'_+(\frac{H}{n}).$$

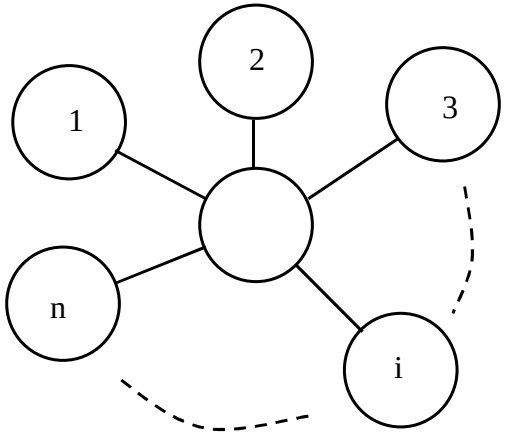
Then the situation in which each generator offers in the function

$$T(q) = q^{n-1}[a + (n-1)\int_q^{\frac{H}{n}} x^{-n} C'_+(x)dx]$$

is a symmetric Nash equilibrium.

[Rudkevich, Duckworth and Rosen, 1998, Anderson & Philpott, 2001]

Equilibrium in a transmission network



Let all the demand occur at the centre, and assume n identical generators with constant costs. Suppose to receive x at centre we must send $x + bx^2$ from node i. Then a Nash equilibrium occurs when each generator offers in the function

$$T(q) = \frac{aq^{n-1}}{(1+4bq)^{\frac{n}{2}}}.$$

Equilibrium in a transmission network

- Supply-function equilibrium results for more general transmission networks are difficult to obtain.

- With line losses it is advantageous to generate at a node that is a net importer of energy, so a symmetric equilibrium is unlikely.

- Special cases can be found computationally [Hobbs, Metzler, Pang, 1999].

- Nash-Cournot models for generators in transmission systems have been developed and solved as complementarity problems [Wei and Smeers, 1999, Hobbs, 1999].

Optimising expected return in practice

Instead of supply curve $q(p)$ generators submit an offer stack

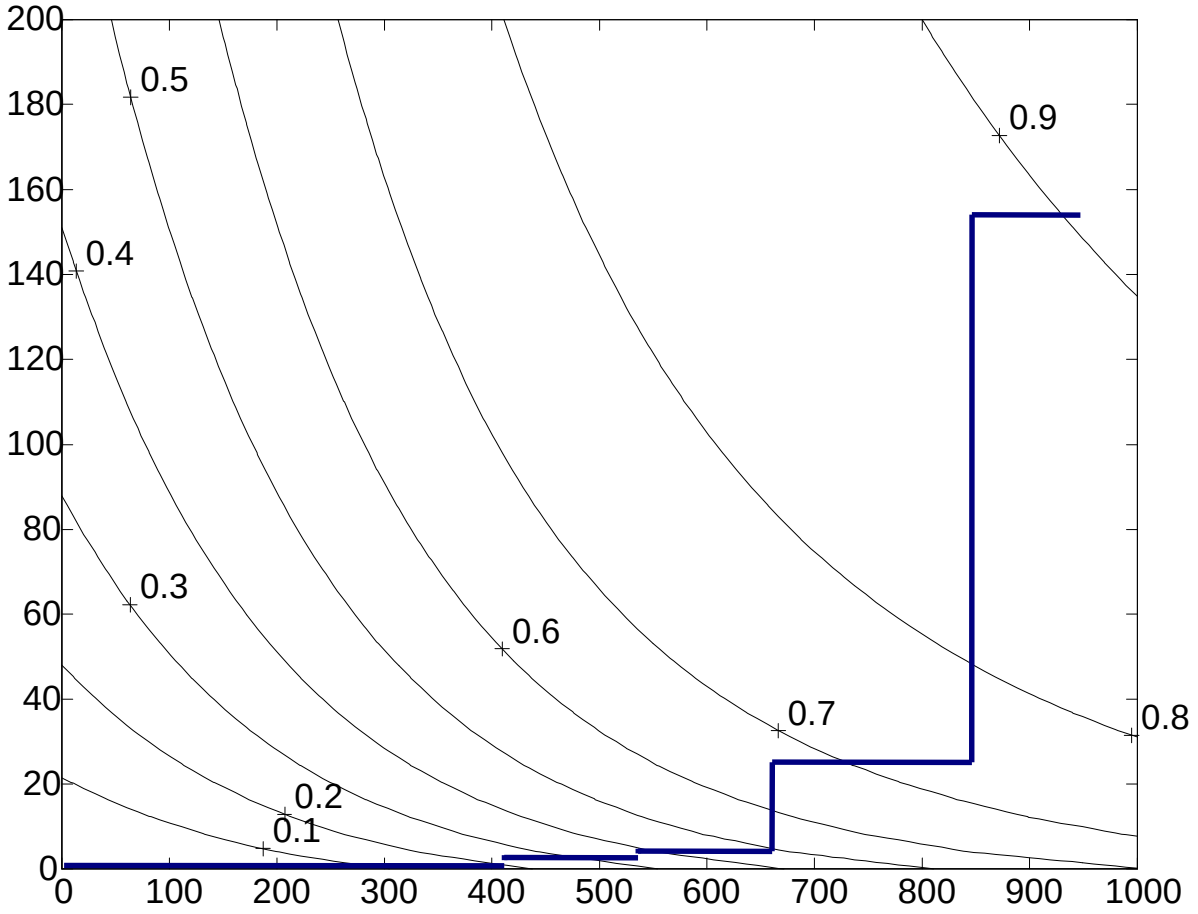
$$\mathcal{S} = \{(x(t), y(t)), \quad t \in [0, T]\}.$$

($q = x(t)$ and $p = y(t)$ are increasing piecewise smooth functions.)

Given ψ the expected return is the line integral

$$\begin{aligned} E[R] &= \int_{\mathcal{S}} R(q,p) d\psi(q,p) \\ &= \int_{\mathcal{S}} R(q,p)\psi_q dq + R(q,p)\psi_p dp \\ &= \int_{\mathcal{S}} R(x(t),y(t))\psi_q x'(t)dt + R(x(t),y(t))\psi_p y'(t)dt. \end{aligned}$$

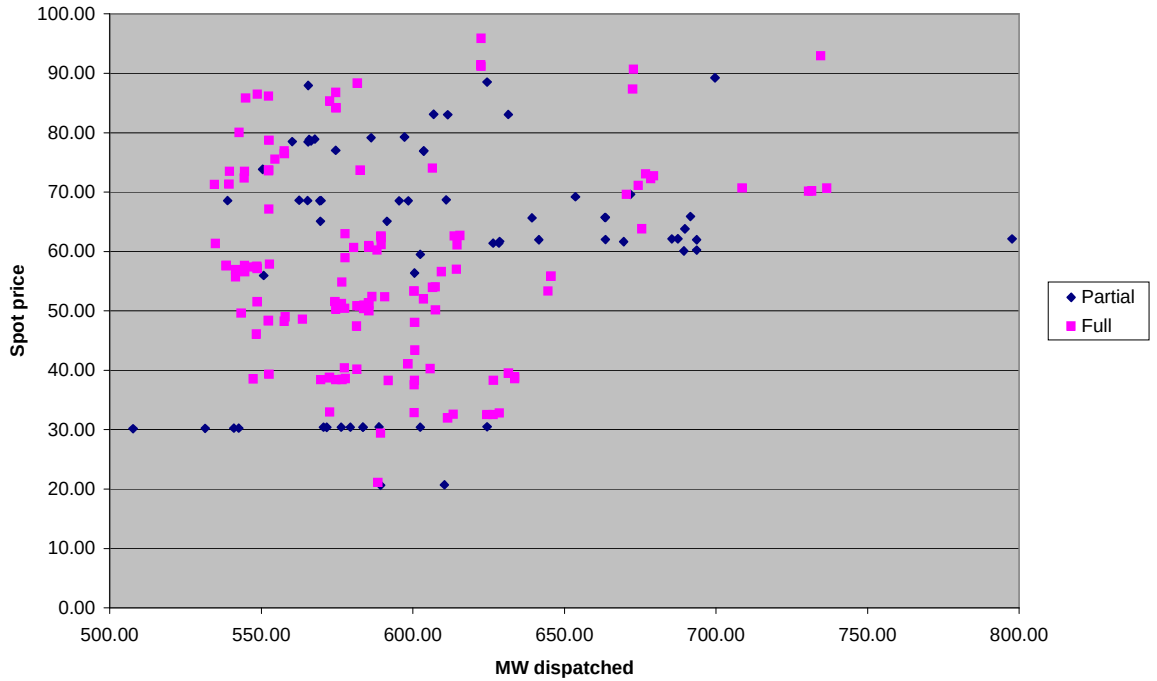
Market Distribution Function $\psi(q, p)$



Bayesian estimation of ψ

Inputs:	Family of ψ functions	$\{\psi^\alpha, \quad \alpha \in A\}$
	Prior density	$\phi_0 : A \to [0,1]$
	Observed dispatches	$\{(q_i, p_i) \mid i \in D\}$ $\{(q_j, p_j) \mid j \in E\}$
Outputs:	Posterior density	$\phi_1 : A \to [0,1]$

Example dispatches



Electricity dispatches in February between 10am and 4pm weekdays

Likelihood function

$D_i(\delta) =$ " generator is partially dispatched in $([q_i, q_i + \delta), p_i)$".

$$\begin{aligned} \Pr[D_i(\delta)|\alpha] &= \psi^\alpha(q_i + \delta, p_i) - \psi^\alpha(q_i, p_i) + o(\delta) \\ &= \delta\psi^\alpha_q(q_i, p_i) + o(\delta) \end{aligned}$$

$E_j(\delta) =$ " generator is fully dispatched in $(q_j, [p_j, p_j + \delta))$".

$$\begin{aligned} \Pr[E_j(\delta)|\alpha] &= \psi^\alpha(q_j, p_j + \delta) - \psi^\alpha(q_j, p_j) + o(\delta) \\ &= \delta\psi^\alpha_p(q_j, p_j) + o(\delta) \end{aligned}$$

Posterior calculation

Posterior density $\phi_1(\alpha)$ after a partial dispatch (q_i, p_i) is

$$\begin{aligned}\phi_1(\alpha) &= \lim_{\delta \to 0} \Pr[D_i(\delta)|\alpha] \frac{\phi_0(\alpha)}{\Pr[D_i(\delta)]} \\ &= \frac{\phi_0(\alpha)\psi_q^\alpha(q_i, p_i)}{\int_A \phi_0(\alpha)\psi_q^\alpha(q_i, p_i)d\alpha}.\end{aligned}$$

Posterior density $\phi_1(\alpha)$ after a full dispatch (q_j, p_j) is

$$\begin{aligned}\phi_1(\alpha) &= \lim_{\delta \to 0} \Pr[E_j(\delta)|\alpha] \frac{\phi_0(\alpha)}{\Pr[E_j(\delta)]} \\ &= \frac{\phi_0(\alpha)\psi_p^\alpha(q_j, p_j)}{\int_A \phi_0(\alpha)\psi_p^\alpha(q_j, p_j)d\alpha}.\end{aligned}$$

$$\phi_1(\alpha) \propto \prod_{i \in D} \psi_q^\alpha(q_i, p_i) \prod_{j \in E} \psi_p^\alpha(q_j, p_j)\phi_0(\alpha).$$

Example

Suppose that if generator offers q at price 0 then the clearing price $P(q)$ has a log normal distribution, so

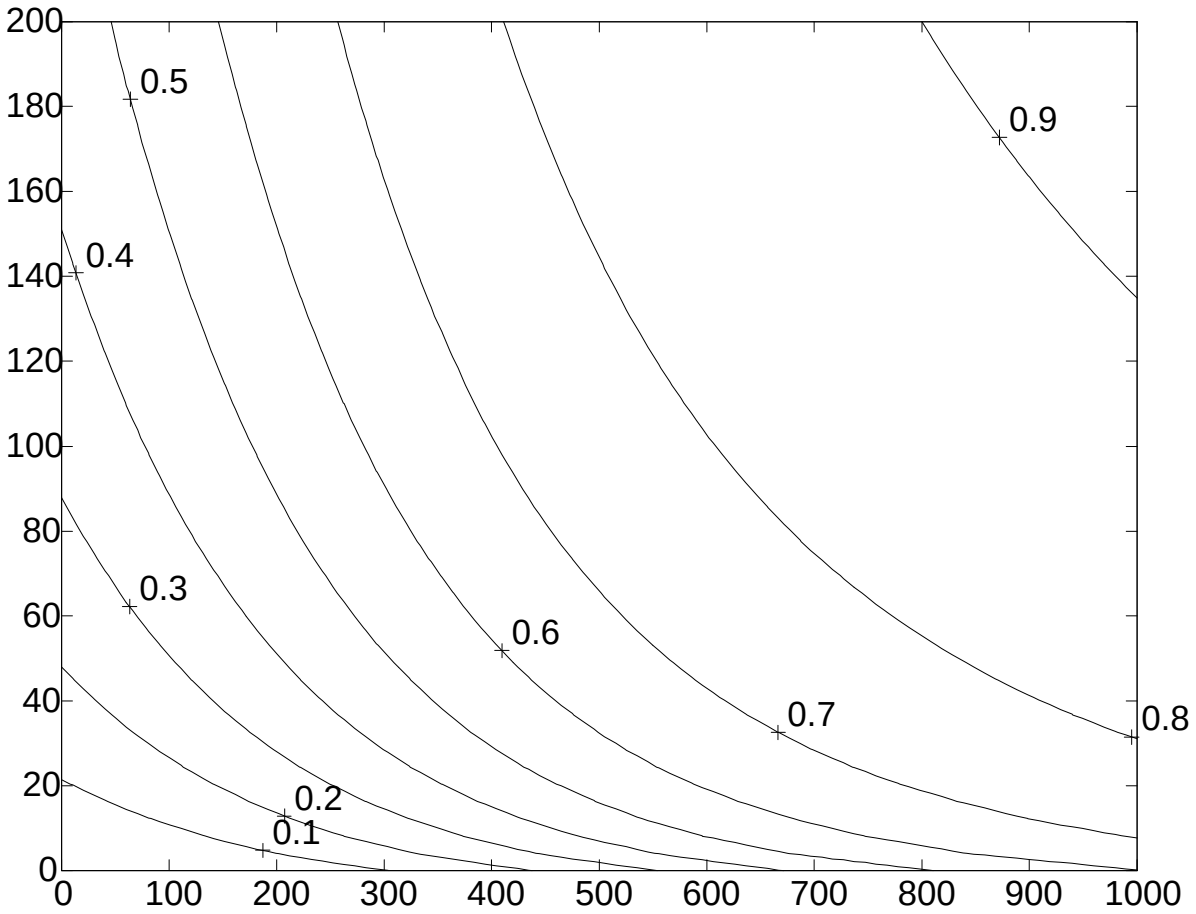
$$\log P(q) \backsim N(\beta - \alpha q, \sigma^2).$$

$$\begin{aligned} \psi^{\alpha,\beta}(q,p) &= \Pr[P(q) < p] \\ &= (2\pi\sigma^2)^{-\frac{1}{2}} \int_{-\infty}^{\log p} e^{-\frac{(z-\beta+\alpha q)^2}{2\sigma^2}} dz. \end{aligned}$$

Given ϕ defined on $(\alpha, \beta) \in A$,

$$\psi(q,p) = (2\pi\sigma^2)^{-\frac{1}{2}} \int_A \int_{-\infty}^{\log p} e^{-\frac{(z-\beta+\alpha q)^2}{2\sigma^2}} \phi(\alpha,\beta) dz d\alpha d\beta.$$

Example: Uniform prior gives this ψ



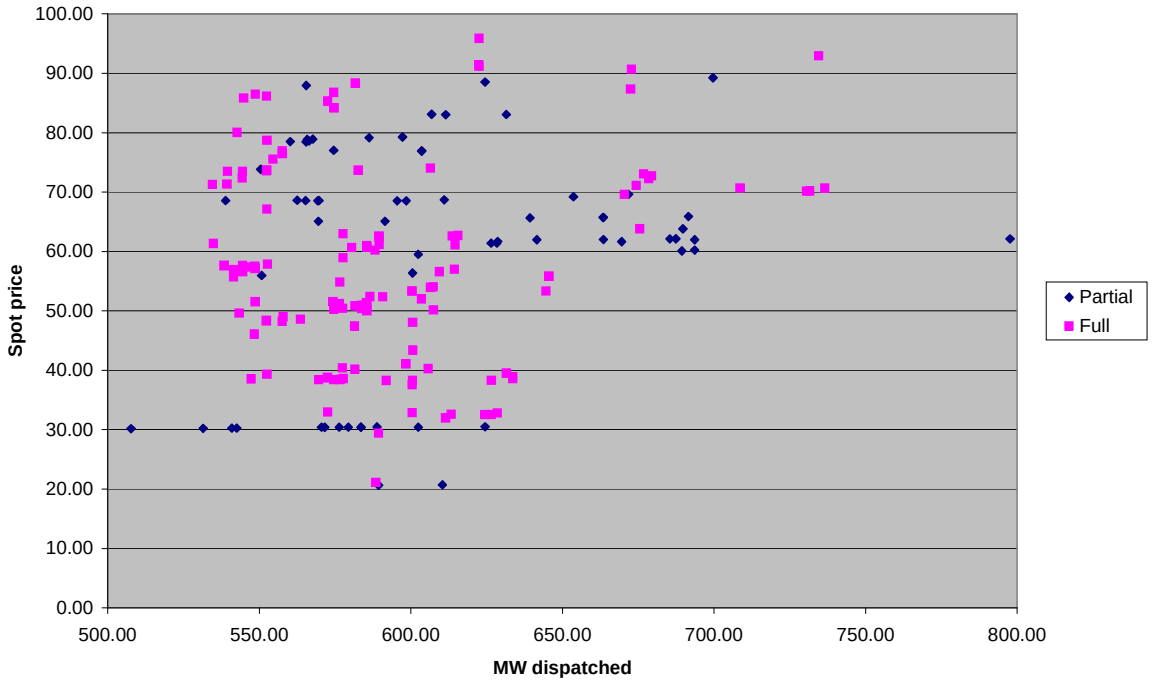
Posterior calculation

$$\psi_p^{\alpha,\beta} = (2\pi\sigma^2)^{-\frac{1}{2}}(1/p)e^{-\frac{(\alpha q-\beta+\log p)^2}{2\sigma^2}}$$

$$\begin{aligned}\psi_q^{\alpha,\beta} &= (2\pi\sigma^2)^{-\frac{1}{2}}\int_{-\infty}^{\log p}\frac{\partial}{\partial q}e^{-\frac{(z-\beta+\alpha q)^2}{2\sigma^2}}dz \\ &= (2\pi\sigma^2)^{-\frac{1}{2}}\alpha e^{-\frac{(\alpha q-\beta+\log p)^2}{2\sigma^2}}\end{aligned}$$

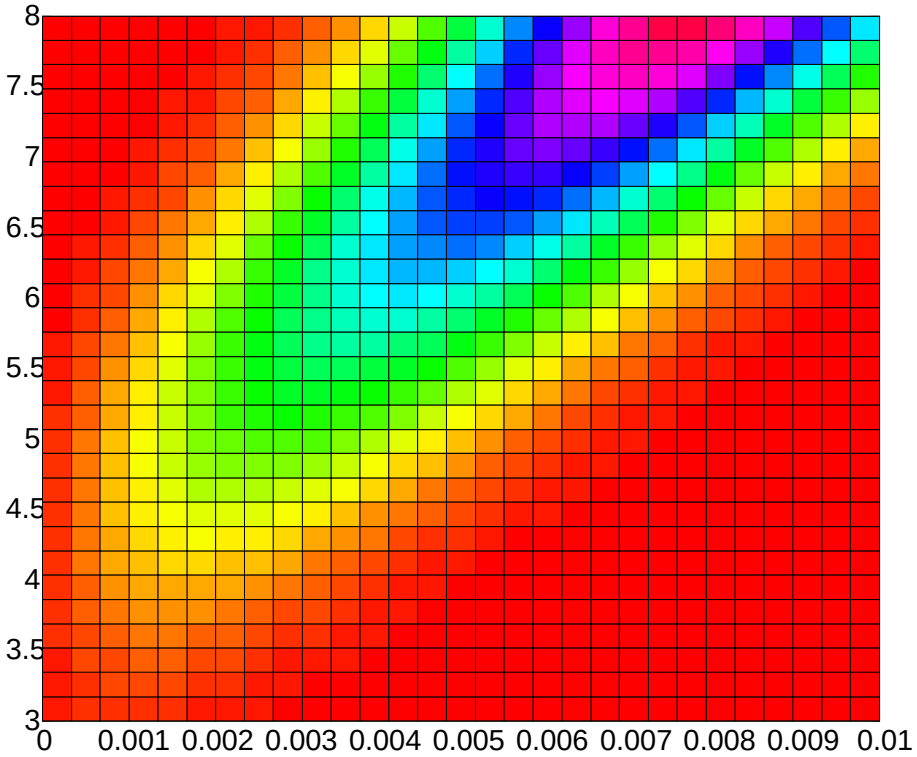
$$\phi_1(\alpha,\beta) \propto \prod_{i\in D}\alpha e^{-\frac{(\alpha q_i-\beta+\log p_i)^2}{2\sigma^2}}\prod_{j\in E}(1/p)e^{-\frac{(\alpha q_j-\beta+\log p_j)^2}{2\sigma^2}}\phi_0(\alpha,\beta)$$

Example dispatches

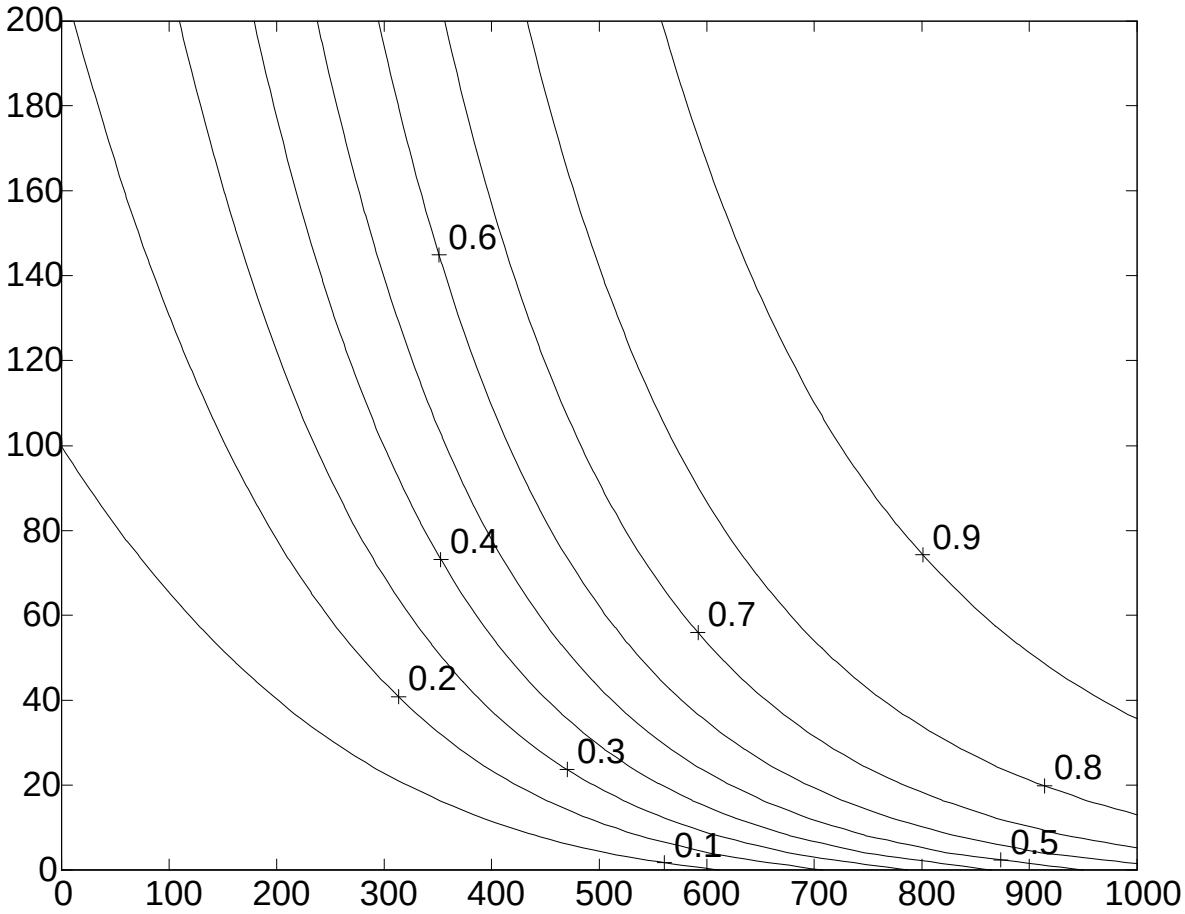


Electricity dispatches in February between 10am and 4pm weekdays

Posterior density of (α, β)



Contours of posterior ψ

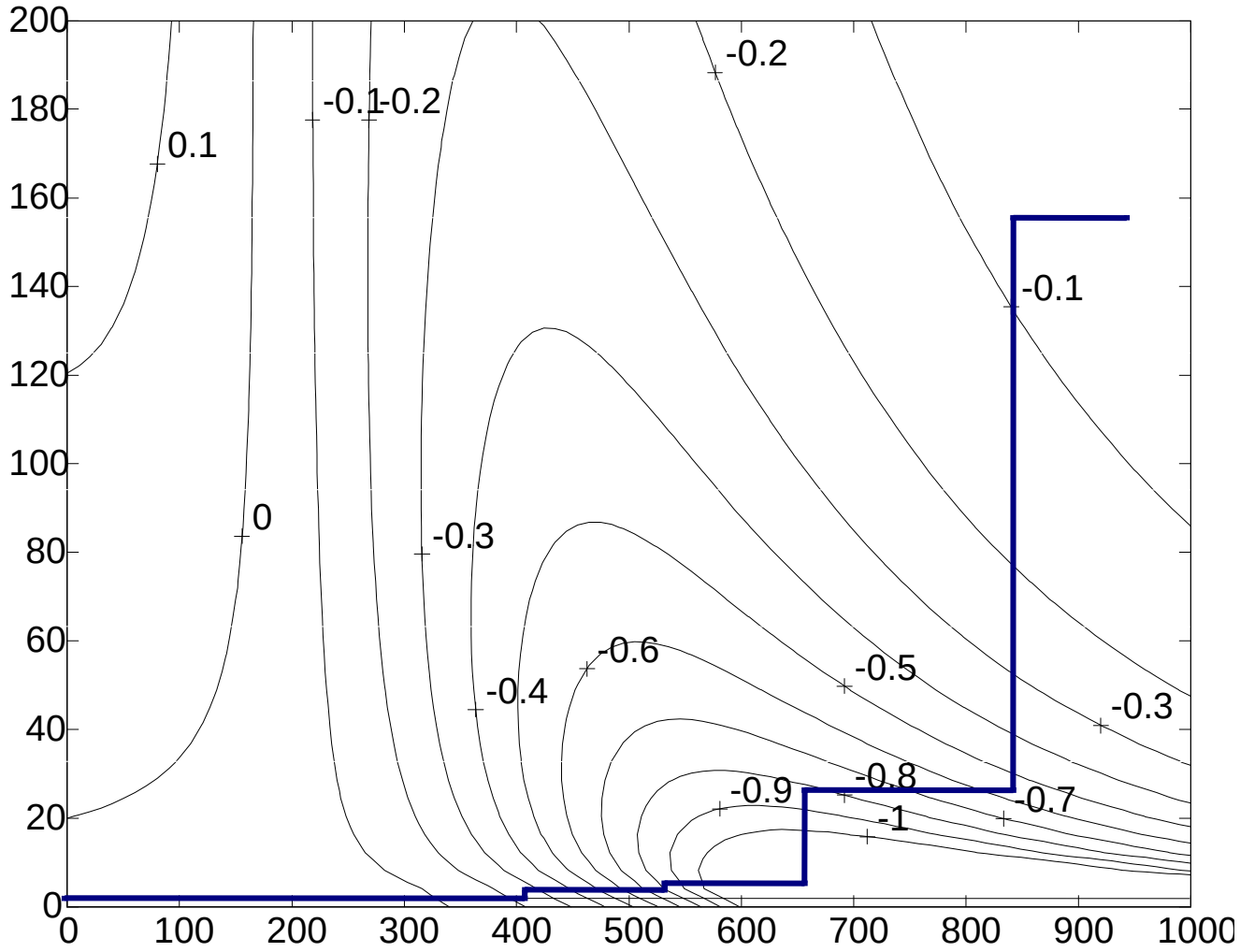


Behaviour of Optimal Supply Curves

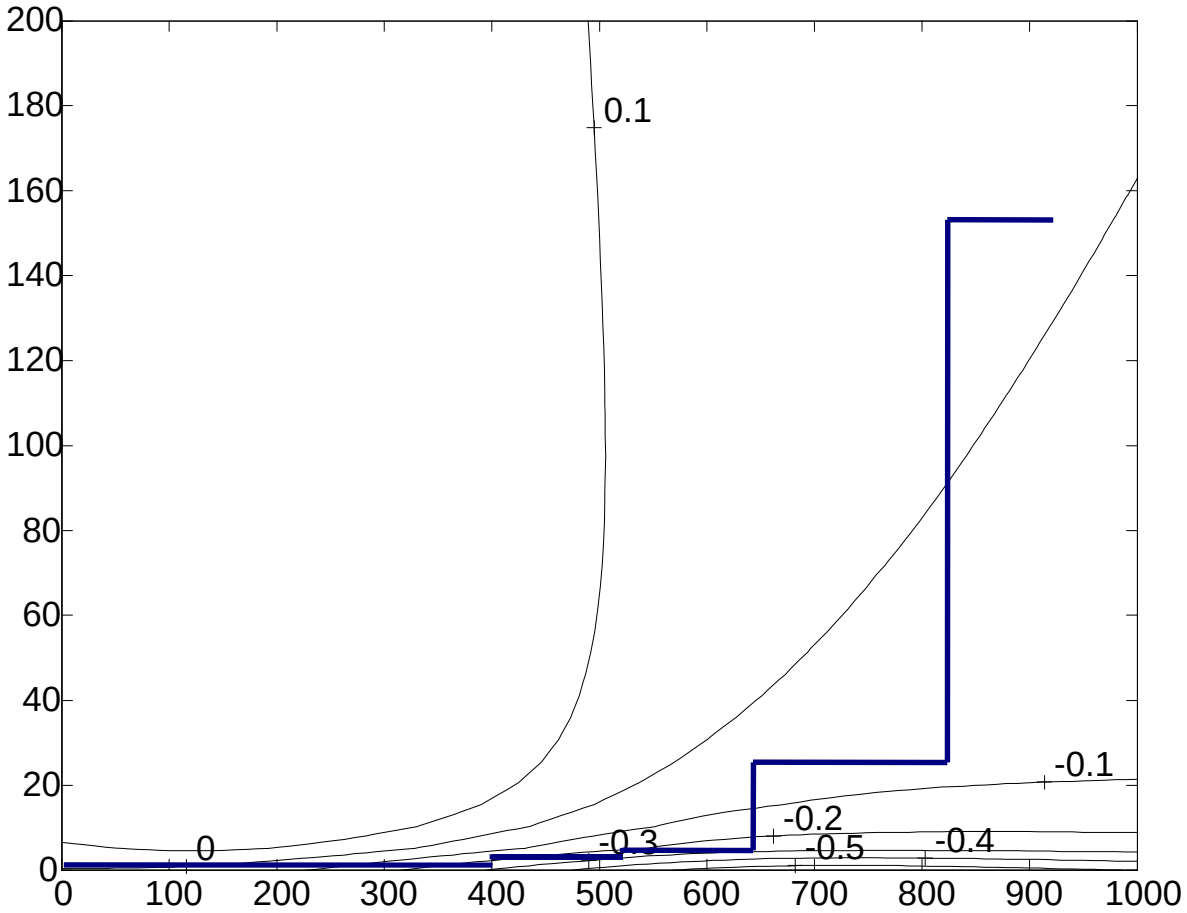
If $Z(q,p)$ is continuous then:

- an optimal curve can have $q'(t) > 0$ and $p'(t) > 0$ only where $Z(q,p) = 0$;
- the rest of the optimal curve is made up of horizontal and vertical sections;
- if $\mathcal{S}$ is any horizontal section then $\int_{\mathcal{S}} Z(q,p)q'(t)dt = 0$;
- if $\mathcal{S}$ is any vertical section then $\int_{\mathcal{S}} Z(q,p)p'(t)dt = 0$.

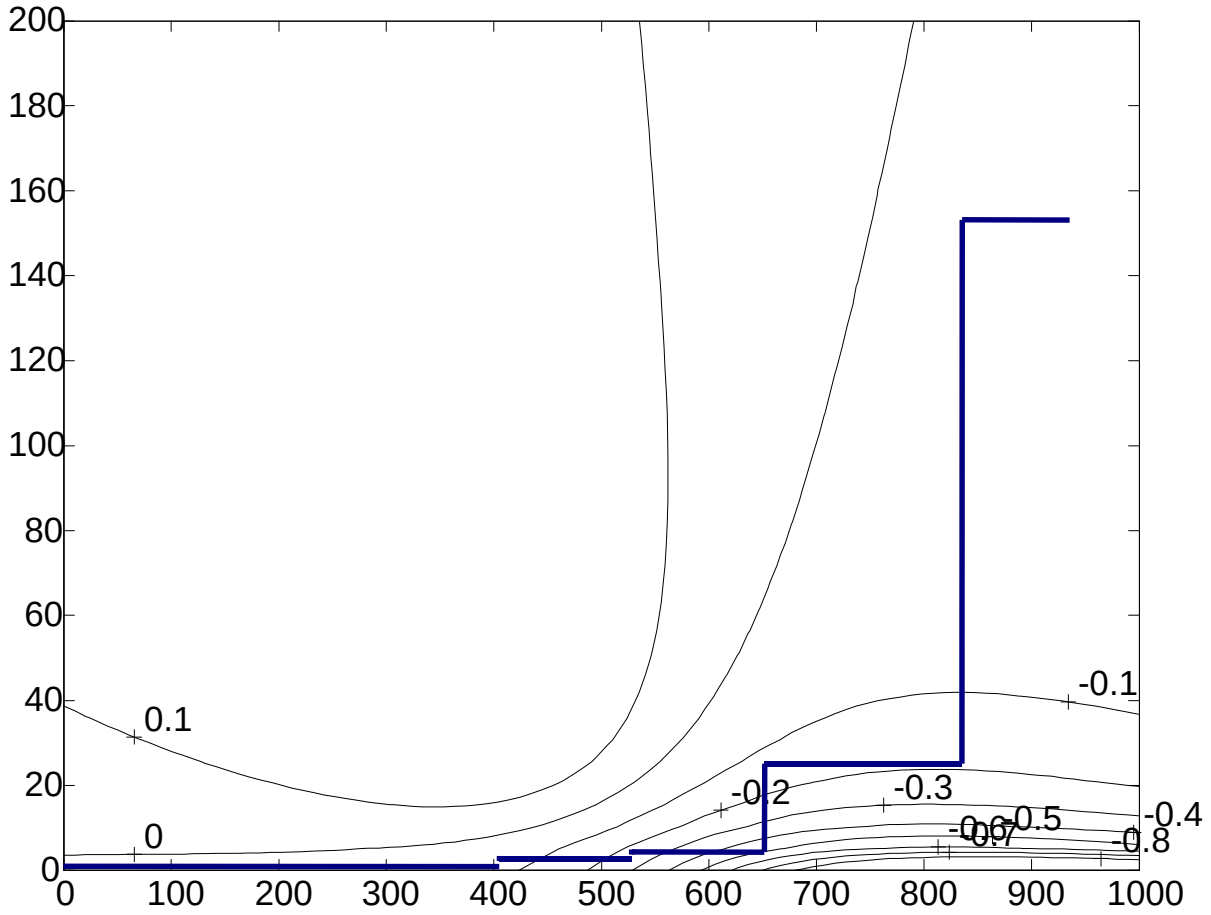
$Z(q,p)$ for posterior ψ ($C'(q)$=20, Q=0)



$Z(q,p)$ for prior ψ ($C'(q)$=20, Q=500)



$Z(q,p)$ for posterior ψ ($C'(q)$=20, Q=500)



Unit commitment in electricity pool markets

- Stochastic integer programming models using Lagrangean relaxation [Renaud (1995), Caroe & Schultz (1999), Takriti et al (2000) Growe-Kuska et al (2001)].

- In a pool market a unit submits an offer stack in each period:

  - its offer prices affect its dispatch;

  -seek a decision rule: dispatch $=$ increasing function of price.

- For a price-taking unit and Markov price process, optimal stacks can be computed using dynamic programming [Neame et al. (2001)]

Hydro-electric reservoir optimisation models

Many papers on centrally-planned systems [Pereira and Pinto (1985, 1991), Jacobs et al (1995)]. Stack optimisation in oligopolies poses some interesting challenges for stochastic programming.

	COMPETITION	OLIGOPOLY
Small reservoir	**Use DP assuming Markov price process (Pritchard & Zakeri)**	• **Use DP and time-varying ψ** • **Cournot model (Bushnell 1998)**
Large reservoir - short time horizon	**Offer at (constant) marginal water value**	**Offer at (constant) marginal water value and time-varying ψ**
Large reservoir - long time horizon	**Aggregation of time periods needed - dispatch is probabilistic**	**Cournot model (Scott & Read 1996) Challenge for SP**