POWER SCHEDULING IN A HYDRO-THERMAL SYSTEM UNDER UNCERTAINTY

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Abstract: A multi-stage stochastic programming model for power scheduling under uncertainty in a generation system comprising thermal and pumped-storage hydro units is developed. For its computational solution two different decomposition schemes are elaborated: Stochastic Lagrangian relaxation and scenario decomposition. Numerical results are reported for realistic data from a German power utility.

Keywords: Power scheduling, uncertain electrical load, stochastic Lagrangian decomposition, scenario decomposition.

1 INTRODUCTION

Uncertainty is an inherent feature of power scheduling. Among the main sources of uncertainty there are load profiles, generator outages, streamflows in water units, and prices or market situations in general. The latter will be of increasing importance due to the ongoing liberalization in the European power industry.

The present paper aims at treating power optimization and uncertainty in a unified framework. Based on the energy situation encountered at the German utility VEAG Vereinigte Energiewerke AG Berlin we develop a multi-stage stochastic programming model for power scheduling under uncertainty of load profiles. This provides challenges from the modelling viewpoint and, at the same time, raises fundamental mathematical questions on the design of solution methods. Inclusion of uncertainty leads to a tremendous increase in the complexity of the traditional power optimization models. The remedy we propose is decomposition which we will elaborate in two different ways. The first is based on stochastic Lagrangian relaxation of coupling constraints and leads to single-unit multi-stage stochastic programs of tractable size. The second aims at decomposing the full model according to scenarios and leads to single-scenario problems that are quite similar to traditional deterministic power scheduling models. For both decomposition approaches we will present the theoretical underpinnings and some initial computational experience.

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2 MODELLING

We consider a power generation system comprising (coal-fired and gas-burning) thermal units, pumped-storage hydro plants and delivery contracts and describe an optimization model for its least-cost operation. In our model, T denotes the number of time intervals of the optimization horizon and $\{\mathbf{d}^t : t = 1, ..., T\}$ the electrical load forming a stochastic process (on some probability space $(\Omega, \mathcal{A}, \mathcal{P})$). We assume that the load is known (at least) during the first time period(s). Denoting by $\mathcal{A}_t$ the $\sigma-$field generated by $(d^1, ..., d^t)$, we obtain a nested sequence (filtration) of $\sigma-$fields: $\mathcal{A}_1 = \{\emptyset, \Omega\} \subseteq \mathcal{A}_2 \subseteq \ldots \subseteq \mathcal{A}_t \subseteq \ldots \subseteq \mathcal{A}_T \subseteq \mathcal{A}$.

Let I denote the number of thermal and J the number of pumped storage hydro units. Delivery contracts are regarded as particular thermal units. According to the stochasticity of the electrical load, the decisions for all units are discrete-time stochastic processes as well:

$$\{\mathbf{x^t} = (\mathbf{p}^t, \mathbf{u}^t, \mathbf{s}^t, \mathbf{w}^t) : t = 1, ..., T\}.$$

Here, the decision variable $\mathbf{u}_i^t \in \{0,1\}$ indicates whether the thermal unit i is in operation at time t, $\mathbf{p}_i^t$ $(i = 1,...,I,\ t = 1,...,T)$ denotes the output of the thermal unit i at time t and $\mathbf{s}_j^t$, $\mathbf{w}_j^t$ $(j = 1,...,J,\ t = 1,...,T)$ are the generation and pumping levels, respectively, of the pumped-storage hydro plant j at time t. The following box constraints reflect output limitations of all units

$$\begin{array}{rcccl} \mathbf{u}_i^t p_i^{min} & \le & \mathbf{p}_i^t & \le & \mathbf{u}_i^t p_i^{max}, \\ 0 & \le & \mathbf{s}_j^t & \le & s_j^{max}, \\ 0 & \le & \mathbf{w}_j^t & \le & w_j^{max}, \\ 0 & \le & \boldsymbol{\ell}_j^t & \le & \ell^{\max}, \end{array} \quad t = 1,...,T, \quad (1)$$

where $p_i^{min}, p_i^{max}, s_j^{max}, w_j^{max}$ denote minimal and maximal outputs of the units and the maximal storage volumes in the upper reservoirs, respectively. The dynamics of the storage volume, which is measured in electrical energy, is modeled by the equations:

$$\begin{array}{rcll} \boldsymbol{\ell}_j^t & = & \boldsymbol{\ell}_j^{t-1} - \mathbf{s}_j^t + \eta_j \mathbf{w}_j^t\ , & t = 1,...,T, \\ \boldsymbol{\ell}_j^0 & = & \ell_j^{\text{in}}, \quad \boldsymbol{\ell}_j^T = \ell_j^{\text{end}}\ , & j = 1,...,J. \end{array} \quad (2)$$

Here, ℓ_j^{in} and ℓ_j^{end} denote the initial and final volumes in the upper reservoir, respectively, and η_j is the cycle (or pumping) efficiency of plant j. The cycle efficiency is defined as the quotient of the generation and of the pumping load that correspond to the same volume of water. The equalities (2) show, in particular, that there occur no in- or outflows with the upper reservoirs and, hence, that the pumped storage plants of the system operate with a constant amount of water. Constraints avoiding simultaneous generation and pumping in the hydro plants are dispensable since it can be shown that such a deficiency can not occur in optimal points.

Further single-unit constraints are minimum up- and down-times and possible must-on/off constraints for each thermal unit. Minimum up- and down-time constraints are imposed to prevent the thermal stress and high maintenance costs due to excessive unit cycling. Denoting by τ_i the minimum down-time of unit i, the corresponding constraints are described by the inequalities:

$$\begin{aligned} & \mathbf{u}_i^{t-1} - \mathbf{u}_i^t \le 1 - \mathbf{u}_i^\tau, \\ & \qquad \tau = t+1, ..., \min\{t + \tau_i - 1,\ T\}, \\ & \qquad t = 1,...,T. \end{aligned} \quad (3)$$

Analogous constraints can be formulated describing minimum-up times.

The subsequent load and reserve constraints couple different power units. The load constraints say that the sum of the output powers is greater than or equal to the load demand in each time period t:

$$\sum_{i=1}^{I} \mathbf{p}_i^t + \sum_{j=1}^{J} (\mathbf{s}_j^t - \mathbf{w}_j^t) \ge \mathbf{d}^t,\ t = 1,...,T. \quad (4)$$

In order to compensate unexpected events within a specified short time period, a spinning reserve, describing the total amount of generation available from all units synchronized on the system minus their present load, is prescribed. The corresponding constraints are given by the following inequalities:

$$\sum_{i=1}^{I} (p_i^{\max} \mathbf{u}_i^t - \mathbf{p}_i^t) \ge R^t,\ t = 1,...,T, \quad (5)$$

where $R^t > 0$ is a specified spinning reserve in period t.

Final constraints model the non-anticipativity of the stochastic decision process. They say that, at time t, decisions $\mathbf{x}^t = (\mathbf{p}^t, \mathbf{u}^t, \mathbf{s}^t, \mathbf{w}^t)$ must not depend on future realizations of the process $\{\mathbf{d}^t : t = 1,...,T\}$. In other words,

$$(\mathbf{p}^t, \mathbf{u}^t, \mathbf{s}^t, \mathbf{w}^t) \text{ is } \mathcal{A}_t\text{-measurable}, \quad t = 1,...,T. \quad (6)$$

The objective function is given by the expected total costs for operating the thermal units over the whole time horizon, i. e.,

$$\begin{aligned} F(\mathbf{x}) &= F(\mathbf{p}, \mathbf{u}, \mathbf{s}, \mathbf{w}) \\ &= I\!\!E \left\{ \sum_{i=1}^{I} \sum_{t=1}^{T} \Big[FC_i(\mathbf{p}_i^t, \mathbf{u}_i^t) + SC_i^t(\mathbf{u}_i) \Big] \right\}, \end{aligned} \quad (7)$$

where $I\!\!E$ denotes expectation, FC_i are the fuel cost functions and SC_i^t are the start-up costs for switching unit i online during period t. We assume that each FC_i is piecewise linear convex (in the first variable) and each SC_i^t is piecewise constant in time reflecting the dependence on the cooling time of the unit.

Altogether, the model (1) - (7) amounts to a mixed-integer multi-stage stochastic program which is loosely coupled with respect to operating units. The number of (stochastic) decision variables of the model, which computes as

$2(I+J)T$, is (very) large even for power systems of medium size. Figure 1 shows a typical weekly (deterministic) load curve and the corresponding cost-optimal hydro-thermal schedule.

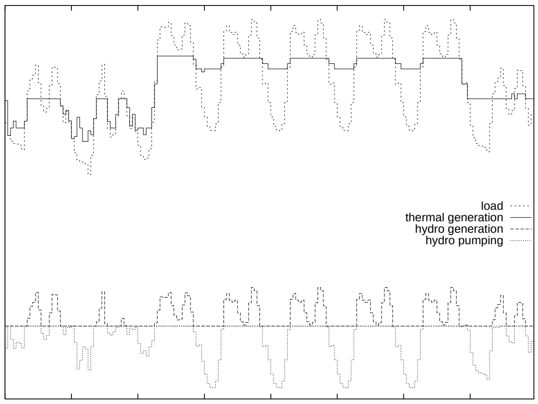

Figure 1: Load and hydro-thermal schedule

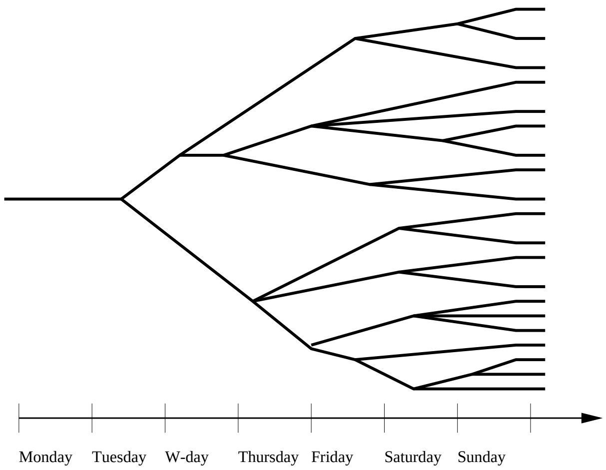

Figure 2: Example of a scenario tree

For the numerical solution of the model we now assume that the stochastic load process has finitely many realizations (or *scenarios*). It is well known (cf. [1, 2]) that the non-anticipativity constraint (6) can be expressed by linear equations for the scenarios $\mathbf{x}^\nu$, $\nu = 1, ..., r$, of the decision vector $\mathbf{x}$ (cf. Section 4) and that (6) leads to a tree structure of the load scenarios (see Figure 2). Since the decision vector $\mathbf{x}$ exhibits the same tree structure as the load, the model may easily become extremely large if the number of nodes in the scenario tree increases. Table 1 shows the dimensions of the model for a weekly time horizon ($T = 168$) and a medium size generation system ($I = 25$ and $J = 7$) and for scenario trees with equidistant binary branches.

Since the huge size of the model (1) - (7) prevents the application of state-of-the-art mixed-integer LP solvers, decomposition techniques may provide a practicable alternative. Earlier decomposition approaches to similar models are described in ([4, 13, 12]). In the following we present two types of (dual) decomposition schemes. The first one is based on dualizing constraints coupling across units (i.e. on a stochastic version of the *Lagrangian relaxation* technique) and the second on a dualization of the non-anticipativity constraints (i.e. on *scenario decomposition*).

3 STOCHASTIC LAGRANGIAN RELAXATION

The first decomposition approach to solving the multi-stage stochastic programming model in Section 2 makes use of the fact that the number of unit coupling (stochastic) constraints (4) and (5) is small compared with the dimension of the (stochastic) decision vector $\mathbf{x} = (\mathbf{p}, \mathbf{u}, \mathbf{s}, \mathbf{w})$. We associate stochastic Lagrange multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ with these constraints, and consider the Lagrangian L and the dual function D:

$$\begin{aligned} L(\mathbf{x}; \boldsymbol{\lambda}, \boldsymbol{\mu}) &= F(\mathbf{x}) \\ &+ \mathbb{E} \sum_{t=1}^{T} \{ \boldsymbol{\lambda}^t (\mathbf{d}^t - \sum_{i=1}^{I} \mathbf{p}_i^t - \sum_{j=1}^{J} (\mathbf{s}_j^t - \mathbf{w}_j^t)) \quad (8) \\ &+ \boldsymbol{\mu}^t (\mathbf{r}^t - \sum_{i=1}^{I} (\mathbf{u}_i^t p_i^{max} - \mathbf{p}_i^t)) \} \end{aligned}$$

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \min_{(\mathbf{p},\mathbf{u},\mathbf{s},\mathbf{w})} L(\mathbf{p}, \mathbf{u}, \mathbf{s}, \mathbf{w}; \boldsymbol{\lambda}, \boldsymbol{\mu}), \qquad (9)$$

where the minimization in (9) is subject to the remaining constraints ((1), (2), (3) and (6). Justified by general duality results for convex multi-stage stochastic programs (see [11], [6]), we consider the dual problem

$$\max_{(\boldsymbol{\lambda}, \boldsymbol{\mu})} D(\boldsymbol{\lambda}, \boldsymbol{\mu}) \qquad (10)$$

where the maximization is subject to the constraints that $\boldsymbol{\lambda}^t$ and $\boldsymbol{\mu}^t$ are $\mathcal{A}_t$-measurable and nonnegative, and have finite first moments, t=1,...,T. In particular, this means that both $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ exhibit the same tree structure as $\mathbf{d}$ and that the dimension of the dual problem (10) is

Scenarios	Nodes	Variables		Constraints	Nonzeros
		binary	continuous		
1	168	4200	6552	13441	19657
10	756	18900	29484	60490	88462
20	1176	29400	45864	94100	137612
50	2478	61950	96642	198290	289976
100	4200	105000	163800	336100	491500

Table 1: Dimension of the model depending on the number of scenarios

twice the number of nodes in the scenario tree. Moreover, the dual function decomposes into the following form

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{i=1}^{I} D_i(\boldsymbol{\lambda}, \boldsymbol{\mu}) + \sum_{j=1}^{J} \hat{D}_j(\boldsymbol{\lambda}) + I\!\!E \sum_{t=1}^{T} [\boldsymbol{\lambda}^t \mathbf{d}^t + \boldsymbol{\mu}^t R^t], \tag{11}$$

where $D_i(\boldsymbol{\lambda}, \boldsymbol{\mu})$ and $\hat{D}_j(\boldsymbol{\lambda})$ are the optimal values of stochastic single-unit thermal and pumped-storage hydro subproblems, respectively. The stochastic single-unit subproblems take the following form:

$$D_i(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \min_{\mathbf{u}_j} \sum_{t=1}^{T} \{ \min_{\mathbf{p}_j^t} [FC_i(\mathbf{p}_i^t, \mathbf{u}_i^t) - (\boldsymbol{\lambda}^t - \boldsymbol{\mu}^t)\mathbf{p}_i^t] - \boldsymbol{\mu}^t \mathbf{u}_i^t p_i^{max} + SC_i^t(\mathbf{u}_i) \} \tag{12}$$

$$\hat{D}_j(\boldsymbol{\lambda}) = \min_{(\mathbf{s}_j, \mathbf{w}_j)} I\!\!E \sum_{t=1}^{T} [-\boldsymbol{\lambda}^t(\mathbf{s}_j^t - \mathbf{w}_j^t)] \tag{13}$$

Here, the inner minimization in (12) with respect to the one-dimensional variable $\mathbf{p}_i^t$ satisfying upper and lower bounds (1) can be carried out explicitly by examining all kinks in the piecewise linear fuel cost function FC_i. Hence, (12) represents a *combinatorial multi-stage stochastic program* where the decision $\mathbf{u}$ satisfies the constraints (3). The subproblem (13) is a *linear multi-stage stochastic program* and the minimization is subject to the constraints (1) and (2).

The algorithm based on this stochastic Lagrangian relaxation technique consists of the following ingredients:

(a) Construction of a scenario tree for the stochastic load process,

(b) maximization of the nondifferentiable concave dual function D by proximal bundle methods using function and subgradient information ([7, 8]),

(c) efficient solvers for the stochastic single-unit subproblems: stochastic dynamic programming for (12) and a descent algorithm for (13),

(d) Lagrange heuristics for determining a feasible approximate solution for the optimal first-stage decision of (1)-(7).

For a detailed discussion of state-of-the-art algorithms, in particular proximal bundle methods, for nondifferentiable optimization problems in the context of Langrangian relaxation we refer to [6]. A description of the algorithms in (c) and their implementations can be found in [9, 10]. After having solved the dual problem (10), its optimal value $\max_{(\boldsymbol{\lambda},\boldsymbol{\mu})} D(\boldsymbol{\lambda}, \boldsymbol{\mu})$ provides a lower bound for the optimal costs of the model (1)-(7). In general, however, the corresponding decision vector $(\mathbf{p}^t, \mathbf{u}^t, \mathbf{s}^t, \mathbf{w}^t)$ violates the constraints (4) and (5). To find a feasible first-stage solution the Lagrange heuristics in (d) begins with taking the mean value function of the stochastic processes $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\mathbf{s}$ and $\mathbf{w}$. This is followed by a water rescheduling procedure in order to find improved hydro schedules and by a thermal Lagrangian heuristics which goes essentially back to [14]. After having the binary decisions fixed for the whole time horizon, a fast economic dispatch algorithm (see [10]) completes the procedure in (d).

Test results for a hydro-thermal power system with T=168, I=25 and J=7, and with randomly generated scenario trees having different numbers of scenarios and nodes are displayed in Table 2. Test runs were performed at an HP 9000 Workstation (770/J280). In addition to CPU-

Scenarios	Nodes	time[s]/gap[%]	Nodes	time[s]/gap[%]
10	781	31.2 / 0.274	1043	52.93 / 0.138
20	1982	89.13 / 0.149	1627	93.62 / 0.101
30	2643	139.71 / 0.528	2643	138.61 / 0.528
50	4530	475.29 / 0.175	4060	274.43 / 0.096
80	6548	537.28 / 0.137	6501	597.04 / 0.114
100	9230	1183.25 / 0.108	9224	1072.18 / 0.131

Table 2: Numerical results

times Table 2 shows the relative optimality estimates (gaps) obtained from the (deterministic) costs of the primal solution and the optimal dual (stochastic) costs.

4 SCENARIO DECOMPOSITION

In this section we view the multi-stage stochastic program from Section 2 as a large-scale mixed-integer linear program (MILP) consisting of single-scenario MILPs coupled by the non-anticipativity constraints. More specifically, the model can be written in the following compound form

$$\min\Big\{\sum_{\nu=1}^{r}\pi^\nu c\mathbf{x}^\nu : \mathbf{x}^\nu \in \mathcal{P}^\nu, \sum_{\nu=1}^{r} H^\nu \mathbf{x}^\nu = 0\Big\}. \quad (14)$$

Here $\mathbf{x}^\nu$ ($\nu = 1,\dots,r$) stands for the decision vector corresponding to the scenario ν. The sets $\mathcal{P}^\nu$ ($\nu = 1,\dots,r$) are the constraint sets corresponding to the individual scenarios. It is important to note that these are solution sets to systems of linear inequalities with integer requirements to certain variables, e.g., to the on/off decisions for the thermal units. The load scenarios enter the mentioned inequality systems as parts of the right-hand sides. By the linear equation $\sum_{\nu=1}^{r} H^\nu\mathbf{x}^\nu = 0$ with suitable matrices H^ν we model the non-anticipativity conditions. Since the load scenarios do not enter the objective function we have the same vector c for all scenarios. Finally, π^ν ($\nu = 1,\dots,r$) denote the probabilities for the individual scenarios.

It is easy to see that problem (14) were decomposable with respect to the scenarios if it wasn't for the coupling constraint $\sum_{\nu=1}^{r} H^\nu\mathbf{x}^\nu = 0$. This motivates to set up a Lagrangian relaxation of the mentioned constraint. To this end we formulate the Lagrangian function

$$\begin{aligned} L(\mathbf{x},\boldsymbol{\lambda}) &= \sum_{\nu=1}^{r}\pi^\nu c\mathbf{x}^\nu + \boldsymbol{\lambda}\sum_{\nu=1}^{r} H^\nu\mathbf{x}^\nu \\ &= \sum_{\nu=1}^{r}(\pi^\nu c\mathbf{x}^\nu + \boldsymbol{\lambda}H^\nu\mathbf{x}^\nu) \\ &= \sum_{\nu=1}^{r} L^\nu(\mathbf{x}^\nu,\boldsymbol{\lambda}) \end{aligned}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier vector from a Euclidean space of suitable dimension and $L^\nu(\mathbf{x}^\nu,\boldsymbol{\lambda}) = \pi^\nu c\mathbf{x}^\nu + \boldsymbol{\lambda}H^\nu\mathbf{x}^\nu$ for all ν.

The Lagrangian relaxation of (14) then reads

$$\begin{aligned} D(\boldsymbol{\lambda}) &= \min\{L(\mathbf{x},\boldsymbol{\lambda}) \,:\, \mathbf{x}^\nu\in\mathcal{P}^\nu,\ \nu=1,\dots,r\} \\ &= \sum_{\nu=1}^{r}\min\{L^\nu(\mathbf{x}^\nu,\boldsymbol{\lambda}) \,:\, \mathbf{x}^\nu\in\mathcal{P}^\nu\}, \end{aligned}$$

and the Lagrangian dual is the optimization problem $\max_{\boldsymbol{\lambda}} D(\boldsymbol{\lambda})$.

From duality in mixed-integer linear programming it is well known that the optimal value of the Lagrangian dual rather provides a lower bound for the optimal value of (14) than coincides with this number. However, the Lagrangian dual is a non-smooth concave maximization problem for which powerful algorithms including implementations are available (see [7, 8]). Moreover, computing $D(\boldsymbol{\lambda})$ benefits from the decomposition indicated in the above formula. Instead of solving full-sized MILPs the far smaller problems $\min\{L^\nu(\mathbf{x}^\nu,\boldsymbol{\lambda}) \,:\, \mathbf{x}^\nu\in\mathcal{P}^\nu\}$ have to be solved. The latter corresponding to the individual scenarios the method is termed scenario decomposition. Note further that the scenario subproblems are very similar to power scheduling problems for each individual load profile. The only difference is in the term $\boldsymbol{\lambda}H^\nu\mathbf{x}^\nu$ that enters the objective. Therefore, experience gathered with deterministic counterparts to our stochastic power scheduling model can be immediately exploited when solving the scenario subproblems.

After having solved the Lagrangian dual we obtain a lower bound to the optimal value of (14) and together with the optimal $\boldsymbol{\lambda}$ we have solutions $\mathbf{x}^\nu$ $(\nu = 1, \ldots, r)$ to the scenario subproblems. In the rare event where in addition it holds $\sum_{\nu=1}^{r} H^\nu \mathbf{x}^\nu = 0$ we know that $\mathbf{x}^\nu$ $(\nu = 1, \ldots, r)$ solve problem (14) as well. In general one has to expect that the non-anticipativity condition is violated by the solutions to the scenario subproblems. Then heuristics are employed using the scenario subproblem solutions as input and yielding feasible non-anticipative points whose objective function values provide upper bounds for the optimal value of (14), see [2] for a more detailed description.

The (relative) difference between the upper bound obtained by the above mentioned heuristics and the lower bound from the Lagrangian dual provides an optimality estimate for the output of the heuristics. This can be further improved by a branch-and-bound scheme on top of the Lagrangian dual whose details are described in [2]. Here the basic idea is to partition the feasible region of (14) and to apply the above scheme (lower bounding by the Lagrangian relaxation of non-anticipativity, upper bounding by heuristics starting from subproblen solutions) to each of the members of the partition. As in traditional LP-based branch-and-bound or global-optimization-related branch-and-bound we obtain tighter and tighter bounds together with feasible points that are closer and closer to the optimum. In theory, convergence to the optimum may be ensured. In practice however, there is a substantial tradeoff between the speed of convergence and the computing time. This tradeoff is very much depending on the concrete problem at hand and has to be explored in test runs. The final part of this section is devoted to a first step in that direction.

In [3] a power scheduling problem with uncertain load is modelled as a two-stage stochastic program. For lack of space we have to refer to [3] for model details. Instead Table 3 gives an impression on the sizes of two model types from [3]. The columns correspond to the numbers of scenarios, constraints, (integer and continuous) variables, integer variables, and the dimension of $\boldsymbol{\lambda}$, respectively.

For each of the model types, load scenarios were generated to cover uncertainty caused by generator failures and by forecast inaccuracy, see [3] for details. In the implementation of our scenario decomposition method we used CPLEX [5] for solving mixed-integer linear programs arising in the Lagrangian relaxation and NOA 3.0 [7, 8], an implementation by K.C. Kiwiel of his proximal bundle method, for solving the Lagrangian dual. Test runs were performed at a Digital Alpha Personal Workstation with 500 MHz processor. Table shows the relative optimality estimates (gaps) achieved after 10 minutes of CPU-time.

Our preliminary results indicate that both stochastic Lagrangian relaxation and scenario decomposition bear some potential in solving large-scale MILPs that arise in power scheduling under uncertainty.

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	Scen.	Constr.	Var.	Int.	Mult.
Type I	4	47159	47327	7560	11424
	10	113639	109775	14616	28560
	16	180119	172223	21672	45696
Type II	4	32049	37257	5880	4704
	10	78369	89625	12936	11760
	16	124689	141993	19992	18816

Table 3: Problem sizes

	Scen.	Gap	
		Generator Failure Instances	Inaccurate Load Forecast Instances
Type I	4	0.8%	0.5%
	10	0.8%	3.3%
	16	2.8%	2.7%
Type II	4	0.1%	0.1%
	10	0.2%	0.3%
	16	0.3%	0.4%

Table 4: Optimality estimates

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