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The $\mathrm{AdS}_5 \times \mathrm{S}^5$ mirror model as a string

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Doing a double Wick rotation in the worldsheet theory of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ light cone superstring results in an inequivalent, so-called mirror theory that plays a central role in the field of integrability in AdS/CFT. We show that this mirror theory can be interpreted as the light cone theory of a free string on a different background. Interestingly, this background is formally related to $\mathrm{dS}_5 \times \mathrm{H}^5$ by a double T-duality, and moreover has hidden supersymmetry. The geometry follows from simple considerations regarding generic bosonic light cone string actions, but can also be extracted from an integrable deformation of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ coset sigma model. As a byproduct we prove the previously observed mirror duality of these deformed models at the bosonic level. While we focus on $\mathrm{AdS}_5 \times \mathrm{S}^5$, our results apply more generally.

Integrability has and continues to be of central importance in furthering our understanding of the AdS/CFT correspondence [1]. Remarkable progress has been made in the spectral problem in particular [2], based on the integrability of the worldsheet theory of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring [3]. Under the assumption that integrability holds at the quantum level, the spectrum of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring can be determined by means of the thermodynamic Bethe ansatz applied to a doubly Wick rotated version of its worldsheet theory [4, 5], as advocated in [6] and worked out in [7–11]. As the light cone $\mathrm{AdS}_5 \times \mathrm{S}^5$ string is not Lorentz invariant however, this double Wick rotation results in an inequivalent quantum field theory, the so-called mirror theory [7]. This mirror theory also appears extensively in the exact description of polygonal Wilson loops or equivalently planar scattering amplitudes [12–14]. The recently developed quantum spectral curve [15, 16] builds on this mirror theory as well, being derived from the thermodynamic Bethe ansatz equations. Given the central importance of the mirror theory, we would like to elevate it beyond the status of a technical tool. This raises the question whether the mirror theory itself can arise directly by light cone gauge fixing a free string on some background. Here we show that this is the case. Our results have an interesting relation to the integrable deformation of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring of [17] and indicate new string backgrounds with hidden supersymmetry.

Our construction is based on a simple observation regarding the bosonic light cone action for a string on a fairly generic class of backgrounds that directly produces the desired 'mirror' metric. In short the doubly Wick-rotated light cone action is given by a formal exchange and inversion of the metric components of the two directions making up the light cone coordinates, combined with a sign flip on the B field. It is not obvious that the resulting metric is part of a string background, but we demonstrate explicitly that the mirror version of $\mathrm{AdS}_5 \times \mathrm{S}^5$ is a solution of type IIB supergravity with nontrivial dilaton and Ramond-Ramond five form. The double Wick rotation of the corresponding Green-Schwarz fermions is compatible with our procedure, as we have explicitly verified at the quadratic level and will report on elsewhere in detail [18]. Interestingly, the mirror space we obtain from $\mathrm{AdS}_5 \times \mathrm{S}^5$ is formally related to $\mathrm{dS}_5 \times \mathrm{H}^5$ by a double T-duality, H^5 being the five-dimensional hyperboloid. Our mirror space has a curvature singularity, but this is not necessarily problematic for the string sigma model. In fact, the mirror sigma model inherits the symmetries and in particular the integrability of the light cone $\mathrm{AdS}_5 \times \mathrm{S}^5$ sigma model, and should hence be well-behaved.

The mirror background has an $\mathfrak{so}(4)^{\oplus 2}$ symmetry that matches the bosonic $\mathfrak{su}(2)^{\oplus 4} \subset \mathfrak{psu}(2|2)^{\oplus 2} \oplus \mathbb{H}$ symmetry of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ light cone string, $\mathbb{H}$ being a central element corresponding to the worldsheet Hamiltonian. The supersymmetry of the mirror model is not realized through superisometries however, as the mirror background admits no Killing spinors. This is natural because the central element of the symmetry algebra of the mirror theory is nonlinearly related to its Hamiltonian ($\mathbb{C} \sim \sinh \frac{\tilde{\mathbb{H}}}{2}$) [7], so that the full superalgebra should be nonlinearly realized on the fermions of the mirror background.

While the mirror sigma model is integrable, this is not obvious from its geometry. Interestingly however, we can obtain (the bosonic part of) this sigma model as a limit of the integrable deformation of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ coset sigma model constructed in [17], in line with the mirror duality observed in [19]. Provided technical complications in extracting the fermions can be overcome, this relation would manifest classically integrability and κ symmetry of the mirror background.

DOUBLE WICK ROTATIONS OF LIGHT CONE GAUGE FIXED STRINGS

We want to understand whether the double Wick rotated light cone worldsheet theory of a string on a given background can be realized by light cone gauge fixing a string on another background. At the bosonic level this works quite elegantly. In this letter we will consider d-dimensional backgrounds with coordinates $\{t, \phi, x^\mu\}$ and metric

$$\mathrm{ds}^2 \equiv g_{MN}\, dx^M dx^N = -g_{tt}dt^2 + g_{\phi\phi}d\phi^2 + g_{\mu\nu}dx^\mu dx^\nu,$$

the metric components depending only on the transverse coordinates x^α, and B fields that are nonzero only in the transverse directions. The string action is given by

$$S = -\tfrac{T}{2}\int \mathrm{d}\tau\mathrm{d}\sigma\, \left(g_{MN}\, dx^M dx^N + B_{MN}\, dx^M \wedge dx^N\right),$$

where T is the string tension. To fix a light cone gauge we introduce light cone coordinates [20]

$$x^+ = t, \qquad x^- = \phi - t,$$

and then fix a uniform light cone gauge (in the first order formalism)

$$x^+ = \tau, \qquad p^+ = 1.$$

The bosonic action for a string on this background then takes the form

$$S = -\tfrac{T}{2}\int \mathrm{d}\tau\mathrm{d}\sigma\, \left(1 - \sqrt{Y} + B_{\mu\nu}\dot{x}^\mu x'^\nu\right),$$

where

$$Y = (\dot{x}_\mu x'^\mu)^2 - (\dot{x}_\mu \dot{x}^\mu - g_{tt})(x'_\nu x'^\nu + 1/g_{\phi\phi}),$$

and dots and primes refer to temporal and spatial derivatives on the worldsheet, having rescaled σ by T. We now observe that a double Wick rotation of the worldsheet coordinates

$$\tau \to i\tilde{\sigma}, \qquad \sigma \to -i\tilde{\tau}.$$

gives an action of the same form, with g_{tt} interchanged for $1/g_{\phi\phi}$ and B for $-B$. This means that we can obtain the double Wick rotated worldsheet theory also directly by gauge fixing a string on a background with a metric with g_{tt} and $1/g_{\phi\phi}$ interchanged and a B field with opposite sign. Note that we generically denote quantities in the double Wick rotated theory (the mirror theory) by tildes.

Taking this construction and feeding it the metric of $\mathrm{AdS}_5\times\mathrm{S}^5$ in global coordinates [21]

$$\mathrm{ds}^2 = -(1+\rho^2)dt^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega_3 + (1-r^2)d\phi^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_3,$$

we directly obtain

$$\mathrm{ds}^2 = \frac{1}{1-r^2}(-dt^2 + dr^2) + r^2 d\Omega_3 + \frac{1}{1+\rho^2}\left(d\phi^2 + d\rho^2\right) + \rho^2 d\Omega_3, \tag{1}$$

the metric that would result in the (bosonic) mirror theory. The transverse directions should not be affected by this transformation, but for the light cone directions this is more subtle and the ϕ direction need not be (taken) compact anymore. For the mirror version of $\mathrm{AdS}_5\times\mathrm{S}^5$ we will take it noncompact in fact, upon considering the relation of our mirror space to the spaces appearing in the deformed sigma models of [17].

STRINGS ON $(\mathbf{AdS_5\times S^5})_\eta$

The family of deformed sigma models of [17] can be labeled by a parameter $\varkappa \in [0,\infty)$, the undeformed $\mathrm{AdS}_5\times\mathrm{S}^5$ string sigma model sitting at $\varkappa = 0$. The corresponding metric is given by [22]

$$\mathrm{ds}^2 = -\frac{f_+(\rho)}{f_-(\varkappa\rho)}dt^2 + \frac{1}{f_+(\rho)f_-(\varkappa\rho)}d\rho^2 + \rho^2 d\Theta_3^\rho + \frac{f_-(r)}{f_+(\varkappa r)}d\phi^2 + \frac{1}{f_-(r)f_+(\varkappa r)}dr^2 + r^2 d\Theta_3^r,$$

where $f_\pm(x) = 1 \pm x^2$ and $d\Theta_3$ is a particular $\varkappa$-dependent deformation of the three-sphere metric in Hopf coordinates

$$d\Theta_3^\rho \equiv \frac{1}{1+\varkappa^2\rho^4\sin^2\zeta}(d\zeta^2 + \cos^2\zeta d\psi_1^2) + \sin^2\zeta d\psi_2^2,$$
$$d\Theta_3^r \equiv \frac{1}{1+\varkappa^2 r^4\sin^2\xi}(d\xi^2 + \cos^2\xi d\chi_1^2) + \sin^2\xi d\chi_2^2.$$

The B field is given by

$$B = \varkappa\Big(\frac{\rho^4\sin 2\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\psi_1\wedge d\zeta - \frac{r^4\sin 2\xi}{1+\varkappa^2 r^4\sin^2\xi}d\chi_1\wedge d\xi\Big),$$

vanishing at $\varkappa = 0$. The tension can be conveniently parametrized as

$$T = g\sqrt{1+\varkappa^2}.$$

The range of ρ is restricted to $[0, 1/\varkappa)$ to preserve the timelike nature of t, with a curvature singularity at $\rho = 1/\varkappa$. At $\varkappa = 0$ there is no singularity but rather the conformal boundary of anti-de Sitter space at $\rho = \infty$.

Now let us introduce rescaled coordinates

$$\tilde{t} = \varkappa t, \quad \tilde{\phi} = \varkappa\phi,$$
$$\tilde{r} = \varkappa\rho, \quad \tilde{\rho} = \varkappa r,$$

and relabeled coordinates

$$\tilde{\xi} = \zeta, \qquad \tilde{\zeta} = \xi,$$
$$\tilde{\chi}_i = \psi_i, \quad \tilde{\psi}_i = \chi_i.$$

If we then also introduce

$$\tilde{\varkappa} = 1/\varkappa,$$

the metric $\mathrm{ds}^2 = \tilde{\varkappa}^2 \tilde{\mathrm{ds}}^2$ becomes

$$\begin{aligned}\tilde{\mathrm{ds}}^2 = & -\frac{f_+(\tilde{\varkappa}\tilde{r})}{f_-(\tilde{r})} d\tilde{t}^2 + \frac{1}{f_+(\tilde{\varkappa}\tilde{r}) f_-(\tilde{r})} d\tilde{r}^2 + \tilde{r}^2 d\Theta_3^{\tilde{r}} \\ & + \frac{f_-(\tilde{\varkappa}\tilde{\rho})}{f_+(\tilde{\rho})} d\tilde{\phi}^2 + \frac{1}{f_-(\tilde{\varkappa}\tilde{\rho}) f_+(\tilde{\rho})} d\tilde{\rho}^2 + \tilde{\rho}^2 d\Theta_3^{\tilde{\rho}} \,,\end{aligned}$$

where now the $d\Theta_3$ factors contain tildes on $\varkappa$ and the angular variables. Up to the tildes and the factor of $\tilde{\varkappa}^2$ this is nothing but the deformed metric we started with, with g_{tt} and $1/g_{\phi\phi}$ interchanged. Similarly, the B field precisely picks up a sign in addition to tildes and a factor of $\tilde{\varkappa}^2$. This overall factor can now be naturally absorbed in the string tension.

Identifying quantities with and without tildes puts us squarely in the situation of the previous section, proving that the deformed bosonic light cone theory at tension $T(g)$ and deformation value $\varkappa$ is equal to the double Wick rotated theory at tension $T(\tilde{\varkappa} g)$ and deformation value $\tilde{\varkappa}$. This is precisely the statement of mirror duality observed in [19], which we have now proven at the bosonic level.

Just as the undeformed limit $\varkappa \to 0$ gives the metric of $\mathrm{AdS}_5 \times \mathrm{S}^5$, the maximally deformed limit $\tilde{\varkappa} \to 0$ gives precisely the mirror metric (1). From this perspective the mirror ϕ coordinate is naturally noncompact. Let us now discuss the mirror space in more detail.

THE $\mathrm{AdS}_5 \times \mathrm{S}^5$ MIRROR BACKGROUND

We obtained the mirror space (1) directly from $\mathrm{AdS}_5 \times \mathrm{S}^5$, but interestingly it is also closely related to $\mathrm{dS}_5 \times \mathrm{H}^5$. Namely, (by construction) our mirror space turns into $\mathrm{dS}_5 \times \mathrm{H}^5$ upon applying a timelike T-duality in t and a noncompact T-duality in ϕ. Indeed after a timelike T-duality the first line of eqn. (1) becomes

$$-(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_3 \,,$$

which is dS_5 in static coordinates. Similarly, T-duality in ϕ turns the second line into

$$(1+\rho^2)d\phi^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega_3 \,,$$

which is H^5 in analogous coordinates. The timelike respectively noncompact nature of these T-dualities makes this a rather formal relation however. So while our original string lived on the maximally symmetric space $\mathrm{AdS}_5 \times \mathrm{S}^5$, its mirrored version does not quite live on the maximally symmetric space $\mathrm{dS}_5 \times \mathrm{H}^5$, but rather its doubly T-dual cousin (1).

At this stage we should mention that based on simpler models [23] it was conjectured that the maximal deformation limit of the models of the previous section should correspond to $\mathrm{dS}_5 \times \mathrm{H}^5$ [17]. Indeed, this space was already extracted in a different $\varkappa \to \infty$ limit combined with two spacelike T-dualities [24]. This limit requires taking one of the coordinates outside its natural range however, thereby changing the coordinate that is timelike, and as such does not appear to be smoothly connected to the general deformed geometry. Like $\mathrm{dS}_5 \times \mathrm{H}^5$, the resulting geometry is formally supported by an imaginary five-form flux. Our limit instead yields a geometry that is smoothly related to the generic case, and naturally results in a real worldsheet theory. We consider our doubly T-dual relation to $\mathrm{dS}_5 \times \mathrm{H}^5$, albeit part timelike and noncompact, ‘close enough’ to the conjecture of [17]. Our mirror space is clearly related to the one obtained in [24] by one time and three spacelike T-dualities.

Our mirror space is a product of two five-dimensional spaces, with curvature

$$R = \left(4\frac{1-2r^2}{1-r^2}\right)\left(-4\frac{1+2\rho^2}{1+\rho^2}\right) \,,$$

showing a (naked) singularity at $r=1$. This means that in strong contrast to $\mathrm{AdS}_5 \times \mathrm{S}^5$, the mirror space is singular. Sigma models on singular backgrounds are not necessarily ill-defined however, and the integrability of our string sigma model is actually a promising indication of good behaviour.

A more pressing question is that of conformality of the sigma model at the quantum level. At one loop this is equivalent to the statement that our metric is part of a solution of supergravity, which we will now demonstrate. As our metric is T-dual to the metric of $\mathrm{dS}_5 \times \mathrm{H}^5$, it is natural to assume that the (type IIB) supergravity solution is supported by only a dilaton Φ and a self-dual five-form flux F, just as $\mathrm{dS}_5 \times \mathrm{H}^5$ formally is [25]. We then have to solve the equations of motion for the dilaton

$$4\nabla^2\Phi - 4(\nabla\Phi)^2 = R \,,$$

the metric

$$R_{\mu\nu} = -2\nabla_\mu\nabla_\nu\Phi + \frac{1}{4\cdot 4!} e^{2\Phi} F_{\mu\rho\lambda\sigma\delta} F_\nu^{\rho\lambda\sigma\delta} \,,$$

and the five-form

$$\partial_\nu\left(\sqrt{-g} F^{\rho\lambda\sigma\delta\nu}\right) = 0 \,.$$

The equation for the dilaton is solved by

$$\Phi = \Phi_0 - \frac{1}{2}\log(1-r^2)(1+\rho^2) \,,$$

where Φ_0 is a constant. The five-form is then

$$F = 2e^{-\Phi_0}\Big(r^3 \sin 2\zeta \, \mathrm{d}\phi \wedge \mathrm{d}r \wedge \mathrm{d}\zeta \wedge \mathrm{d}\psi^1 \wedge \mathrm{d}\psi^2 - \rho^3 \sin 2\xi \, \mathrm{d}t \wedge \mathrm{d}\rho \wedge \mathrm{d}\xi \wedge \mathrm{d}\phi^1 \wedge \mathrm{d}\phi^2\Big)\,,$$

where ζ, ψ^1, ψ^2 and ξ, χ^1, χ^2 are the angular coordinates of the two three-spheres with line elements entering eqn. (1) with factors r^2 and ρ^2 respectively. Although the metric (1) corresponds to a direct product of two manifolds, the five-form has mixed components, four of them being coordinates on one of the manifolds and the fifth on the other. This corresponds to the exchange of the differentials $\mathrm{d}t \leftrightarrow \mathrm{d}\phi$ under double T-duality applied to the imaginary five-form that supports $\mathrm{dS}_5 \times \mathrm{H}^5$. If we introduce would-be volume forms ω_i for these two sets of five coordinates, we can represent the five-form as

$$F = 4e^{-\Phi}\left(\omega_1 - \omega_2\right).$$

Since the dilaton and the five-form are real, the string sigma-model defined on this background defines a unitary theory. While the unboundedness of the dilaton raises questions regarding the mirror background in interacting string theory, this is no immediate problem for the sigma model of a free string.

This background has no conventional supersymmetry as it does not admit Killing spinors. This follows from the variation of the dilatino λ

$$\delta_\epsilon \lambda = \partial_\mu \Phi \, \Gamma^\mu \epsilon\,,$$

which does not vanish for any nonzero chiral spinor ϵ with our dilaton. Nevertheless, the string sigma model on our mirror background has a hidden form of supersymmetry, inherited from the $\mathrm{AdS}_5 \times \mathrm{S}^5$ string.

(SUPER)SYMMETRY OF THE MIRROR MODEL

Since double Wick rotations preserve symmetries, our model must have the manifest $\mathfrak{psu}(2|2)^{\oplus 2}$ symmetry of the light cone $\mathrm{AdS}_5 \times \mathrm{S}^5$ string. This symmetry need not be linearly realized however, and in fact the action of the supercharges cannot be. We can understand this from the form of the relevant superalgebras.

The on shell symmetry algebra of the light cone $\mathrm{AdS}_5 \times \mathrm{S}^5$ string is $\mathfrak{psu}(2|2)^{\oplus 2} \oplus \mathbb{H}$, where $\mathbb{H}$ is a central element corresponding to the worldsheet Hamiltonian. Considering one of the two copies of $\mathfrak{psu}(2|2)$, the supercharges Q and $Q^\dagger$ satisfy

$$\{Q_\alpha^{\ a}, Q^{\dagger\beta}_b\} = \delta^a_b R_\alpha^{\ \beta} + \delta^\beta_\alpha L_b^{\ a} + \frac{1}{2}\delta^a_b \delta^\beta_\alpha \mathbb{H}\,,$$

where L and R generate the two bosonic $\mathfrak{su}(2)$'s. We see that as usual the generators of the superisometries of the light cone $\mathrm{AdS}_5 \times \mathrm{S}^5$ string anticommute to the generators of isometries, in particular time translations [26]. For the mirror theory we instead have [7]

$$\{\tilde{Q}_\alpha^{\ a}, \tilde{Q}^{\dagger\beta}_b\} = \delta^a_b R_\alpha^{\ \beta} + \delta^\beta_\alpha L_b^{\ a} + T\delta^a_b \delta^\beta_\alpha \sinh \frac{\tilde{\mathbb{H}}}{2}$$

where the rest of the anticommutators vanish when we interpret the mirror theory as an on shell string with zero worldsheet momentum. Now were these mirror supercharges to correspond to linearly realized superisometries of a string background, they ought to commute to the associated Hamiltonian, not its hyperbolic sine. Put differently, identifying the mirror Hamiltonian as the generator of time translations, we are simply no longer dealing with a Lie superalgebra. Moreover, in the deformed sigma models the symmetry algebra of the worldsheet theory is expected to be a quantum deformed version of $\mathfrak{psu}(2|2)^{\oplus 2}$, as supported by the S-matrix computations of [22]. This symmetry does not appear to be realized in a geometric fashion on the deformed background however, and there is no reason to assume that it should be linearly realized on the worldsheet fermions even in the maximally deformed limit where the algebra is no longer deformed. Note that the bosonic part of the symmetry is nonetheless realized linearly, cf. the two three-spheres in eqn. (1).

We should also briefly address the off shell symmetry algebra of the mirror theory. It is well known that the worldsheet symmetry algebra of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ string picks up a central extension $\mathbb{C} \sim \sin \frac{\mathbb{P}}{2}$ when going off shell [27], allowing the exact S-matrix to be determined. From the point of view of a string-based mirror theory, we find the following scenario plausible. We start with a manifest worldsheet symmetry of $\mathfrak{psu}(2|2)^{\oplus 2} \oplus \sinh \frac{\tilde{\mathbb{H}}}{2}$, where we can match $\sinh \frac{\tilde{\mathbb{H}}}{2}$ with $\mathbb{C}$ through analytic continuation. Going off shell by relaxing the level matching condition, the algebra should pick up a central extension matching the analytic continuation of $\mathbb{H}$. In this way the off shell symmetry algebras of both theories, and hence their exact S-matrices, would be related by the expected analytic continuation.

OUTLOOK

We have given the supergravity background in which a free string has a light cone worldsheet theory identical to the $\mathrm{AdS}_5 \times \mathrm{S}^5$ mirror theory. With this direct interpretation of the mirror model we can ask new types of questions, the main one being whether we can give meaning to our geometric mirror transformation in the context of AdS/CFT. The fact that a double Wick rotation on the worldsheet has an interpretation in terms of free string theory leads us to wonder whether a 'similar analytic continuation' can be implemented in planar $\mathcal{N} = 4$ SYM. If possible, the end result should have an interesting relation to our string. It might be fruitful to approach this through the deformed sigma models which continuously connect $AdS_5 \times S^5$ and the mirror background, where it may be possible to implement the deformation (perturbatively) in planar $\mathcal{N} = 4$ SYM. The unbounded dilaton in our mirror background does not bode well for attempts at extending this beyond the sigma model of a free string (planar gauge theory), but then a double Wick rotation loses its simple physical interpretation on a higher genus Riemann surface to begin with.

Our considerations clearly apply to many spaces other than $AdS_5 \times S^5$, though they are of course mainly of interest in cases where a mirror model comes into play [28]. Our procedure for example immediately applies to $AdS_2 \times S^2 \times T^6$, and $AdS_3 \times S^3 \times M^4$ supported by pure Ramond-Ramond fluxes. More interesting in this light are $AdS_4 \times \mathbb{CP}^3$, and $AdS_3 \times S^3 \times M^4$ supported by mixed fluxes [29], since the (light cone) metric of $AdS_4 \times \mathbb{CP}^3$ (see e.g. [30]) and the B field of $AdS_3 \times S^3 \times M^4$ do not fit the simplifying assumptions made in this letter. We will relax our assumptions on the metric and B field when explicitly adding fermions [18].

Returning to the mirror version of $AdS_5 \times S^5$, it would be interesting to investigate possible consequences of the singular nature of our background, starting for example with the analysis of classical string motion in our mirror space. Also, extracting the explicit fermionic couplings in the deformed coset sigma models is a complicated but relevant question which may simplify in the maximal deformation limit. Finally, while we now see that the deformed geometry interpolates between two solutions of supergravity, demonstrating that it is one at any $\varkappa$ remains an interesting open problem.

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