SINGULARITY THEORY, HOMEWORK SHEET NO. 1

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Problem 1

What is your favourite singularity? Think about examples of singularities that you have encountered before.

Problem 2

Find a smooth parametrisation $t \mapsto (x(t), y(t))$ of a curve in the plane with a double point that has no critical points.
Can you find a way to fit the "double point singularity" into our framework, i.e. is there a map $f : M \to N$ that has a critical point at a double point? Use *Wolfram Alpha* (or your favourite software) to experiment by plotting some small perturbations of f.

Problem 3

Calculate the rank of the differential of $p : S^2 \to \mathbb{R}^2$, the vertical projection from the sphere to the plane, to find the set of critical points of p.

Problem 4

Proof the following two preparation theorems.

Let $f : M \to N$ with $m := \dim M$, $n := \dim N$ and let $x \in M$ be a regular point of f. If

- *$m > n$, then there are local coordinates $(x_1, \ldots, x_m)$ around x and $(y_1, \ldots, y_n)$ around $f(x)$, such that f takes the form $f(x_1, \ldots, x_m) = (x_1, \ldots, x_n)$.*
- *$m < n$, then there are local coordinates $(x_1, \ldots, x_m)$ around x and $(y_1, \ldots, y_n)$ around $f(x)$, such that f takes the form $f(x_1, \ldots, x_m) = (x_1, \ldots, x_m, 0, \ldots, 0)$.*

What about the case $m = n$?