SINGULARITY THEORY, HOMEWORK SHEET NO. 5

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Problem 1

Let $f : M \to N$ be smooth map and $m = n$. Find the smallest values for m such that f can have a singularity of type $\Sigma^{2,1}$ or $\Sigma^{2,2}$.

Problem 2

Compute the intrinsic derivative for a map $f : \mathbb{R}^m \to \mathbb{R}^n$ (i.e., compute D of $df : T\mathbb{R}^m \to f^*T\mathbb{R}^n$).

Hint: W.l.o.g. assume that

$$df(x) = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} \in M_{n,m}(\mathbb{R}).$$

For more hints, see page 151 in the book by Guillemin and Golubitsky.

Problem 3

Show that the intrinsic derivative $D(df)(x)$ is determined by the 2-jet of f at x.

Problem 4

Consider the two maps $f_\pm : \mathbb{R}^4 \to \mathbb{R}^4$,

$$f_+(x_1, x_2, x_3, x_4) := (x_1, x_2, x_3^2 + x_4^2 + x_1x_3 + x_2x_4, x_3x_4)$$

and

$$f_-(x_1, x_2, x_3, x_4) := (x_1, x_2, x_3^2 - x_4^2 + x_1x_3 + x_2x_4, x_3x_4).$$

Show that

(1) both maps have a $\Sigma^{2,0}$-singularity at the origin.
(2) both maps are one-generic.
(3) they are not equivalent.

Are there any singularities of type $\Sigma^{i,,j}$ for $j > 0$?