SINGULAR INTEGRALS. EXERCISE SHEET NO. 5

1

For $f(t) = \int_\sigma \frac{dz}{z^2-t}$ where σ is a circle of radius $\frac{1}{2}$ around $z = 1$ find the associated fiber bundle of pairs $\pi : (X \times T, S) \to T$ (cf. Proposition 6.2) and determine its Landau variety L.

2

Find a fiber bundle that is not locally trivial.

3

Construct an example of a stratification by stratifying your favourite space.

4

Discuss the analytic continuation of the function $f(t) = \int_\sigma \frac{dz}{z^2-t^3}$ where σ encircles $z = 1$ by studying the associated fiber bundle of pairs $\pi : (X \times T, S) \to T$. First, show that the singular set S is not a submanifold of $X \times T$, then find a stratification of S and the Landau variety of π viewed as a stratified fiber bundle.

5

For $1 \neq a \in \mathbb{C}$ let
$$f(t) = \int_{\sigma_t} \frac{dz}{(z^2 - t)(z - a)}$$
where σ_t is a closed contour encircling $z = \sqrt{t}$ and $z = a$. Discuss the analytic structure of f by two methods: First, by analytic continuation, second, by computing the integral. Compare the two results.