On the scale singularity set of Lorentzian Almost Einstein manifolds

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An almost Einstein (AE) manifold $(\bar{M}, \bar{g}, \sigma)$ is a natural generalisation of an Einstein manifold, i.e. a manifold (M, g) for which $\mathrm{Ric} - \frac{\tau}{n} g = 0$, τ being the scalar curvature. It can be seen as a conformal compactification of such an Einstein manifold with $(M \simeq \bar{M} \backslash \Sigma, g = \sigma^{-2}\bar{g})$ and boundary $\Sigma = \sigma^{-1}(0)$. Since for a given AE-manifold the rescaled data $(\varphi^2 \bar{g}, \varphi\sigma)$ are also AE, the concept of AE manifolds is conformally invariant. By denoting the induced metric on Σ by $[\gamma] = [\bar{g}|_{\Sigma}]$, the following question arises. Given $(\Sigma, [\gamma])$, does there exist an AE-manifold $(\bar{M}, \bar{g}, \sigma)$ with $(\Sigma, [\gamma])$ as conformal boundary?

The focus of my talk will be on Lorentzian AE-manifolds which are conformally Ricci flat. It turnes out that for such an AE structure the boundary Σ locally must be a geodesic lightcone or a lightlike surface and the gradient of σ has certain properties which will be presented. In addition, special conditions for $\bar{g}$ near the conformal boundary will be derived. The resulting Cauchy problem is that on a characteristic cone.