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Function Space Optimization – WS 2007/08
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1. Let $\phi : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be twice differentiable. Determine the generalized Euler-Lagrange equation for the variational problem with the functional $F : C_0^2([0,1] \to \mathbb{R}^n,$

$$\min \; F(x) = \int_0^1 \phi(x(s), \dot{x}(s), \ddot{x}(s))\, ds$$

2. Find the Euler-Lagrange equation for the variational problem

$$\min \; I(y,z) = \int_0^1 \left[y''(x)\, z(x) + x\, y(x)\, z''(x) + z'''(x) y(x)^2 \right] \, dx$$

3. Minimize the integral $I(x) = \int_0^4 x''(x)^2\, dx$ over the set

$$S = \left\{ x \in C^2([0,4] \to \mathbb{R}) : \; x(1) = -1,\, x(2) = 2,\, x(3) = 1 \right\}.$$

4. Find the extremal values over $C^1([a,b] \to \mathbb{R})$, $0 < a < b$ fixed, for the integrals

$$\text{a)} \int_a^b \frac{y'(x)^2}{x^3}\, dx \quad \text{and b)} \quad \int_a^b \left(y'(x)^2 + y(x)^2 + 2y(x)\, e^x \right) \, dx\, .$$

5. Find the extramal path for the integral

$$I(y) = \int_0^1 \frac{\sqrt{1 + y'(x)^2}}{x}\, dx \text{over the set} S = \left\{ y \in C^1([0,1] \to \mathbb{R}) : \; y(0) = 0,\, y(1) = 1 \right\}$$

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